Morgan

THE

# PRINCIPLES AND DOCTRINE

OF

# ASSURANCES,

# ANNUITIES ON LIVES,

AND

# CONTINGENT REVERSIONS,

*STATED AND EXPLAINED.*


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ACTUARY TO THE SOCIETY FOR EQUITABLE ASSURANCES ON LIVES AND SURVIVORSHIPS.



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# PREFACE.

THE first edition of this work having been published above forty years ago has long been out of print, and my leisure hours for several years have been employed in preparing the present edition; in which not only much additional matter is contained, but the solutions of all the different problems are derived from the real probabilities of life rather than from an hypothesis, which in many cases is found to be so defective as not even to approximate to the truth.

Before the year 1788, when I communicated my first paper on the subject to the *Royal Society*, no attempt, to my knowledge, had been made to solve any problem, involving more than one life in the computation, from the real probabilities of life; but the solutions were derived from the hypothesis of an equal decrement, partly from the want of accurate tables of observation, and partly on account of the facility which it afforded of investigating the most complicated problems in the doctrine of life annuities. In the case of a single life, provided the age neither falls short of twenty nor ex-

ceeds sixty years, the errors produced by this hypothesis are inconsiderable; but at earlier and later periods, and when more than one life is concerned, these errors in many instances render the rules altogether unfit for use.

In the former edition of this work, the rules for determining the values of contingent reversions depending on three lives were for the most part derived from two or three problems in Mr. *Simpson's* Select Exercises; but the solutions of those problems having M. *De Moivre's* hypothesis for their basis were incorrect, and in consequence the above-mentioned rules, so far as they were connected with them, were equally so. It should, however, be observed that having been deduced from plain reasoning and depending on no particular hypothesis, the rules would have been perfectly correct if the solutions of Mr. *Simpson's* problems had been derived from the real probabilities of life. These rules were, indeed, rendered in some degree less inaccurate by adopting the *expectations* of life as deduced from real observations instead of assuming them, agreeable to the hypothesis, to be always equal to half the difference between 86 and the age of the given life. Still, however, they were founded on the supposition that the decrements of life were equal; so that the series expressing the several contingencies were made to decrease in arithmetic progression, while the series expressing the expectation decreased in

a different proportion, and thus made it impossible that a rule founded on such discordant principles should be strictly true.

The values of reversions depending on two lives are as readily determined from real observations as from *De Moivre's* hypothesis, and the rules are as simple: but in regard to those reversions which depend on three lives, it must be acknowledged that the operations are much more complicated and laborious. In some cases the values depend on ten or twelve contingencies, and consequently the series expressing those values are more than twice as numerous. To reduce so many quantities, therefore, into any form that shall not render the computation of them a work of great labour, becomes a matter of considerable difficulty. It is possible that the rules which have been given in this work might have been a little simplified by omitting or altering some of the expressions which appear to be of inconsiderable value; but the results would then have been only approximations to the truth, and the subject would have been left in the same uncertainty as if the solutions had been derived from the hypothesis which had been found to be so defective.

My chief purpose in the present work has been to give solutions strictly correct according to any table of observations deduced from the real probabilities of life; and this purpose would have been defeated, if I had omitted any series, how-

ever insignificant, with the view of rendering the process more simple. I know not how far I have succeeded in rendering the rules intelligible to the general class of readers, by giving them in words at length; but the mathematical reader will find no difficulty by referring to the notes at the end of the work; where those rules with their demonstrations are given algebraically. By a little management in arranging and conducting the operations, they will be found, however, to be less formidable than they appear; and though they may be attended with more trouble, than by the method of approximation, they should not be objected to on that account, since the additional labour they produce will be abundantly compensated by the greater accuracy of the computation.

In the first and second chapters the nature of assurances and annuities on lives is stated and explained, together with the principles on which the computations of their values are founded. Being intended as an introduction to the subject, these chapters contain nothing that is either difficult or abstruse, but are written in a plain and familiar manner with the view of preparing the uninformed reader for understanding the solutions of the subsequent problems. Having deduced all those solutions from the real probabilities of life, the examples by which the reasoning in the first and second chapters are illustrated, have been derived from the same source, rather than from the hy-

pothesis of M. *De Moivre*; and with this exception, these two chapters are the same with those in the former edition. It should, however, be observed, that the second section of the first chapter, as well as the introduction written by my invaluable friend and relative Dr. *Price* to that edition, are now entirely omitted, having no particular connection with the subjects of the present work, and containing observations no longer applicable to the circumstances of the Society to which they were addressed.

In the year 1779, when those observations were written, the business of assurances on lives was but little understood and but little practised. Excepting the Society in Serjeants Inn, which assured lives at all ages under 45 at the same annual premium, and never exceeded £300 on the same life, and the *Royal Exchange* Office, which made a few assurances for a single year at the general premium, I believe, of £5 *per cent.*; the *Equitable* Society had no competitors, and were the only Society which varied their premiums according to the age of the person assured. The assurances for the benefit of surviving families at this period were but few in comparison with those which were made on the lives of those improvident persons who, in the disposal of their property, seemed to have as little consideration for their families as for themselves; and as the price of an annuity on a life, however young, very rarely exceeded seven years purchase, the assurances were seldom made for a longer

term, so that a very small proportion was made on the whole continuance of life, or with any other view than to secure a purchaser from the risk of losing the price of his annuity. Still, however, though the progress of the Society was slow, their number continued to increase and their capital to accumulate. By persevering in a course of strict economy in the management of their affairs; by guarding against the introduction of bad lives, and abstaining carefully from any measure that had a direct tendency to alienate their funds, they found themselves in the year 1782 in the possession of such a surplus as to justify them in adopting a new and more correct set of tables, which reduced their premiums in some instances to *one half* of their original amount, and at the same time in making an addition to the claims of £1 10*s.* *per cent.* for every payment which had been made prior to that year. During the following nineteen years four other additions were made to the claims; and in order to prevent the too frequent recurrence to these additions in future, it was determined in 1800 that none should thereafter be made without a previous investigation of the affairs of the Society —that such investigation should take place at the end of every ten years—and that the amount of the additions in present value should never exceed *two thirds* of the *surplus stock* of the Society. The beneficial effects of these wise regulations have been fully manifested in the decennial investiga-

tions of 1810 and 1820: the additions in those two periods exceeding in present value the sum of three millions sterling, and increasing the sums assured in the *old* policies to five times their original amount. These repeated operations in favour of the members, and the immense increase of the society in numbers, wealth, and credit, have naturally excited a desire of forming similar institutions, or rather *trading companies*, with the view of appropriating the whole, or a greater part of those profits among the partnership which have been so liberally shared in common by all the members of the Equitable Society.

Without entering into the merits or demerits of these companies (for they are of very different descriptions) it is impossible to contemplate without great satisfaction the happy effects produced by the success of the Equitable Society. The multiplication of assurance offices, to which this success has given rise, necessarily increases the number of assurances, and makes the subject more widely known and better understood. Thousands of families are now secured in the enjoyment of comforts, of which they would otherwise have been rendered destitute by the death of their friends or relatives. The establishment therefore of these new offices, if properly conducted, appears to be a considerable public benefit, and the only danger of their not continuing so, arises from the disposition which some of them manifest to make pre-

mature inroads upon their capital. It should be remembered, that the probabilities of life are generally higher in the infancy, than they are in the more advanced state of a Society, and consequently that any measure founded on the expectation that events will prove more and more favourable as the Society becomes longer established, will most probably terminate in disappointment. The Equitable Society proceeded quietly in the same course for near twenty years before they ventured upon any change. During that period the decrements of life among the members had attained such a degree of regularity that very little risk was incurred in founding any new measure on the presumption that they would continue to be equally so. The experience of the last forty years has confirmed this opinion, and the probabilities of life among the members of the Society may now be considered as uniformly regular as they are among the general mass of inhabitants in this country.

The erroneous conclusions deduced from the higher probabilities of life in the first years of a Society, are in danger of deriving additional force from a spirit of competition, and an eager desire of increasing their business. By publishing a delusive account of their prosperity founded on these probabilities, they are holding forth advantages which are the less likely to be realized in proportion to their magnitude and the earlier period of

their commencement. It should be particularly observed, that the motives which induce persons to become members with such promises to allure them, will always urge them to press for greater advantages, and the Society will find that the higher the expectations of such persons are raised, the more difficult it will be either to satisfy or to moderate them.

In this commercial country almost every thing becomes a matter of speculation, and the prospect of advantages, however distant, never fails to stimulate adventurers to the pursuit of them. It is no wonder therefore that the great success of one office in the business of assurance on lives should give rise to the establishment of so many new ones. In the course of a few years the number has been increased by about fifteen additional offices, in which are included the whole nearly of the *Fire* Offices, whose business seems to have been extended to assurances on lives with the hope of repairing the losses they have sustained by a ruinous competition with each other in the business of fire-assurances; but I am sorry to say that in some of them this new branch of business does not appear to have been entered upon with a more liberal spirit than they have manifested in their manner of conducting the old one *. To the public in-

* See particularly the prospectus of the *Sun* Life Assurance Society, in the year 1820, in which it is stated " that the managers

deed it is a matter of little moment in what terms they display their superiority to each other, but it is of consequence that they should be convinced of the danger to which they are exposed, by adopting injudicious measures from too eager a pursuit after *monopoly*. As long as they proceed in a safe and honourable course, they have my warmest wishes for their success, as I consider every assurance made for the purpose of providing for a surviving family, in whatever office it is effected, not only as a private but as a public good. It is far from my intention to extend these observations to all the offices which have been established within the last twenty years. Some of them are highly respectable, and cannot fail to prove beneficial to themselves and the public.

" being satisfied that the premiums which have hitherto been re-
" quired for assurances were not properly *graduated* according to
" the ages, have had others calculated proportional to the real values
" of the risks at the different periods of life, and having adopted
" these new rates, now offer assurances to the public on these
" terms, which will be found to be more *liberal* than any hitherto
" granted, the premiums having been, in many cases, reduced
" twenty and thirty per cent." If we may judge from the great irregularities in the increase and decrease of the difference between the premiums at any two successive ages, for a single year, we shall have no great reason to think very favourably of the superior corectness of this new graduation; nor shall we think more favourably of the greater *liberality* of their terms if we compare them with those of other offices; for instead of being 20 and 30 per cent. *lower*, their premiums for the whole continuance of life at all ages after 45, are really *higher* than the premiums required by any other office in London.

In the former edition of this work I gave a particular account of the *Equitable Society*, which being then almost the only society of the kind, I was induced to enter more minutely into the different methods of ascertaining the real state of that, or of any other similar institution; and concluded with observing, that the accounts of the Equitable Society had been lately investigated by each of those methods, and found to be so highly favourable as to justify the measures then adopted. But on more mature consideration, I have seen reason to change my opinion of those measures; and happily the Society, convinced of their pernicious tendency, have never repeated them. In all the subsequent investigations a more prudent and beneficial course has been pursued. By the method of making additions to the claims, no immediate invasion of the capital takes place; but the profits are divided so gradually, as neither to alienate the capital too suddenly, nor even to prevent the accumulation of it. The effect also of its operation for the first years is almost insensible, so that the credit and consequence of the Society are not impaired by the abrupt diminution of their capital; nor is there any danger of such a diminution at any future period, provided, as in the case of the Equitable Society, the present value of those additions is never suffered to exceed a certain portion of the surplus. Indeed I know

of no other safe and effectual method of appropriating this surplus. The immediate division of it is too ruinous in its consequences to be mentioned, or even thought of. The reduction of the premiums, although it were liable to no other objection, is altogether inadequate in many cases; for if all the annual premiums be remitted in some very old assurances, the present value of them during the remainder of the life assured would not be equal to the present value of a single addition of £1 per cent. In order, therefore, to give the old members their due share in the surplus, it would, in this case, be necessary to supply the deficiency by an immediate payment of money; or in other words, by an immediate invasion of the capital. But the method of making additions to the claims is exposed to none of these difficulties. Every member participates, as nearly as circumstances will admit, in due proportion to his age and the number of his payments;—the annual income of the society is not reduced, and the purpose of the assurance is answered in the best manner, by increasing that provision for a family which it was originally intended to secure for them. The long experience of the Equitable Society has fully demonstrated the excellence of this method in the success with which it has been attended. From the time in which it was first adopted to the last decennial investigation in 1819, the number of

members has increased more than ten-fold, the annual income more than one hundred fold, and the capital in a still higher degree. Many millions have been added to the sums originally assured, and the prospect of still further benefits is now secured by the admirable regulations which were formed in the year 1816, for limiting the number of those who are to share in them at one and the same time. These regulations exclude none from the expectation of partaking in those benefits, as some of the societies lately established have very improperly represented. They only *postpone* the enjoyment of them in order to render them ultimately more secure and valuable to the new members.

Having now been engaged in conducting the affairs of the Equitable Society for almost half a century, I cannot take my leave of the subject without expressing the pleasure I feel in witnessing the success of an institution (begun in obscurity, without any capital, and formed on the narrowest scale), exalted by prudence and economy to the highest summit of wealth and credit, extending its benefits to thousands of families, and affording the most solid reason to hope, that those which it has already conferred are only pledges of still greater benefits to thousands of families hereafter. With an ardent wish that this hope may be realized, and impressed with a lively sense of gratitude for

the many instances of kindness which I have experienced during a long series of years from the members of this Society, I shall now conclude this preface with submitting the following work to the candid perusal of the reader.

*Equitable Assurance Office,*
*May* 19*th*, 1821.

*Lately published by the same Author,*

MEMOIRS of the Life of Dr. PRICE.

# A
# TREATISE
# OF
# ASSURANCES ON LIVES.

## CHAPTER I.

*An Introduction to the Doctrine of Assurances on Lives.*

The value of an assurance is computed more or less easily, according to the number of lives involved in the calculation. In order, therefore, to give a clear idea of this subject, and the principles from which the more complicated questions are deduced, it will be necessary to begin with those in which only *one* life is concerned.

The simplest of this kind is the assurance of a single life for the term of one year. In this case nothing further is required than to determine the probability of the life's failing in the year; and in proportion as this probability is greater or less, the value of the assurance will be greater or less; so that were it *certain* that the life would fail, this value would be equal to the sum assured; were it

an *even* chance whether the life failed or not, it would be equal to *half* the sum assured; were there only *one* chance out of *twenty* for the life's failing, the value would only be a twentieth of the sum assured; and so on.

Suppose, for example, that 50 persons aged 39 agreed to assure £100 each on their lives for one year. By the *Northampton* Table* of Observations it appears that one of them will die in the year. The value of the assurance therefore will be $\frac{1}{50}$ of £100, or £2, from each person; for this contribution will just furnish money enough to pay the claim. But since the money is supposed to be advanced at the beginning, and the claim not to be paid till the end of the year, this sum must be discounted for a year, so that the several contributions, together with their interest, may be just equal to the sum assured. Had these persons been 60 years of age, two of them would have died in the year, and the contributions should have been twice as much; had they been 73 years of age, four of them would have died, and the contributions should have been four times as much. At a more advanced age, a still greater number would have died; so that the value of an assurance for one year, by thus continually increasing with age, must at the extremity of life become equal to the sum assured.

* See Tab. I.

Let the term now be two, three, or any certain number of years, the sum as before £100, the value of the assurance to be paid down immediately, and the sum assured at the end of the year in which the life becomes extinct. The investigation in this case, being more complicated than in the former, may be explained in the following manner: Suppose the age of the person to be assured is 30 years, and the probabilities of life as given in the *Northampton* Table. The number of persons living in that table at the age of 30 is 4385,—at the age of 31 it is reduced to 4310,—at the age of 32 to 4235,—at the age of 33 to 4160, and so on. The chances therefore for his living one year will be 4310 out of 4385 and expressed by the fraction $\frac{4310}{4385}$. The chances for his living two years will be 4235 out of 4385 and expressed by the fraction $\frac{4235}{4385}$. The chances for his living three years will be 4160 out of 4385 and expressed by the fraction $\frac{4160}{4385}$. The chances for his living 4, 5, &c. years, will in like manner, be expressed by the fractions $\frac{4085}{4385}$, $\frac{4010}{4385}$, &c. The difference between the number of persons living at the beginning of each year, and the number living at the beginning of the succeeding year, will denote the number of lives that have failed in the year. The difference between 4385 and 4310—between 4310 and 4235—between 4235 and 4160, and so on to the age of 40, being constantly 75, it follows

that $\frac{75}{4385}$ will express the probability of the life's failing in the first year; $\frac{75}{4310}$, multiplied into $\frac{4310}{4385}$, or $\frac{75}{4385}$, will express the probability of its failing in the second, after having survived the first year; $\frac{75}{4235}$ multiplied into $\frac{4235}{4385}$ $\left(\text{or } \frac{75}{4385}\right)$ will express the probability of its failing in the third, after having survived the 1st and 2d years, and this fraction will continue invariably the same, while the probabilities of life decrease in an arithmetical progression, so that the value of the assurance in the 1st year will be the sum discounted for a year, and diminished in the ratio of $\frac{75}{4385}$; the value of the assurance in the 2d year will be the sum discounted for two years, and diminished in the same ratio; the value of the assurance in the 3d, 4th, 5th, and four following years, will be the sum discounted for 3, 4, 5, &c. years, diminished still in the same ratio; but after the 9th year the probabilities of life do not decrease in the same arithmetical progression, and consequently the discounted sums must be respectively diminished in a different ratio:—thus in the 10th year the probability of the life's failing will be $\frac{76}{4385}$, in the 11th year $\frac{77}{4385}$, in the 12th year $\frac{78}{4385}$, and so on. The amount of the several sums so discounted and diminished for 2, 3, 4, &c. years, or to the end of life, according as the assurance is for a term of 2, 3, 4, &c. years, or for the *whole* of life, is equal to the value of such assurance. This amount, however, may be determined in a more compendious way than by

the continued addition of the several values in each year; but as my present design is only to give a general idea of the *principles* from which the values of assurances are investigated, the consideration of this part of the subject must be referred to a subsequent chapter.

If, instead of the value of an assurance for any term of years in a *single* payment, the same value were required in *annual* payments, the answer may be obtained in the following manner. Suppose 56 persons, each aged 30, had agreed to assure £100 each on their lives from year to year, for any certain term; that is, by equal contributions to discharge whatever claims may happen in the course of every year. By the *Northampton* Table one of them will die in the first year, and become a claimant at the end of the year. The sum therefore to be contributed from each person will be £1 15*s.* 9*d.* nearly; but if the money has been advanced at the beginning of the year, this contribution will be less, or only £1 15*s.* 9*d.* discounted for a year. In the second year there will be 55 persons alive, and one in 55 will die by the same table; consequently one again will become a claimant in the course of the second year. But a contribution of £1 15*s.* 9*d.* each from 55 persons will not be sufficient to discharge such a claim, and therefore at the beginning of the second year they ought to contribute a little more than they did at the beginning of the first year. For the same rea-

son, those that live to the beginning of the *third* year ought to contribute more than they did in the *second* year in order to make good the claim, and so on for every year during the whole term. If the value of the assurance is paid, not in annual contributions *increasing* from year to year, but in one *fixed* annual contribution, it is obvious that it should be higher than the contribution in the first year of the term, and lower than the contribution in the last year: In other words, it should be such a *mean*, as, being multiplied into the value of the life for the given term, shall produce a sum just equal to the value of the assurance in a single payment. But this will be better understood when the principles on which the values of life annuities are founded, have been explained in the following chapter.

Had the ages of the 56 persons, instead of 30, been 57, two of them would have died by the *Northampton* Table in the first and following years, and the contributions must consequently have been *doubled*. But since at this advanced age the probabilities of dying increase more rapidly than at the earlier age of 30, the contributions necessary to discharge the claims in each year must of course increase in the same proportion, so that the difference between the first and last contribution in the one case must be more considerable than it is in the other; and hence may be inferred the reason why the annual payment for assuring an old life

for 6 or 7 years exceeds the payment for one year in a much greater proportion than it does in the case of a young life. Were the annual payments to be continued during the *whole* of life, that is, were it an assurance of £100 to be paid whenever the life fails, the reasoning would be the same in all respects; observing, however, that the *mean* of all the payments would be much more considerable; for the contributions in this case must increase annually till they become equal to the sum assured. This follows from that invariable law of nature, which, rendering human life more and more precarious, lessens the number of survivors in each year, till at the extreme of life they are reduced to a single person, whose contribution, if he lives to the beginning of the last year, must equal the claim to be paid on his decease. It may not be improper to observe that the *mean* between the first and last payment is not to be understood as a simple *arithmetical* mean, or the *half* of the sum of those payments, but such a constant annual sum, which must so far exceed the value of the assurance for the first year as to produce a surplus, that, in consequence of being improved at compound interest, shall form a capital which shall be sufficient to discharge the claims as they become payable; so that when the last life drops, just so much of this capital shall remain as shall be equal to one claim.

From these principles the values of all assurances on single lives, on the joint continuance of the lives, or the longest of any number of lives are deduced. It should, however, be observed that as, in supposing any two events, the chance is universally *less* for the happening of *both* of them than of *one*, so the value of an assurance on two joint lives which depends on such a contingency must be *greater* than the value of an assurance on the single life of one cf them; for in the one case the claim is paid if they do not *both* live to the end of the term, that is, if *two* events do not happen; in the other case, it is paid if *one* does not live to the end of the term, or if *one* event does not happen. For the same reason, the value of an assurance on the longest of two or any number of lives, will be *less* than the value of an assurance on a single life; for in the former the claim is not paid unless two or more events *do* happen, that is, unless two or more lives fail in the term; hence it follows that the greater the number of *joint* lives that are assured, the *greater* will be the value of the assurance; and on the contrary, the *greater* the number assured during the longest liver, the *less* will be the value of the assurance.

In regard to assurances on *survivorships*, especially those cases in which the contingencies are extended to more than two lives, the reasoning is much more complicated, and leads to the most abstruse parts of the doctrine of chances. At present,

therefore, I shall confine myself to those cases in which only two lives are concerned; not only as being the most common, but as the same reasoning, if rightly understood, may be applied to other cases in which a greater number of lives are involved in the computation. To begin with the most simple: Let us suppose the assurance made only for one year on the contingency of one life's surviving another. Let the age of the person to be survived be 50, and of the person who is to be the survivor 30; that is, a given sum is to be paid in case the former should die before the latter in one year. The payment of the claim in this case depends upon either of two events; first, that they shall both die in the year, and the youngest shall die last: secondly, that the oldest person *only* shall die in the year. By the *Northampton* Table * the probability that a person aged 50 dies in the year is $\frac{81}{2857}$, and the probability that a person aged 30 dies in the year is $\frac{75}{4385}$, the probability therefore that they both die is expressed by these two fractions multiplied into each other, or $\frac{81 \times 75}{2857 \times 4385}$ †; but that one in particular shall die before the other is very nearly an

* Tab. 1.

† In this and the following chapter, it is taken for granted that the reader is so far acquainted with the doctrine of chances as to know, that the probability that any number of independent events will all happen, is the product of all their probabilities, separately taken, multiplied by one another.

equal chance. *Half* of this fraction, therefore, or $\frac{81 \times 75}{2 \times 2857 \times 4385}$, will be the value of the expectation on the first of the two events. The second, which is by far the most considerable, will be expressed by the probability of the older person's *dying*, or $\frac{81}{2857}$, multiplied into the probability of the younger person's living, or $\frac{81 \times 4310}{2857 \times 4385}$. The sum of these two fractions therefore, or $\frac{81 \times 75}{2 \times 2857 \times 4385}$ added to $\frac{81 \times 4310}{2857 \times 4385}$, will express the ratio in which the sum proposed to be assured, after being discounted for a year, is to be diminished. By reasoning in the same manner the value of the assurance may be determined, if the life to be survived is younger than the life to be the survivor. If the term is extended to two, three, four, &c. years, or to the extremity of life, the principles from which the solution is derived are exactly similar, provided the value is required in *one present payment.* Thus the chance that the older life shall die in each of the following ten years is constantly $\frac{82}{2857}$, and the chance that the younger life shall die in each of those years is constantly $\frac{75}{4385}$ *. Half the fraction $\frac{82 \times 75}{2857 \times 4385}$ will therefore, during this period, invariably express the value of the expectation on the contingency of their both dying in the same year, and one in particular dying *last.* The other expectation after the first year depends on the younger person's living two, three, four, &c. years,

* See page 9.

and the older person's having died in the first, second, third, &c. years, according as the assurance is to be continued for two, three, four, &c. years, and is determined by multiplying $\frac{82}{2857}$ (expressing the latter probability) into the several fractions expressing the former probability, which appear by the *Northampton* Table to be $\frac{4235}{4385}\ldots\frac{4160}{4385}\ldots\frac{4085}{4385}$, &c.; these fractions being severally multiplied into $\frac{82}{2857}$ and added to $\frac{82 \times 75}{2 \times 2857 \times 4385}$, expressing the expectation on the first contingency, will give the ratio in which the sum to be assured is to be diminished, after having been previously discounted for two, three, four, &c. years respectively. The sum of all these expressions, continued for as many terms as are equal to the number of years for which the assurance is to be made, will give its whole value in one present payment. But if the assurance is not to cease till the claim is determined, those terms must be continued to the extremity of the older life. The repeated multiplications and additions which would be necessary by these means to ascertain the value of the assurance would be attended with great labour, especially if the operations were to be continued during life, or even during a period of eight or ten years. A more concise method, therefore, is given in the fourth chapter of this work, for determining the sum of those terms in all cases, and reducing them into general rules.

In computing the values of assurances on survivorships in *annual* payments from the same principles with those on single lives, the reasoning becomes much more complicated. Suppose a given sum to be paid on the death of a person aged 56, should that happen before the death of another aged 30, and that 58 persons all aged 56 had agreed to assure £100 each on their lives, dependent on this contingency. By the *Northampton* Table one will die out of 58 at the age of 30 in the first year, and two out of the same number at the age of 56. *One* claim, therefore, will become due in this year, towards which a contribution of £1 14*s.* 6*d.* from each person must be paid. That the other does not become due depends upon two events; first, that the younger life which drops was one of those assured against either of the two older lives which failed in this year; secondly, that provided this was the case, the younger person also died *before* the older one in the year. That the first event happens, there are two chances out of 58, or a probability which may be expressed by the fraction $\frac{2}{58}$ or $\frac{1}{29}$; that the second event happens is very nearly an *even* chance which is expressed by $\frac{1}{2}$. These two fractions being multiplied into each other will produce $\frac{1}{58}$, expressing the whole probability that the second claim does not become due; that is, $\frac{1}{58}$th or £1 14*s.* 6*d.* must be added to the former contribution of £1 14*s.* 6*d.* which will make the whole

of the first year's assurance equal to £3 9*s.* and as this is supposed payable at the beginning, and the claim not till the end of the year, it must be discounted for one year. In the second year, there will be very nearly three assurances less than in the first year; and as the deaths among the old and young lives continually bear the same proportion to each other, there will again be nearly two claims to be paid in this year, which being among fewer persons will render the contributions from each so much the higher. In the third year there will be nearly six assurances less than in the first, and three less than in the second year. The claims, however, will be much the same, and consequently the contributions higher than in the preceding year. By reasoning in the same manner, it will be found that the annual payments are continually increasing till they become greatest in the last year; in order therefore to reduce these increasing annual payments to one *fixed* sum which shall be the same during every year of the assurance, it will be necessary to find such a *mean* between the highest and lowest contribution as shall provide a sufficient surplus, with its accumulating interest, to discharge the last claim. To determine with perfect accuracy *how much* the contributions increase and the claims diminish in every year would lead to a train of reasoning so perplexed and complicated as to be altogether unfit for the present purpose, and it will be

sufficient only to observe from this general view of the subject, since the contributions appear to increase much faster than the claims diminish, that the last contribution must necessarily be the greatest, and consequently that the fixed annual payment must be such a mean as I have described.

As the annual payments in the case of survivorships are made to cease whenever *either* of the lives fails, it is evident that these payments must be such a *mean*, as being multiplied into the value of an annuity on the *joint* lives shall be equal to the value of the assurance in one payment: or in other words, the annual payment will be the quotient arising from the value of the assurance in one payment divided by the value of an annuity on the two joint lives. But this will be better understood when the principles on which the values of annuities on single and joint lives shall have been explained in the following chapter.

## CHAPTER II.

### *An Introduction to the Doctrine of Annuities on Lives.*

THE value of a life annuity, like that of an assurance, is more or less easily determined in proportion to the number of lives on which the annuity depends. When only *one* life, therefore, is concerned, the operation will be most simple, and if the annuity is limited to a single year, it becomes the exact converse of the operation for an assurance during the same time. In the former, a given sum is to be paid if the life *exists* one year, in the latter the sum is to be paid if the life *fails* in the year; so that if it were an *even* chance whether it would exist through a year, or not, the value of an annuity and of an assurance would be equal; and the former will exceed the latter, in proportion as the chance of *living* exceeds that of *dying* in the year. Suppose, for example, that A. and his heirs were entitled to 50 annuities of £100 each on the single lives of as many persons, whose common age was 39; that is, that they were to receive £100 from every one of 50 persons of this age, at the end of every year while they lived. By the *Northampton* Table of Ob-

servation* it appears that 49 out of the 50 will live one year, and therefore the value of the first payment of those annuities is £100 multiplied by 49 (or £5,000, the sum of all the payments lessened in the ratio of 49 to 50) which is equal to £4,900. But as this money is not payable till the *end* of the year, it must be discounted for a year, so that the present value with the interest may in this time just accumulate to £4,900. Hence it follows that the value of the first payment of an annuity, in the present instance, exceeds that of an assurance for a year, in the ratio of 49 to one, the premium for assuring those several sums on 50 such lives being only £100 discounted for a year†. Had the annuity been on 50 single lives, whose common age was 60, two persons would have died by the *Northampton* Table in the year, so that the first payment would have been £100 multiplied into 48 (or £5,000 lessened in the ratio of 48 to 50) which is equal to £4,800; and in the former chapter‡ it is proved that the premium for assuring those sums is £200§, hence the first payment of an annuity on a life of 60 exceeds an assurance for a year on the same life only in the ratio of 48 to 2 or 24 to 1. At the age of 94 the number of those that die exceeds the number of those that live one year, and con-

* Tab. 1. † See page 2. ‡ Ibid.

§ It will be remembered that both these sums of £4,800 and £200 are to be discounted for a year.

sequently the value of the first payment of an annuity becomes *less* than the assurance for a year of a sum equal to that annuity. It may be observed from these calculations that the sum of the values of the first payment of an annuity, and of an assurance for one year, is always equal to such first payment of the annuity to be received *certainly* at the end of the year; for if a given sum is to be then received, whether a life *does* or *does not* exist, one or other of these two events must happen, and it is obvious that the expectation on *both* of them is the expectation of a certainty. But the value of the several payments of a life annuity for two, three, or any number of years, may perhaps be determined more simply in the following manner: Suppose the annuity to be confined to one person of the age of 66, and the value of it to be computed from the *Northampton* Table. The number of persons living in that table at the beginning of the first year is 1552, and the number living at the beginning of the following year is 1472. The probability therefore of the person's living in this case, to the end of the first year, is in the proportion of 1472 to 1552, and the value of £1 to be then received on that contingency is the sum discounted for a year, and multiplied into the fraction $\frac{1472}{1552}$. In the second year, the number living at the beginning of the third year being 1392, the probability of the person's living to that time will be in the proportion of 1392 to 1552, and therefore the value of the

second payment of the annuity will be £1 discounted for two years, and multiplied into the fraction $\frac{1392}{1552}$. By reasoning in the same manner, £1 discounted for three years, and multiplied into the fraction $\frac{1312}{1552}$, will give the value of the third payment of the annuity. If the operations are continued for any number of terms, the sum of those terms will be the value of the annuity for an equal number of years; and if in the present case they be continued to 30 terms, it appears from the table that the numerator of the fraction, which constitutes the 31st term, vanishes, and consequently the sum of the different products for 30 terms will express the value of the annuity for the whole duration of a life of 66. The subject may be otherwise explained in the following manner. Suppose a life annuity of £1 payable yearly to every one of 5132 persons, all now aged 20, the first payment of which is to be made a year hence. It appears from the *Northampton* Table of Observations, that only 5060 of these persons will be living at the end of a year, and consequently that the money to be then paid will be only £5,060. The present value therefore of the first payment of the annuities will be the sum which now put out to interest will increase in a year to £5,060, that is, it is £5,060 discounted for a year, or £4,865 7s. 6d.; for this sum added to its interest for a year at £4 per cent. will just make up £5,060. From the same table it appears further, that of 5132 persons living at 20 years of

age, only 4985 will be living at the end of two years. The present value therefore of the second payment of the annuities will be the sum which being now put out to compound interest at £4 per cent. will increase in two years to £4,985. This sum is £4,609. In like manner 4910, 4835, 4760, &c. being the number living at the end of 3, 4, 5, &c. years, the value of the 3d, 4th, 5th, &c. payments of the annuities will be £4,910, £4,835, £4,760, &c. discounted for 3, 4, 5, &c. years respectively. These sums so discounted and continued to the year in which all the lives become extinct will amount to £82,282, and will be sufficient if improved at £4 per cent. to make good the payment of an annuity for life of £1 to every one of 5132 persons aged 20, according to the *Northampton* Table of Observations. The value therefore of such an annuity payable only to one such person must be the 5132d part of £82,282, or £16 0s. 8d.

Were the probabilities of life to decrease in arithmetical progression according to an hypothesis invented by M. De *Moivre** the sum of the

* In this hypothesis the limit, or utmost extent of life, is confined to the age of 86; and supposing any number living at a given age, an equal number of them is supposed to die in every year, till at the age of 86 all the lives become extinct. Thus, if there are 50 persons living at the age of 30, one of them by this hypothesis will die annually during the term of 56 years, at which period the last surviving life will have failed; or, in other words, 55 will be living at the end of the 1st year, 54 at the end

*two joint* lives of the same age, as the money is to be received, in the first case, if only *one* event happens, and, in the second case, it is not to be received, unless that event and *another* happen. For the same reason, an annuity upon *two joint* lives is worth more than an annuity upon *three joint* lives of the same ages, inasmuch as the money in the one case is to be received, if only *two* events happen; in the other it is not to be received, unless those two events and *one more* all happen. Universally, the greater the number of lives, the less will be the value of the annuity on their *joint* continuance; and this number may be so increased, as to render such an annuity nearly of *no* value. When the number of lives exceeds *two*, the computations, in the manner I have described, become so laborious, as to render it necessary to have recourse to methods for approximating to the true values. For this purpose Mr. Simpson in his Select Exercises (prob. IX.) has given a rule for determining the value of three joint lives, from the given value of two joint lives, which has been rendered sufficiently correct in most cases by those tables of the values of single and joint lives, which have since been computed from the real probabilities of human life.*

* In the year 1783 Dr. *Price* published tables of single and joint lives from the *Northampton* Table of Observations, and also from the probabilities of life in the whole kingdom of *Sweden*.

With respect to annuities on the *longest* of any number of lives, the reasoning is less simple, and if more than two lives are concerned, it becomes so complicated, as not to admit of being explained in a few words. I shall therefore confine myself at present to the single case of two lives, which being derived from the same principles with those from which the value of a greater number of lives is determined, will, if properly understood, lead to a general knowledge of the subject in all cases. Suppose the ages of the two persons, during whose lives the annuity is to be enjoyed, are 50 and 60. By the *Northampton* Table the probability of the former's living one year, is expressed by the fraction $\frac{2776}{2857}$, and the probability of the latter's living the same time by the fraction $\frac{1956}{2036}$. The probabilities of their severally *dying* in the year, are $\frac{2776}{2857}$ subtracted from unity, and $\frac{1956}{2036}$ subtracted from unity, that is, they are $\overline{1-\frac{2776}{2857}}$ and $\overline{1-\frac{1956}{2036}}$. *. These two expressions being multiplied into each other will give the probability of their *both* dying

About the same time Mr. Baron *Maseres* also published similar tables from the probabilities of life, as deduced by M. *De Parcieux*. These have been computed at different rates of interest, and are indeed a very important addition to the doctrine of annuities. The Northampton and Sweden tables are inserted in this volume.

* A certainty is always denoted by unity, so that the difference between the probability of an event's *happening* and this certainty, must express the probability of the same event's *not* happening.

in the year*, which is equal to $1 - \frac{2776}{2857} - \frac{1956}{2036} + \frac{2776 \times 1956}{2857 \times 2036}$. If this expression again be subtracted from unity, the remainder will give the probability that they will *not* both die in the year; that is, that one of them at least will live to the end of it. In order therefore to subtract the above expression, nothing more is necessary than to change the different signs in it, and then add the whole to unity; in which case we shall have $\frac{2776}{2857} + \frac{1956}{2036} - \frac{2776 \times 1956}{2857 \times 2036}$ for the probability required. In the same manner the probability of one of them at least, living to the end of the second year may be found to be $\frac{2694}{2857} + \frac{1874}{2036} - \frac{2694 \times 1874}{2857 \times 2036}$, the probability of one of them at least living to the end of the third year $\frac{2612}{2857} + \frac{1793}{2036} - \frac{2612 \times 1793}{2857 \times 2036}$, and so on for a number of years equal to the difference between the age of the younger of the two lives, and that of the oldest life in the table of observations. If those several expressions be respectively multiplied into the annuity discounted for 1, 2, 3, &c. years, and the products be added together, the sum will be the whole value of the annuity for the continuance of the *longest* of two lives aged 50 and 60 years. By proceeding in the same manner the value of an annuity on the longest of any other two lives, and by any table of observations may be determined. But if the values of the single and joint lives are given, it appears from what has been observed in

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* See note, page 9.

the beginning of this chapter that the value of an annuity during the continuance of the *longest* of them may be obtained without difficulty. For since $\frac{2776}{2857}+\frac{2694}{2857}+\frac{2612}{2857}$, &c. multiplied into the annuity discounted for 1, 2, 3, &c. years, is shown to be the value of an annuity on a single life of 50, $\frac{1956}{2036}+\frac{1874}{2036}+\frac{1793}{2036}$, &c. so discounted, to be the value of an annuity on a single life of 60, and $\frac{2776\times 1956}{2857\times 2036}+\frac{1874\times 2694}{2857\times 2036}+\frac{2612\times 1793}{2857\times 2036}$, &c. discounted in like manner to be the value of an annuity on two joint lives aged 50 and 60,* it follows that if "from the sum "of the values of an annuity on the single lives, "the value of an annuity on the joint lives be "subtracted, the remainder will give the value of "an annuity on the continuance of the longest of "two such lives."† If the lives are increased to three, four, five, or any other number, the value of an annuity on the *longest* of them may still be found from the values of the single and joint lives; but the operation becomes much more

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* See page 20.

† EXAMPLE. Suppose the value to be required of an annuity on the longest of two lives aged 40 and 50, reckoning interest at 4 per cent. and computing from the probabilities of life at *Northampton*. By Tab. 3. the value of a single life of 40 is 13.197, and by the same table the value of a single life of 50 is 11.264. Their sum is 24.461; from which 8.834 (the value of the joint lives of 40 and 50 by Tab. 4.) being subtracted, we have 15.627 for the number of years purchase required.

complicated, and if the number exceeds three, almost impracticable, especially as there are no tables of the values of *four* joint lives, nor even any tolerably correct method of approximating to them.

## CHAPTER III.

*On the method of calculating tables of the values of annuities on single and joint lives.*

WERE it *certain* that a person of a given age would live to the end of a year, the value of an annuity of £1 on such a life would be the present sum that would increase in a year to the value of a life one year older, together with the value of the single payment of £1 to be made at the end of a year; that is, it would be £1 together with the value of a life one year older than the given life, multiplied by the value of £1 payable at the end of a year. Let the value of a life one year older than the given life be denoted by N. and the value of £1 payable at the end of a year by $\frac{1}{r}$ *, then will the value of the annuity on the given life on the supposition of a certainty be $\frac{1}{r} + \frac{1}{r} \times N$. But, in fact, it is *uncertain* whether the life will or will not exist to the end of the year; this value, therefore, must be diminished in the proportion of that uncertainty; that is, it must be multiplied by the probability that the given life will survive

* If $r$ represent £1 increased by its interest for a year, $\frac{1}{r}$ will be the present value of £1 to be received at the end of a year. See the Appendix at the end of the Tables.

one year, or supposing $\frac{b}{a}$* to express this probability it will be $\frac{b}{ar} \times \overline{1 + N}$. This theorem has been otherwise demonstrated by Mr. *Simpson* in his book on Life Annuities (prob. I. cor. 7th,) and by Dr. *Price* in his Treatise on Reversionary Payments (Note N. Appendix.) Its great utility will appear from the following examples: Suppose the probabilities of life to be as they have been given by Dr. *Price* from the Bills of Mortality at *Northampton*, and the rate of interest to be 3 per cent., or that $r$ is equal to 1.03. By reasoning, as in page 17, the value of a life aged 95 will be expressed by the fraction $\frac{1}{4} \times \frac{1}{1.03} = .242718$. The value of a life one year younger by this theorem will be $\frac{4}{9 \times 1.03} \times 1.242718 = .536232$. The value of a life two years younger will be by the same theorem $\frac{9}{16 \times 1.03} \times 1.536232 = .838961$. The value of a life three years younger will be $\frac{16}{24 \times 1.03} \times 1.838961 = 1.190262$, and if we proceed in this manner the value of every younger life may be deduced from that next proceeding; nor will the number of operations necessary to determine the values of all the lives much exceed the number of those which must otherwise be used for the single value of the youngest life. In computing a table of the values of annuities according to this

* If $a$ denote the number living at the age of the given life, and $b$ the number living in the following year, $\frac{b}{a}$ will express the probability that such life exists one year. (See page 17.)

or any other theorem, it cannot but be particularly agreeable to have the truth of our operations proved as we go on. I do not know that any rule has been delivered for this purpose, on which account I shall beg leave to offer the following: "Find the present value of £1 payable if the "youngest life in the table should live to that of "the oldest. Find next the present value of £1 "payable if the youngest life should live to an age "one year less than that of the oldest. Multiply "this last value by the value of an annuity on the "oldest life but one in the table, found in the "manner just directed; and if the product is "equal to the value of £1 payable on the first "mentioned contingency in this rule, a demon-"stration arises of the value of the life's being "right. Again, find the value of £1 payable if "the youngest life should live to an age two years "less than the oldest life in the table. Multiply "this by the value of an annuity of £1 on a life "two years younger than the oldest life, (found "from the value just calculated of a life one year "younger,) and if the product is equal to the sum "of the two values of £1 payable if the youngest "life should continue to the age of the oldest; "and to the age of the life one year less than the "oldest, the value of this second life will also be "right. In the same manner let the three values "of £1 payable if the youngest life should con-

"tinue to the several ages of the oldest, the secon
" and the third lives be added together, and
" they are equal to the product arising from th
" multiplication of the value of an annuity on th
" life three years younger than the oldest, (four
" from the value last calculated of a life two yea
" younger) into the value of £1, payable if th
" youngest life should continue to an age also thre
" years less than the oldest, a demonstration wi
" be given of the correctness of the value of th
" third life. Universally; if the several values o
" £1 payable if the youngest life in the table
" should live to the age of the oldest, the second,
" third, fourth, &c. lives be added together, and
" are found respectively equal to the products
" arising from the value of a life one, two, three,
" &c. years, younger than the oldest life multiplied
" into the value of £1 payable if the youngest life
" should continue to an age one, two, three, four,
" &c. years less than the oldest life, a proof will
" arise of the exactness of the operations as far as
" they have proceeded; and when we have gone
" in this manner through all the ages in the table
" and the sum of the several values of £1 payable
" if the youngest life should continue to the ages
" of all the older lives, is found equal to the value
" found last of all, of the youngest life, we have a
" conclusive proof that the values of the lives at
" all ages in the table have been computed withou

"committing any mistake, and are exact at least "as far, or to as many places of decimals as the "equality reaches." This rule will be better understood from the following examples. Let the probabilities of life be as they are in the *Northampton* Table, and the rate of interest 3 per cent. By reasoning as in page 17, the value of £1 payable if a life aged one year lives to the age of 96, is expressed by the fraction $\frac{1}{8650}$ multiplied into £1 discounted for 95 years, or .06032,* which is equal to .00000697. Again the value of £1 payable if the same life continues to the age of 95 years is $\frac{4}{8650}$ multiplied into £1 discounted for 94 years, or .06213, which is equal to .00002873. The value of an annuity on a life of 95 has been found in page 28, to be equal to .242718, and this being multiplied into .00002873 gives .00000697, which has likewise been just found to be the value of £1 payable if a life aged one year lives to 96, and therefore the value of the annuity is proved to be right. The value of £1 payable if a life of one year lives to be 94 is $\frac{9}{8650}$ multiplied into £1 discounted for 93 years, or .063994, which is equal to .00006658. The value of a life aged 94 calculated from the value of a life aged 95 in the manner directed in page 27, is .536232. These multiplied into each other produce .000035703,

* See Tab. 18.

which is also the sum of the values of £1 payable if a life of one year continues to the several ages of 95 and 96; consequently the value of this annuity is also right. Again, the sum of the three values of £1 payable if a life of one year continues to the several ages of 94, 95, and 96 is .00010228. The value of a life aged 93 calculated from the value of a life aged 94 is .838961 and the value of £1 payable if a life aged one year continues to the age of 93 is $\frac{16}{8650}$ multiplied into £1 discounted for 92 years, or .065914 which is equal to .00012192, and .838961 multiplied into this number produces .00010228, or the *sum* of the values just found. Hence it follows that .838961 is the true value of an annuity of £1 on a life aged 93 years. By proceeding in this manner through every age, it will appear at last that the sum of all the values of £1 payable if a life aged one year continues to the several ages of 96, 95, 94, 93, 92, &c. years is 16.0214, which corresponds *exactly* with the value of an annuity of £1 on the same life found in consequence of computing upwards from the oldest to the youngest life according to the theorem in this chapter, and therefore the values of all the lives are strictly true.*

In making these calculations it will be conve-

* See the demonstration of this rule in Note II.

nient to dispose the operations into the following order:

| Age. | Values of Lives. | Values of £1 payable. | Sums. |
|---|---|---|---|
| 96 | 0.0000 | .00000697 | 0.000000 |
| 95 | .242718 | .00002873 | .00000697 |
| 94 | .536232 | .00006658 | .00003570 |
| 93 | .838961 | .00012192 | .00010228 |
| | &c. | &c. | &c. |

In the first of these columns is the age. In the second the value of a life at that age agreeable to the *Northampton* Table of Observations, found by calculating upwards from the oldest to the youngest, in the manner already directed; that is, (reckoning interest at 3 per cent.) $.00000 = \frac{0}{1} \times \frac{1}{1.03}$ ... $.242718 = \frac{1}{4} \times \frac{1}{1.03}$ ... $.536232 = \frac{4}{9} \times \frac{1}{1.03} \times \overline{1 + .242718}$ ... $.838961 = \frac{9}{16} \times \frac{1}{1.03} \times \overline{1 + .536232}$, &c. &c. In the third column is the value of £1 payable to a life aged one year (being the youngest life to which the operations are to be carried), provided it should exist to the ages of 96, 95, 94, &c. that is, .00006032 is the probability that a life aged one shall exist till 96, multiplied by the value of £1 payable at the end of 95 years, or $.00000697 = .06032 \times \frac{1}{8650}$. In like manner $.00002873 = .06213 \times \frac{4}{8650}$ ... $.00006658 = .063994 \times \frac{9}{8650}$, &c. &c. In the fourth column are the sums of all the values in the third column *above* the values even with those sums, or (which comes to the same thing) the sums of the two numbers immediately above them,

and even with one another in the 3d and 4th columns; that is, .00000697=.00000697+.00000 ....00003570=.00000697+.00002873.... .00010228 =.00006658+.00003570, &c. &c. In consequence of this arrangement it will be always found that the product of every number in the second column, multiplied by the number even with it in the third column, gives the number even with both in the fourth column; and this forms a proof that there has been no error during the whole progress of the calculations in any of the preceding ones, and that the value of every life, as it comes out, may be depended on to as many places of decimals, as those in the product just mentioned are the same with the decimal figures in the correspondent number in the 4th column. For example; .838961 (the value of a life aged 93) multiplied by .00012192, gives .00010228, which being the same to the last place of decimals with .00010228 in the 4th column, proves that the values of the lives aged 93, 94, and 95 may be depended upon to the six places of decimals. This proof makes a little more labour necessary in calculating the values of lives; but this addition of labour is more than compensated by the time and trouble that must be saved in consequence of detecting every error as it arises, and also by the satisfaction attending a series of operations, every one of which, if right, proves the truth of all that have been made before it. The following expedient, however, will remove one half the labour. In

order to find the numbers in the third column one multiplication is necessary and one division. The time employed in the latter may be saved by making the number 10,000 the divisor instead of the number in the Table of Observations opposite to the youngest age. For example; in all the operations which bring out the numbers in the third column, the divisor is 8650, but it might have been taken to be 10,000, and then the first number in that column, instead of .00000697, would have been $.000006032 = \frac{1}{10,000} \times .06032$. The second number, instead of .00002873, would have been $\frac{4}{10,000} \times .06213 = .000024852$. The third, instead of .00006658, would have been $\frac{9}{10,000} \times .063994 = .000057595$, &c. &c. And the correspondent sums in the fourth column would have been .000000 . . . .000006032 . . . .000030884 . . . .00008848, &c. The table then would have been as follows;

| Age. | Values of Lives. | Values of £1 payable, &c. | Sums. |
|---|---|---|---|
| 96 | 0.00000 | .000006032 | .000000 |
| 95 | 0.242718 | .000024852 | .000006032 |
| 94 | 0.536232 | .000057595 | .000030884 |
| 93 | 0.838961 | .000105462 | .000088479 |
| 92 | 1.19026 | .000162938 | .000193941 |
| 91 | 1.50103 | .000237755 | .000356879 |
| 90 | 1.79474 | .000331324 | .000594634 |
| 89 | 2.01312 | .000459953 | .000925958 |
| 88 | 2.18521 | .000634219 | .001385911 |
| 87 | 2.31236 | .000873614 | .002020130 |
| 86 | 2.46181 | .001175442 | .002893744 |
| 85 | 2.62012 | .001553044 | .004069186 |

| Age. | Values of Lives. | Values of £1 payable, &c. | Sums. |
|---|---|---|---|
| 84 | 2.79372 | .002012447 | .005622230 |
| 83 | 2.98226 | .002560019 | .007634677 |
| 82 | 3.22934 | .003156904 | .010194696 |
| 81 | 3.49933 | .003815466 | .013351600 |
| 80 | 3.78150 | .004539732 | .017167066 |
| 79 | 4.07717 | .005323980 | .021706798 |
| 78 | 4.37250 | .006181998 | .027030778 |
| 77 | 4.65192 | .007139610 | .033212776 |
| 76 | 4.92544 | .008192664 | .040352386 |
| 75 | 5.19970 | .009336205 | .048545050 |
| 74 | 5.49113 | .010540896 | .057881255 |
| 73 | 5.79384 | .011809462 | .068422151 |
| 72 | 6.10373 | .013144757 | .080231613 |
| 71 | 6.41788 | .014549414 | .093376370 |
| 70 | 6.73417 | .016026595 | .107925784 |
| 69 | 7.05104 | .01757936 | .12395238 |
| 68 | 7.36731 | .01921085 | .14153174 |
| 67 | 7.68210 | .02092433 | .16074259 |
| 66 | 7.99473 | .02272330 | .18166692 |
| 65 | 8.30467 | .02461154 | .20439022 |
| 64 | 8.61152 | .02659250 | .22900176 |
| 63 | 8.91001 | .02868621 | .25559426 |
| 62 | 9.20550 | .03088146 | .28428047 |
| 61 | 9.49288 | .03319977 | .31516193 |
| 60 | 9.77738 | .03562933 | .34836170 |
| 59 | 10.0587 | .03817484 | .38399103 |
| 58 | 10.3368 | .04084093 | .42216587 |
| 57 | 10.6115 | .04363262 | .46300680 |
| 56 | 10.8826 | .04655507 | .50663942 |
| 55 | 11.1500 | .04961362 | .55319449 |
| 54 | 11.4138 | .05281375 | .60280811 |
| 53 | 11.6739 | .05616140 | .65562186 |
| 52 | 11.9302 | .05966213 | .71178326 |
| 51 | 12.1828 | .06332250 | .77144539 |
| 50 | 12.4360 | .06712522 | .83476789 |
| 49 | 12.6937 | .07105091 | .90189311 |
| 48 | 12.9508 | .07512666 | .97294402 |
| 47 | 13.2028 | .07938308 | 1.0480707 |
| 46 | 13.4498 | .0838272 | 1.1274538 |
| 45 | 13.6920 | .0884664 | 1.2112810 |
| 44 | 13.9295 | .0933086 | 1.2997474 |
| 43 | 14.1625 | .0983616 | 1.3930560 |
| 42 | 14.3911 | .1036341 | 1.4914176 |

| Age. | Values of Lives. | Values of £1 payable, &c. | Sums. |
|---|---|---|---|
| 41 | 14.6200 | .1091036 | 1.5950517 |
| 40 | 14.8480 | .1147766 | 1.7041553 |
| 39 | 15.0753 | .1206588 | 1.8189319 |
| 38 | 15.2978 | .1267911 | 1.9395907 |
| 37 | 15.5156 | .1331824 | 2.0663818 |
| 36 | 15.7290 | .1398432 | 2.1995642 |
| 35 | 15.9380 | .1467840 | 2.3394074 |
| 34 | 16.1427 | .1540151 | 2.4861914 |
| 33 | 16.3433 | .1615482 | 2.6402065 |
| 32 | 16.5400 | .1693945 | 2.8017547 |
| 31 | 16.7328 | .1775664 | 2.9711492 |
| 30 | 16.9218 | .1860757 | 3.1487156 |
| 29 | 17.1072 | .1949363 | 3.3347913 |
| 28 | 17.2891 | .2041607 | 3.5297276 |
| 27 | 17.4675 | .2137634 | 3.7338883 |
| 26 | 17.6426 | .2237584 | 3.9476517 |
| 25 | 17.8144 | .2341606 | 4.1714101 |
| 24 | 17.9830 | .2449856 | 4.4055707 |
| 23 | 18.1486 | .2562495 | 4.6505563 |
| 22 | 18.3111 | .2679682 | 4.9068058 |
| 21 | 18.4708 | .2801601 | 5.1747740 |
| 20 | 18.6385 | .2926677 | 5.4549341 |
| 19 | 18.8208 | .3053867 | 5.7476018 |
| 18 | 19.0131 | .3183594 | 6.0529885 |
| 17 | 19.2184 | .3315248 | 6.3713479 |
| 16 | 19.4359 | .3448725 | 6.7028727 |
| 15 | 19.6577 | .3585243 | 7.0477452 |
| 14 | 19.8728 | .3726845 | 7.4062695 |
| 13 | 20.0814 | .3873722 | 7.7789540 |
| 12 | 20.2837 | .4026052 | 8.1663262 |
| 11 | 20.4800 | .4184041 | 8.5689314 |
| 10 | 20.6633 | .4349416 | 8.9873355 |
| 9 | 20.8123 | .4527261 | 9.4222771 |
| 8 | 20.8857 | .4728130 | 9.8750032 |
| 7 | 20.8538 | .4962093 | 10.3478162 |
| 6 | 20.7275 | .5231724 | 10.8440255 |
| 5 | 20.4735 | .5552155 | 11.3671979 |
| 4 | 20.2109 | .5899005 | 11.9224134 |
| 3 | 19.5757 | .6391743 | 12.5123139 |
| 2 | 18.5995 | .7070875 | 13.1514882 |
| 1 | 13.8585* | 1.0000000 | 13.8585757 |

* True value 16.0214.

In this way the same proof of all the operations is obtained; nor has any error arisen except in the value of the life last calculated, which is easily corrected by taking the number of the living at the age of this life, agreeable to the table of observations, instead of supposing it 10,000; that is, by making the value of the life aged one year $\frac{7283}{8650} \times \frac{1}{1.03} \times \overline{1 + 18.5995}$, instead of $\frac{7283}{10,000} \times \frac{1}{1.03} \times \overline{1 + 18.5995}$.

The use of logarithms will further expedite the computations, by the assistance of which the values of single lives at all ages agreeable to any given table of observations may be calculated in a few hours, without the possibility of committing any mistake.

By a similar process the values of annuities on the *joint continuance* of two lives may be computed with equal facility and correctness. But this part of the subject will require no explanation, as it may easily be understood from what has been already said, in regard to the values of annuities on *single* lives. Thus, let the value of any two joint lives be denoted by M. the probability that two lives, each one year younger, will exist together one year by $\frac{nb}{am}$, and let $\frac{1}{r}$, as before, be the value of £1 payable at the end of the year; then by reasoning as in page 27, the value of an annuity on the joint continuance of those two younger lives will be expressed by $\frac{nb}{amr} \times \overline{1 + M}$. This, together with the necessary proof of the operations,

will be sufficiently explained by the following specimen.

| Ages. | | Values of Joint Lives. | Values of £1 payable, &c. | Sums. |
|---|---|---|---|---|
| 96 | 62 | 0.00000 | .000003088146 | .000000000000 |
| 95 | 61 | 0.23254 | .000013279908 | .000003088146 |
| 94 | 60 | 0.51044 | .000032066397 | .000016368054 |
| 93 | 59 | 0.79297 | .00006107974 | .000048434451 |
| 92 | 58 | 1.1173 | .00009801823 | .00010951419 |
| 91 | 57 | 1.3989 | .0001483509 | .0002075324 |
| 90 | 56 | 1.6618 | .0002141533 | .0003558833 |
| 89 | 55 | 1.8531 | .0003076044 | .0005700366 |
| 88 | 54 | 2.0021 | .0004383541 | .0008776410 |
| 87 | 53 | 2.1110 | .0006233915 | .0013159951 |
| 86 | 52 | 2.2418 | .0008651009 | .0019393866 |
| 85 | 51 | 2.3811 | .001177799 | .002804488 |
| 84 | 50 | 2.5353 | .001570730 | .003982287 |
| 83 | 49 | 2.7043 | .002053371 | .005553017 |
| 82 | 48 | 2.9262 | .002599382 | .007606388 |
| 81 | 47 | 3.1666 | .00322295 | .01020577 |
| 80 | 46 | 3.4157 | .00393150 | .01342872 |
| 79 | 45 | 3.6748 | .00472411 | .01736022 |
| 78 | 44 | 3.9316 | .00561718 | .02208433 |
| 77 | 43 | 4.1723 | .00663941 | .02770151 |
| 76 | 42 | 4.4065 | .00779328 | .03434092 |
| 75 | 41 | 4.6417 | .00907742 | .04213420 |
| 74 | 40 | 4.8924 | .01046763 | .05121162 |
| 73 | 39 | 5.1531 | .01196935 | .06167925 |
| 72 | 38 | 5.4185 | .01359201 | .07364860 |
| 71 | 37 | 5.6861 | .01534261 | .08724061 |
| 70 | 36 | 5.9542 | .01722868 | .10258322 |
| 69 | 35 | 6.2214 | .0192581 | .1198119 |
| 68 | 34 | 6.4868 | .0214389 | .1390700 |
| 67 | 33 | 6.7497 | .0237799 | .1605089 |
| 66 | 32 | 7.0099 | .0262900 | .1842888 |
| 65 | 31 | 7.2667 | .0289788 | .2105788 |
| 64 | 30 | 7.5200 | .0318562 | .2395576 |
| 63 | 29 | 7.7653 | .0349521 | .2714138 |
| 62 | 28 | 8.0075 | .0382597 | .3063651 |
| 61 | 27 | 8.2422 | .0418121 | .3446248 |
| 60 | 26 | 8.4741 | .0456020 | .3864369 |

| Ages. | | Values of Joint Lives. | Values of £1 payable, &c. | Sums. |
|---|---|---|---|---|
| 59 | 25 | 8.7031 | .0496420 | .4320389 |
| 58 | 24 | 8.9290 | .0539458 | .4816809 |
| 57 | 23 | 9.1518 | .0585274 | .5356267 |
| 56 | 22 | 9.3714 | .0634013 | .5941541 |
| 55 | 21 | 9.5878 | .0685832 | .6575554 |
| 54 | 20 | 9.8067 | .0740449 | .7261386 |
| 53 | 19 | 10.0316 | .0797670 | .8001835 |
| 52 | 18 | 10.2600 | .0857681 | .8799505 |
| 51 | 17 | 10.4934 | .0920313 | .9657186 |
| 50 | 16 | 10.7353 | .0985301 | 1.0577499 |
| 49 | 15 | 10.9847 | .1052627 | 1.1562800 |
| 48 | 14 | 11.2310 | .1123271 | 1.2615427 |
| 47 | 13 | 11.4704 | .1197755 | 1.3738698 |
| 46 | 12 | 11.7033 | .1276258 | 1.4936453 |
| 45 | 11 | 11.9301 | .1358977 | 1.6212711 |
| 44 | 10 | 12.1467 | .1446616 | 1.7571688 |
| 43 | 9 | 12.3408 | .1541081 | 1.9018304 |
| 42 | 8 | 12.4879 | .1646335 | 2.0559385 |
| 41 | 7 | 12.5739 | .1766009 | 2.2205720 |
| 40 | 6 | 12.6052 | .1901732 | 2.3971729 |
| 39 | 5 | 12.5608 | .2059850 | 2.5873461 |
| 38 | 4 | 12.5106 | .2232773 | 2.7933311 |
| 37 | 3 | 12.2268 | .2467213 | 3.0166084 |
| 36 | 2 | 11.7285 | .2782389 | 3.2633297 |
| 35 | 1 | 3.5415* | 1.0000000 | 3.5415686 |

In the third column the values are $\frac{1793}{1874} \times \frac{0}{1} \times \frac{-}{1}$ $=.00000 \ldots \frac{1874}{1956} \times \frac{1}{4} \times \frac{1}{1.03} \times \overline{1.+0} = .23254 \ldots \frac{1956}{2038}$ $\frac{4}{9} \times \frac{1}{1.03} \times \overline{1+.23254} = .51044 \ldots \frac{2038}{2120} \times \frac{9}{16} \times \frac{1}{1.03}$ $\overline{1+.51044} = .79297$, and so on. In the four column the numbers of the living at each of tl youngest ages to which the values are to be foul (which in the present case are 1 and 35) are su

* True value 10.2102.

posed to be 10,000; so that .000003088146 = $\frac{1874}{10,000} \times \frac{1}{10,000} \times .164789$ (the value of £1 payable at the end of 61 years); .000013279908 = $\frac{1956}{10,000} \times \frac{4}{10,000} \times .169733$ (the value of £1 payable at the end of 60 years), or $\frac{4}{10,000} \times .03319977$ (the number in the third column of the table of the values of single lives opposite the age of 61), . . . .000032066397 = $\frac{9}{10,000} \times \frac{2038}{10,000} \times .174825$, or $\frac{9}{10,000} \times .03562933$, (the number in the third column of the abovementioned table, opposite the age of 60), and so on. Having therefore previously computed a table of the values of single lives, and preserved the numbers in the third column, one half the trouble of computing the values in the fourth column of the present table may be saved. In the fifth column are the sums respectively of .000003088146 and .00000000 . . . of .000013279908 and .000003088146 . . . of .000032066397 and .000016368054, &c. And the equality of the products of the values even with one another in the third and fourth columns, with the number in the same line in the fifth column, forms the same proof of the accuracy of the operations with that already explained in the account of the method of computing the values of single lives. The calculation of the values of joint lives, in exact agreement to a given table of observations, was formerly considered as a task so laborious, that with the exception of Mr. *Simpson*, who computed the values of joint lives from the London Table,

I do not know that it was ever undertaken. But these values being computed only to one place of decimals, and deduced from a table of observations representing the rate of mortality among the inhabitants of London taken in the gross, and consequently giving the probabilities of life much too low for the middle and higher sorts of people in London itself, were rendered of little use. Recourse therefore was had to the values of joint lives founded on M. *De Moivre*'s hypothesis of an equal decrement of human life, which in the middle stages agrees tolerably well with the probabilities of life deduced from real observations, but in the earlier and later stages corresponds so ill with those probabilities, that the values of joint lives derived from it, are much too incorrect to be applied to the solution of problems which involve two or three lives in the computation. The inaccuracy of this hypothesis and the defectiveness of Mr. *Simpson*'s tables induced me to be more minute, than I should otherwise have been, in describing the safest and most expeditious method of calculating tables of the values of single and joint lives, from any given table of observations; and since the first publication of this work about forty years ago, I have had the satisfaction of seeing those methods adopted, and such tables computed as have entirely superseded the use of M. *De Moivre*'s hypothesis in all cases, and laid the foundation for correct solutions of all the

different problems in the doctrine of Life Annuities and Reversions. It may not be improper to observe that the several tables of the values of single and joint lives in the present volume have been computed in this manner, and that those tables which have been derived from the probabilities of life at *Northampton*, have not only been adopted by the legislature in order to determine the duties on contingent legacies, and for other purposes, but further that they have been applied to the computation of the different premiums of assurance required during the last 38 years by the Equitable society; which premiums, with very little variation, have also been assumed by most of the other assurance offices lately established in this country.

## CHAPTER IV.—Sect. I.

### *An Account of the Method of computing the Values of contingent and other Annuities, the Values of Reversionary Payments, &c.*

#### Introduction.

In the preceding chapters I have endeavoured to explain the principles from which the values of assurances and annuities are deduced, which, being properly understood, may be applied without much difficulty to the solution of the more complicated cases of contingent reversions and survivorships. Being now possessed of tables of the values of single and joint lives, computed from real observations, it no longer becomes necessary to have recourse to any rules for approximating to these values, and therefore the rules given for that purpose in the former edition of this work, are now omitted, as well as all those problems whose solutions are derived from M. *De Moivre*'s hypothesis of an equal decrement of life. There are however many problems introduced into the present, which were not given in the former edition; but there are no problems inserted, whose solutions are not founded on principles altogether independent of any *hypothesis*, and which are not strictly true, according to any table of observations. In some cases, particularly those in which three lives are concerned, the rules are necessarily more compli-

cated than those derived from M. *De Moivre's* hypothesis; but their superior correctness more than compensates for the greater length of the operations which they require.

In this chapter I have confined myself to a description of the rules, without entering into their demonstrations; many of which are very abstruse and complicated. These have all been inserted in Notes at the end of the Volume, for the information of the mathematical reader, being aware that the *practical* part of this work ought not to be encumbered with analytical operations, which would serve rather to perplex than to inform the reader who is not a mathematician.

The solutions of the greater number of the problems involving three lives in the computation were communicated by myself to the Royal Society, and published in the Philosophical Transactions about 25 years ago, and were I believe the first attempt to give correct rules for determining the values of those contingent reversions, according to the real probabilities of life. By a slight alteration in some of the symbols, these rules have been simplified in a small degree; but they are still rather complicated, nor do I see how they can be rendered less so, without at the same time rendering them less accurate, and thus reducing them to be mere *approximations*, like those which were founded on the erroneous hypothesis of M. *De Moivre.*

## PROBLEM I.

by .49.

To find the *expectation* of a given life, according to any table of observations; or the time which, one with another, a body of people will live at the age of the given life; some of them living as much *longer* as others live *less* than that time.

SOLUTION. Find the sum of all the living in the table, from the age of the given life; divide this sum by the number of the living at that age, and the quotient, lessened by half-unity, will be the required expectation.

EXAMPLE. The sum of all the living from the age of ten, in the *Northampton* Table of Observations, is 228599; this sum divided by 5675, the number living at the age of ten, gives 40.28, which being lessened by half-unity, the remainder, or 39.78 will be the expectation of a life of ten, according to the probabilities of life at *Northampton*. The *expectation* of a life is the same with the value of an annuity on such life, supposing money not to bear interest; and therefore having computed the expectation of any given life, a table of the expectations of all the younger lives may be obtained in the same manner as the values of annuities are obtained from those on the older lives; that is, if E. be the computed expectation of any life, and $\frac{b}{a}$ the probability that a life one year younger shall exist a year, the expectation of such younger life will be $\frac{b}{a} \times \overline{1 + E}$. Thus,

the expectation of a child aged 10 is 39.78, and the probability that a child aged nine exists one year, is $\frac{5675}{5735}$, the expectation therefore of this younger child will be $\frac{5675}{5735} \times 40.78 = 40.36$.*

## PROBLEM II.

To determine the value of an annuity on a given life, for any given number of years.

SOLUTION. Find the value of a life as many years older than the given life as are equal to the term for which the annuity is proposed; multiply this value by £1 payable at the end of the term, and also by the probability that the life will continue so long †. Subtract the *product* from the

giv. Valu Defe Ann Pri p.1 Ba p.

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* See Tab. 2.

† The probability that a life shall continue any number of years, or attain to a *given age*, is the fraction, whose numerator is the number living in the table of observations opposite that age, and denominates the number opposite to the present age of the given life: Thus 2038 is the number opposite to 60, and 4385 the number opposite to 30, $\frac{2038}{4385}$ therefore is the probability that a person aged 30 shall attain to 60, or live 30 years.

In the same manner the probability that any two lives shall continue in being together any number of years, or *both* attain to given ages, is the fraction whose *numerator* is the product arising from the numbers of living in the table, opposite to each of those given ages multiplied by one another, and *denominator* the product arising from the numbers opposite to the present ages, multiplied into each other. Hence the probability, that two lives aged 20 and 25 shall *both* continue in being 10 years is $\frac{4385 \times 4010}{5132 \times 4760} = \frac{1758385}{2442832}$.

present value of the given life, and the remainder multiplied into the annuity will be the answer.

Example. Let the annuity be £10, the age of the life 30 years, and the proposed term 15 years. The value of a life aged 45, (or 15 years older than the given life) appears by Tab. 3. at 3 per cent. to be 13.692. The value of £1 payable at the end of 15 years, by Tab. 18, is .64186, and the probability that the life will exist so long by Tab. 1. is $\frac{3248}{4385}$. These three quantities, multiplied into each other, produce 6.510, which being subtracted from

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Again, the probability that a life shall *fail* in any number of years, or *not* attain to a given age, is the fraction expressing the probability of his *living* so long, subtracted from unity, agreeable to what has been observed in page 23: In other words, it is the fraction whose denominator is the number of the living opposite to the present age, and whose numerator is the difference between this denominator and the number of living at the given age. Thus, $\frac{2038}{4385}$ is the probability that a life of 30 shall attain the age of 60; $\frac{2347}{4385}$ therefore will be the probability that he shall *not* attain this age; For the same reason $\frac{4385}{5132}$ and $\frac{4010}{4760}$ being the probabilities that two lives aged 20 and 25 shall severally exist 10 years, $\frac{747}{5132}$ multiplied into $\frac{750}{4760}$ ($=\frac{560250}{24428320}$) will be the probability that they shall *both die* in 10 years; and this fraction again being subtracted from unity, will give $\frac{23868070}{24428320}$ for the probability that they shall not both die in this time, that is, that one of them at least shall *live* 10 years. Further $\frac{17584350}{24428320}$ being the probability that the two lives of 20 and 25 shall exist together 10 years, if the numerator be subtracted from the denominator, we shall in this case have $\frac{6843970}{24428320}$ for the probability that they shall *not both* live so long; that is, that one of them at least shall *die* in this time.

16.922 (the present value of the given life by Tab. 3.) leaves 10.412, and this remainder multiplied into 10 gives £104.12 or nearly £104 2*s.* 5*d.* for the value required.

Corollary 1st. In the same manner may the value of an annuity for a given term upon two *joint* lives be determined; let the age of each of the two persons be 30 years, the term 15 years, and the rate of interest 3 per cent. The value of two joint lives, each aged 45, by Tab. 4. is 9.776, which being multiplied into .64186 (the value of £1 at the end of 15 years by Tab. 18.) and also into $\frac{3248 \times 3248}{4385 \times 4385}$, the probability that *both* lives shall exist so long by Tab. 1., will produce 3.443, and this subtracted from 12.589, the whole present value of the joint lives by Tab. 4., leaves 9.146 for their value during the term proposed. And since the value of the longest of two lives is equal to the joint lives subtracted from the sum of the two single lives *; if from twice 10.412 we subtract 9.146 we shall have 11.678 for the value of the longest of two equal lives aged 30, for the term above mentioned.

Corollary 2d. The value of an annuity for the remainder of a term certain after any life or lives is immediately obtained from this problem, or the foregoing corollary, being only the difference be-

* See page 25, and also Prob. III.

tween the value of the life or lives found above and the value of an annuity certain for the proposed term. Hence if this term be 15 years, it value at 3 per cent. by Tab. 19. is 11.938, from which 10.412, or 9.146, or 11.678 being subtracted we have either 1.526, 2.792, or .260 for the value of an annuity during the remainder of 15 years after a single life of 30, or after two *joint* lives of the same age, or after the *longest* of two such lives.

Corollary 3d. If the value of an annuity on the life or lives for any given term be subtracted from the value of the annuity for the whole life or lives, the difference will be the value of the annuity during the remainder of such life or lives after the expiration of that term. Or it is equal in the case of *single* and *joint* lives to the *product* described in the solution of this problem; thus 6.5 is the value of an annuity of £1 during the remainder of a life of 30 *after* 15 years, and 3.44 the value of the like annuity during two *joint* lives after the same term. *

* In the case of the longest of two lives, it is evident that the value can only be found by taking the *difference* mentioned in this corollary; so that 21.255 being the whole value of the longest of two lives aged 30 at £3 per cent., found by Prob. III., and 11.678 being the value for 15 years, if the latter be subtracted from the former, we shall have 9.577 for the value of the longest of those two lives, *after* the term just mentioned.

### PROBLEM III.

To find the value of an annuity during the longest of two lives. Baily p. 38

SOLUTION. From the sum of the values of the two single lives, subtract the value of the two joint lives, and the remainder will be the value required. *

EXAMPLE. Let the ages of the two lives be 40 and 50, and the rate of interest 5 per cent. By Tab. 3. the value of the single life of 40 is 11.837, the value of the single life of 50 is 10.269, and the value of the two joint lives by Tab. 4. is 8.177, which being subtracted from 22.106, the sum of the two former values, leaves 13.929 for the number of years purchase required.

### PROBLEM IV.

To approximate to the value of an annuity on the joint continuance of three lives, A. B. and C. † Baily 2nd l. p 37

SOLUTION. Let A. be the youngest, and C. the oldest of the three lives; take the value of the two joint lives B. and C. and find the age of a single life D. of the same value nearly; then find the value of the joint lives A. and D. which will be the answer.

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* See the demonstration in page 23.

† A general account of the method of computing the values of two joint lives from the real probabilities of life has been already given in page 20.

EXAMPLE. Supposing the value were required at £4 per cent. of an annuity on the joint continuance of three lives aged 10, 17, and 47 years. By Tab. 4. the value of the two oldest joint lives is 10.208, answering to a single life D. of 55 years, by Tab. 3., and the value of two joint lives aged 10 and 55, by Tab. 4. is 9.256, which is the answer.

## PROBLEM V.

To find the value of an annuity upon the longest of the three lives A. B. and C.

SOLUTION. From the sum of the values of the three single lives added to the value of the three joint lives, found by the preceding problem, subtract the sum of the two joint lives of A. and B. of A. and C., and of B. and C., and the remainder will be the answer. *

EXAMPLE. Let the ages of the three lives respectively be 20, 30, and 60, interest £3 per cent. By Tab. 3. the values at £3 per cent. of the several single lives of A. B. and C. are 18.638, 16.922, and 9.777. By the preceding problem the value of the three joint lives is 7.444, these four values added together make 52.781. By Table 4, the several values of the joint lives of A. and B., of A. and C., and of B. and C. are 13.286, 8.597, and 8.378; this sum, or 30.261, subtracted from 52.781

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* See the demonstration, Note III.

leaves 22.52 for the number of years purchase required.

SCHOLIUM. By proceeding in the same manner, and computing at £4 per cent., the *exact* values of three equal lives, each aged 60, may by the help of the table of three joint lives (Tab. 5.) be found = 13.194; . . . the *exact* value of three equal lives aged 70 = 9.817, and the *exact* value of three equal lives aged 75 = 7.960. The computation of the values of three joint lives for all ages, is a work of such immense labour, that it is never likely to be accomplished; but the approximation in Prob. IV. renders such labour in a great measure unnecessary. Thus the several values above mentioned, by having recourse to this approximation, instead of the correct values of the three joint lives, appear to be 13.251, 9.853, and 7.967, which are sufficiently near the truth for any useful purpose.

## PROBLEM VI.

To find the value of an annuity granted upon three lives, A. B. C., on condition of its ceasing as soon as any two of them become extinct.

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p. 3

SOLUTION. From the sum of the values of the two joint lives A. B., A. C., and B. C., subtract twice the value of the three joint lives, A. B. and C., and the remainder will be the answer. *

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* See the demonstration, Note IV.

EXAMPLE. Let the ages of A. B. and C. re spectively be 20, 30, and 60, and the rate o interest £4 per cent. By Tab. 4. the values of th joint lives of A. B., A. C. and B. C. are 11.87 7.995, and 7.802, the sum of these numbers 27.670. By Prob. IV. the value of the three join lives is 6.986, the double of which is 13.972, an therefore 13.698 is the value required.

SCHOLIUM. The value of an annuity for an given term, upon the continuance of two lives ou of three, is determined from the solution of th second problem, by substituting in the precedin rule the value of the joint lives *for the given term* instead of the values for their *whole duration.*

## PROBLEM VII.

To determine the sum to which any give annuity, forborn and improved at compound in terest, will amount during the continuance of an given life.

SOLUTION. If the life exists one year, the ac cumulation will be just £1. If it exists two yea the accumulation will be £1 increased by it interest for a year. If it exists three years th accumulation will be £1 increased by its interes for two years, and so on for the other years, s that the whole accumulation will be expressed b the sum of the series arising from the multipl cation of these several amounts of £1, into th respective probabilities that the life exists one

two, three, &c. years; thus, supposing the life to be 30 and the rate of interest £4 per cent. the accumulation will be $\frac{4310}{4385}=.9829$ in the 1st year, the accumulation in the 2d year will be $\frac{4235}{4385}\times 1.04 = 1.0044$, the accumulation in the 3d year will be $\frac{4160}{4385}\times 1.0816 = 1.0261$, and so on. But as this would be a very tedious way of obtaining the amount in every separate case, the following theorem is given, by which a table of the amounts during the lives of persons at all ages may be computed with no more labour than would be required to compute the amount during the continuance of the youngest life. Let Q. be the amount of the annuity computed in the manner described above during the oldest life. Let $\frac{b}{a}$ be the probability that a life one year younger exists a year, and let $r$ be £1 increased by its interest for a year; then will the amount during such younger life be $\frac{b}{a}\times\overline{1+r\,Q.}$ * From the amount, thus obtained, may in like manner be computed the amount during the next younger life, and so on during every other younger life in the table of observations.

## PROBLEM VIII.

To find the value of an annuity during a given life of £1 payable at the end of the 1st year, £2 at the end of the 2d year, £3 at the end of the

* See the demonstration and the Table in Note V.

3d year, and so on; the annuity uniformly i creasing every year till the extinction of the give life.

SOLUTION. This, like the preceding probler will not admit of a solution strictly accurat except by a separate computation of the value each payment of the annuity; so that in order find the value of the first payment, £1 discount for a year must be multiplied into the probabili that the life exists a year; to find the value of tl 2d payment, £2 discounted for two years mu be multiplied into the probability that the li exists two years; to find the value of the payment, £3 must be discounted for 3 yea and multiplied into the probability that the l exists three years, and so on with regard to t other payments till the life becomes extinct the table of observations. Thus, supposing t age of the life to be 35 and the rate of inter £4 per cent. The value of the first payment v be $.9569^{*} \times \frac{3935}{4010} = .9290$, the value of the 2d pa ment will be $.9157 \times \frac{3860}{4010} \times 2 = 1.7629$, the value the 3d payment will be $.8763 \times \frac{3785}{4010} \times 3 = 2.48$ and if these operations be continued to the end life, the whole value will be 176.967. In order avoid these tedious operations in every particu case, a table may be constructed by the followi theorem of the values of this annuity at all ag

* See Tab. 1 and 18.

with no more trouble than would be required to compute the value of the like annuity on the youngest life. Let *b*, *a*, and *r*, denote the same quantities as in Prob. VII.; let R be the value of this annuity computed as above, and N the value of an annuity on the given life; then will the value of an increasing annuity on a life one year younger than the given life, be equal to $\frac{b}{ar} \times \overline{1+R+N}$.* From this value may be computed the value of an annuity on the next younger life, and so in succession of every other younger life in the table.

COROLLARY 1st. If the annuity is to commence with a larger sum than £1, and in the same manner to go on increasing £1 every year, add to the preceding value the value of the given life, multiplied into the first payment lessened by unity, and the *sum* will be the answer. Thus if instead of £1, £2, £3, &c. the annuities had been £15, £16, £17, &c., the value, supposing the life to be 35, and the rate of interest £4 per cent., would have been 176.967 added to 14.039 multiplied into 14, or £373.513.

COROLLARY 2d. From a table computed by the rule in this problem, the value may be determined of an annuity uniformly *decreasing* every year till the life shall become extinct, provided the annuity does not vanish in the meantime.† Let $\alpha$ be the

* See the demonstration, Note VI.

† It should be observed that unless $\frac{\alpha}{\beta}$ be greater than the difference between the age of the given life, and that of the oldest life in the table, this rule fails.

first payment of the annuity, $\overline{\alpha-\beta}$, the second payment, $\overline{\alpha-2\beta}$, the third payment; then will the value of such annuity be $\alpha A-\frac{\beta b}{ar}\times R$, $A$ being the value of an annuity on the given life, $R$ the amount of an increasing annuity on a life one year older, and $\frac{b}{a}$ the probability that the given life exists one year. Thus, if it were required to find the value of an annuity of £70 decreasing £1 annually during the life of a person aged 3- at £4 per cent., it would be equal to $14.195\times$ — $-\frac{4010\times 1}{4085\times 1.04}\times 176.967=993.65-167.03=$ £826.6-

Corollary 3d. By the assistance of the same table the value may be found of £1, payable a life *fails* in the first year, £2 if it fails in the second year, £3 if it fails in the third year, and so on, increasing £1 every year during the whole continuance of such life. Let $A$ be the value of an annuity on the given life, $M$ the value found by this problem of an increasing annuity on the said life, and the required value will be $\frac{1+A}{r}-\frac{\overline{r-1}.M}{r}$ equal, if the age of the life be 35, and the rate of interest £4 per cent., to $\frac{15.039-.04\times 176.967}{1.04}$ $=$ £7.653.*

## Problem IX.

To find the value of an annuity during the remainder of the life of A. after the decease of B.

Solution. From the value of an annuity during the life of A. subtract the value of an annuity o-

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* See the demonstration of this and the second corollary Note VI.

the joint lives of A. and B., and the remainder will be the answer.*

EXAMPLE. Let the respective ages of A. and B. be 35 and 45, and the rate of interest £4 per cent. By Tab. 3. the value of an annuity on the life of A. is 14.039. By Tab. 4. the value of an annuity on the joint lives of A. and B. is 10.622; the difference therefore, or 3.417, is the number of years purchase required.

### PROBLEM X.

To find the value of an annuity on the life of A. after the extinction of the two lives of B. and C.

SOLUTION. From the *sum* of the values of an annuity on the life of A. and of an annuity on the three joint lives, subtract the *sum* of the values of the two joint lives A. B. and of the two joint lives A. C., and the remainder will be the number of years purchase required.†

EXAMPLE. Let the respective ages of A., B. and C. be 15, 50, and 65, and the rate of interest £5 per cent. By Tab. 3. the value of an annuity on the life of A. is 14.588. By Prob. IV. the value of the three joint lives may be found equal to 5.631, the sum of these two values is 20.219. By Tab. 4. the values of the joint lives of A. B. and of A. C. are 9.076 and 6.705, deducting their sum, or 15.781, from 20.219, we have 4.438 for the answer.

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* See the demonstration, Note VII.

† See the demonstration, Note VIII.

## PROBLEM XI.

To find the value of the reversion of the longest of the two lives A. and B. after the decease of C.

SOLUTION. From the value of the longest of the three lives subtract the value of the life of C. and the remainder will be the value required.*

EXAMPLE. Let the ages of the three lives, A. B. C., be 20, 30, and 60 respectively, and the rate of interest £3 per cent. By Prob. v. the value of the longest of the three lives is 22.52. By Tab. 3. the value of an annuity on the life of C. is 9.777, the difference therefore between these two sums, or 12.743, is the answer.

## PROBLEM XII.

To find the value of an annuity during the life of A. after the extinction of the *joint* lives of B. and C.

SOLUTION. Subtract the value of the three joint lives from the life of A. and the remainder will be the answer.*

EXAMPLE. Let the value be required of an annuity of £1 at £4 per cent. during the life of A. aged 30, after the joint lives of B. and C. aged 25 and 60. By Prob. IV. the value of the three joint lives may be found = 6.920. By Tab. 3. the value of the single life of A. is 14.781, the difference therefore, or 7.861, is the required value.

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* See the demonstration, Note VIII.

## Problem XIII.

To find the value of an annuity during the joint lives of A. and B. after the decease of C.

Solution. From the value of the joint lives of A. B. subtract the value of the three joint lives, and the remainder will be the value required. *

Example. Let the ages of A. B. and C. respectively be 40, 50, and 60, the annuity £1, and the rate of interest £5 per cent. By Prob. IV. the value of the three joint lives is 5.690. By Tab. 4. the value of the joint lives of A. B. is 8.177; the former subtracted from the latter, leaves 2.487 for the answer.

## Problem XIV.

To find the value of an annuity during the joint lives of A. and B. which is to be reduced *one half* during the life of the survivor.

Solution. This annuity is equal to half the sum of the values of the two single lives.†

Example. Let the ages of A. and B. respectively be 25 and 32, the annuity £40 which is to be reduced to £20 on the extinction of the joint lives during the life of the survivor, and let the rate of interest be £4 per cent. By Tab. 3. the values of those lives are 15.438, and 14.495, and 14.9665, half their sum multiplied into 40 gives 598.66 for the value required.

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* See the demonstration, Note VIII.

† See the demonstration, Note IX.

If £1,000 were sunk to purchase such an annuity, it should be £66.8 $\left(\text{or } \frac{1000}{14.9665}\right)$ during the joint lives to be reduced to £33.4 during the life of the survivor.

## Problem XV.

Supposing an annuity to be equally enjoyed by A. and B., and that the survivor is to enjoy the whole of it during the remainder of his life, after the decease of the other; to find the value of the interest of each in that annuity.

Solution. Subtract half the value of the joint lives from the value of the life of A. or B. and the remainder will be the value of A. or B.'s interest. *

Example. Let the age of A. be 30, that of B. 45 years, the annuity £1, and the rate of interest £5 per cent. By Table 4. the value of the joint lives is 9.135. By Tab. 3. the value of the life of A. is 13.072, and the value of the life of B. is 11.105, therefore half 9.135 or 4.567, subtracted from 13.072 gives 8.505 for the value of A.'s interest, and 4.567 subtracted from 11.105 gives 6.538 for the value of B.'s interest in the annuity.

## Problem XVI.

An annuity is equally enjoyed by A., B. and C during their joint lives, on the decease of either i

* See the demonstration, Note X.

is in like manner to be equally enjoyed by the two survivors during their *joint* lives, and after their extinction it is to be enjoyed by the last survivor during his life, to find the value of A.'s interest in this annuity.

Solution. From the value of the life of A. added to $\frac{1}{3}$ the value of the three joint lives, subtract half the sum of the values of the joint lives A. B. and A. C., and the remainder will be the value of A.'s interest. *

Example. Supposing the ages of A., B. and C. respectively to be 35, 45, and 50, and the rate of interest £4 per cent. By Tab. 3. the value of the life of A. is 14.039. By Prob. iv. the value of the three joint lives is 7.318, and by Tab. 4. the values of the joint lives of A. B. and of A. C. are 9.706 and 9.110, therefore the value of A.'s interest will be $14.039 + \frac{7.318}{3} - \frac{9.706 + 9.110}{2} = 7.270$ years purchase.

## Problem XVII.

Supposing a given annuity after the decease of A. to be equally divided between B. and C. during their *joint* lives, and then to be enjoyed entirely by the last survivor for life, to find the value of B.'s expectation.

Solution. From the value of an annuity during the remainder of the life of B. after A. found by

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* See the demonstration, Note XI.

Prob. IX. deduct half the value of an annuity during the joint lives of B. and C. after A. found by Prob. XII. and the remainder will give the value of B.'s expectation. *

EXAMPLE. Let the respective ages of A., B. and C. be 45, 35, and 50, and the rate of interest £- per cent. By Prob. IX. the value of an annuity during the life of B. after A. is 3.417. By Prob. XI. the value of an annuity during the joint lives of B. C. after A. is 1.792, therefore $3.417 - \frac{1.792}{2}$ give 2.521 for the number of years purchase required.

## PROBLEM XVIII.

To find the value of an annuity *after* the extinction of any two of the lives, A. B. C., during the continuance of the life of the last survivor.

SOLUTION. To the sum of the values of the three single lives, add three times the value of the three joint lives; and from this sum let twice the sum of the joint lives of A. and B., B. and C., and C. and A. be deducted, and the remainder will be the number of years purchase required. †

EXAMPLE. Supposing the ages of A., B. and C. to be 15, 50, and 65, and the rate of interest £5 per cent. By Tab. 3. the respective values of the three single lives are 14.588, 10.269, and 7.276. By Prob. IV. the value of the three joint lives is 5.631.

* See the demonstration, Note XII.

† See the demonstration, Note XIII.

By Tab. 4. the values of the joint lives of A. B., B. C., and A. C. are 9.076, 5.897, and 6.705; adding therefore the sum of the single lives to three times the value of the three joint lives, we have 49.026, from which deducting twice the amount of the two joint lives (or 43.356) we have 5.69 for the number of years purchase required.

This may in fact be considered as a corollary to the 10th problem; and when the three lives are of equal age, the value of the annuity is immediately obtained from it, being equal to three times the value of A.'s expectation, as found by that problem.

### Problem XIX.

Supposing an annuity to be enjoyed during the life of A., at whose decease B. or his heirs have the nomination of a successor, who is likewise to enjoy the annuity during his life; it is proposed to find the value of the succeeding life.

Solution. From the perpetuity subtract the value of life of A., multiply the remainder by the value of the life to be nominated, divide the product by the perpetuity, and the quotient will be the answer. *

Example. Suppose the age of A. to be 65, and that B. has the power of nominating a life at his decease *then* worth $11\frac{1}{2}$ years purchase; to find the present value at £6 per cent. of such successive life. By Tab. 3. the value of the life of A. is

* See the demonstration, Note XIV.

6.841 which being deducted from 16.667 (the value of the perpetuity), and 9.826 the remainder multiplied into 11.5, we have 112.999; dividing the sum by 16.667, the perpetuity, 6.78, the quotient will be the answer.

COROLLARY I. If instead of a *life* to be nominated, it were required to determine the present value of the reversion of an annuity for a *certain term of years* after the death of A. the operation will be precisely the same; only substituting the value of the annuity for the term of years instead of the value of the succeeding life; so that if in the above example the term had been 26 years, the value of the reversion at the same rate of interest after the death of A. would have been 9.826 multiplied into 13.00 (the value of the annuity certain by Tab. 19.) and divided as above by 16.667 or 7.664.

COROLLARY II. The same rule may be applied to the solution of this problem, when the annuity is enjoyed during the longest of two or three lives or during the joint continuance of two or three lives, and B. and his heirs have the right of nominating one, two, or three lives at their decease. In all cases "the value of the life or lives in possession is to be deducted from the perpetuity, the remainder is to be multiplied into the value of the life or lives *at the time of nomination*, and the product is to be divided by the perpetuity;" so that if the annuity were enjoyed during the longest of two lives aged 40 and 50, and the value of the

two lives nominated at their decease were equal to 15 years purchase, the reversion at £5 per cent. would be $\overline{20-13.93}$ multiplied into $\frac{15}{20}$, or 4.55. *

## PROBLEM XX.

Supposing an estate to be held during the longest of two or three lives, and on the extinction of those lives, that the lease were always to be renewed during other two or three lives at a certain fine; to determine the present value of those fines for ever.

SOLUTION. Find by Prob. III. or Prob. V. the value of an annuity on the longest of two or three lives; find also by the same prob. the value of the lives to be nominated whenever a vacancy happens; subtract the former from the perpetuity, multiply the remainder into the given fine, and the product, divided by the lives to be nominated, will give the value required.

EXAMPLE I. Supposing the lease to be held during two lives of 30 and 45 years, that the ages of the lives to be nominated are always to be 15 years, the rate of interest £5 per cent., and the fines £1000. By Prob. III. the value of two lives of 30 and 45 years is 15.042, the value of two lives each aged 15, is 17.216; the former deducted from 20 leaves 4.958, which being multiplied into 1000,

* It can hardly be necessary to observe, that if the value of the life or lives in possession be added to the value of the life or lives to be nominated at their decease, the *whole* value of the annuity will be obtained.

and then divided by 17.216, gives £2878 for th value required.

EXAMPLE II. Supposing the lease is held durin three lives aged 30, 45, and 60; that the lives t be nominated are to be each 15 years of age, th fines always £1000, and the rate of interest £ per cent.; then the value of the three lives fir mentioned, by Prob. v., will be 15.379, the value the other lives will be 18.244, the former of thes deducted from 20, the perpetuity, leaves 4.62 which being multiplied into 1000, and the produ divided by 18.244, will give £2532 for the answe

## PROBLEM XXI.

Supposing a copyhold estate to be held durin any number of lives, and that a stated fine wer always to be paid upon replacing each of them they become extinct, to find the present value those fines for ever.

SOLUTION. Subtract the sum of the values of al the single lives upon which the copyhold is hel from the perpetuity multiplied by the number o those lives, and reserve the remainder. Find by Table 3. the value of the life to be put in at eac renewal, multiply the reserved remainder int the fine, and the product divided by the value o the life to be nominated will give the value sought.

EXAMPLE. Let the number of lives be three, o the several ages of 35, 45, and 55; let the ag of the life to be always nominated be 12, the fin £1000, and the rate of interest £4 per cent. B

Tab. 3. the values of each of the three lives are 14.039, 12.283, and 10.201; their sum, or 36.523 being deducted from 75, three times the perpetuity, leaves 38.477 for the remainder to be *reserved.* By Tab. 3. the value of a life of 12 years of age is 17.251; 38.477 therefore multiplied into 1000, the stated fine, and divided by 17.251, gives £2230 for the value required.

Corollary. If it were required to determine the annual rent which should be paid as an equivalent for the above fines, their present value must be multiplied into the interest of £1 for a year, and the product will be the answer. Thus, supposing the lives to be as above £2230, their value in one payment being multiplied into .04 (or which is the same thing into $\frac{4}{100}$) will produce £89 4*s.* 0*d.* for the rent required.

## Problem XXII.

Supposing D. to enjoy an annuity during the life of A., E. to have the right of nominating a successor (B.) on the death of A., and F. to have the like right of nominating (C.) the successor of A. and B.; it is required to determine the values of D. E. and F.'s separate interests in this annuity.

Solution. The value of D.'s interest is immediately obtained from the table of the values of annuities on single lives. The value of E.'s interest may be found by Problem xix.; and in

order to find the value of F.'s interest, subtr the sum of the two values above mention from the perpetuity, multiply the remainder in the assumed value of the life of C., and the p duct divided by the perpetuity will be the answe

EXAMPLE. Let the age of A. (as in Prob. XI be 65, and the rate of interest £6 per cent., th will D.'s interest by Tab. 3. be 6.841; E.'s inter by Prob. XIX. will be 6.78, and their sum will 13.621, which being deducted from 16.667 (t perpetuity) and 3.046, the remainder multipli into 11.5 (the assumed value of C.'s life), and t product again divided by 16.667, we have 2.1 1 the value of C.'s interest. These three values add together give 15.721 for the *whole* value of t annuity.*

PROBLEM XXIII.

To find the value of a given sum payable on t decease of A.; or in other words to find the val
2. of an *assurance* of any given sum on the wh duration of the life of A.

(ay SOLUTION. Subtract the value of an annuity
7. the life of A. from the perpetuity, multiply t
124.

* Mr. *Simpson* in his Select Exercises has given a rule determining by one operation the values of all the successive li deduced from the same principles with those on which the pres solution is founded. But as this problem is generally appl towards determining the separate values of alternate presentati to livings, the rule given above is so far preferable. The ope tions in both rules may be continued during any number of s cessive lives..

remainder into the given sum, divide the product by the perpetuity increased by unity, and the quotient will be the answer in a *single* present payment; and this quotient divided by the value of the life, will give the answer in *annual* payments during the continuance of the life, supposing the first of those payments to be made at the *end* of the year; but if the annual payments are to be made at the *beginning* of each year, the above mentioned quotient must be divided by the value of the life increased by unity.

If, instead of a *sum* the value of the reversion of an *estate* were required after the death of A., the value of an annuity on A.'s life must be deducted from the perpetuity, and the remainder multiplied into the annual produce of such estate.

Example. Let it be required to determine the value of an assurance of £1000 on the life of A. aged 20, at £4 per cent., either in a single or in annual payments, to be made at the beginning of each year. By Tab. 3. the value of an annuity on the life of A. is 16.033, which being deducted from 25 (the perpetuity) leaves 8.967; multiplying this number by 1000, and dividing the product by 26 (the perpetuity increased by unity), we have 344.884 for the value of the assurance in *one payment;* and dividing 344.884 by 17.033 (the value of the life increased by unity) we have 20.224 for the same value in *annual* payments. Had it been

required to find the value of the reversion of an *estate* of £40 per annum after the life of A., at the same rate of interest, it would have been equal to 8.967 multiplied into 40, or £358.68. *

COROLLARY. The value of a given sum payable on the extinction of any two or three *joint* lives or of the *longest* of two or three lives, is determined in the same manner; only substituting the value of the *joint* lives, or the *longest* of the lives, instead of the value of the *single* life. Let the value be required, at £3 per cent., of £100 payable on the extinction of two *joint* lives aged 35 and 56. By Tab. 4. the value of the joint lives is 8.9608, which being subtracted from 33.333, the perpetuity, leaves 24.3725; multiplying this number by 100, and dividing the product by 34.333, we have £70.9 for the value sought. Again, let it be required to determine the value of £100 at £3 per cent., payable on the extinction of the longest of two lives aged 42 and 63. By Tab. 3. the values of the two single lives are 14.391 and 8.910. By Tab. 4 the value of the joint lives is 7.372, the value therefore of the *longest* of the two lives is 15.929, which being subtracted from 33.333 leaves 17.404. Hence the required value will be 17.404 multiplied into 100, and then divided by 34.333, or £50.692.

* See the demonstration in Note XV. and in Note E. of the Appendix to Dr. *Price's* Treatise on Reversionary Payments.

## Problem XXIV.

To find the value of a given sum payable at the decease of A., should that happen within a given term; or in other words, to find the value of an *assurance* of any given sum on the life of A. for any given term.

Solution. Find the value of an annuity on the life of A. for one year less than the term by Prob. II. Add unity to this value, and divide the sum by £1, increased by its interest for a year. From the quotient subtract the value of the life for the whole term, found by the same problem, multiply the remainder by the given sum, and the product will be the value required in *one* payment. And if this payment be divided by the annuity found above, for one year less than the term, and increased by unity, we shall have the value of the assurance in *annual* payments to be made at the *beginning* of each year of the term. But if those payments are to be made at the *end* of each year, which is seldom the case, the single payment must be divided by the value of the annuity on the life of A. for the whole term.*

Example. Let the value be required of an assurance of £1000 on the life of A. aged 30, for 7 years at £3 per cent. By Prob. II. the value of the life for 6 years is found equal to

* See the demonstration, Note XVI.

5.1009; hence 6.1009 divided by 1.03 is=5.9223. The value of the life for 7 years, by the same problem, is 5.8166, which being subtracted from 5.9223, and the remainder (.10574) multiplied by 1000 will give £105.74, for the value of the assurance in one payment. And 105.74 divided by 6.1009, or 5.8166 will give either 17.32 or 18.18 for the *annual* payments, according as they are to be made at the *beginning* or the *end* of each year; but subject in both cases to failure, if the life should fail.

Corollary i. In like manner may the value of an assurance on any two or three *joint* lives, or on the *longest* of two or three, or two out of three lives for any term be computed, only substituting the value of the *joint* lives, or of the longest of the lives, or of two out of three lives, instead of the single life in the foregoing rule. This will be better understood from the following examples.

Example I. Let the value be required either in *one*, or in *annual* payments of an assurance of £1000 on two joint lives, each aged 30, for the term of 15 years, at £4 per cent. The value of the joint lives for 14 years, and also for 15 years, may be found by Cor. 1. Prob. ii. to be equal respectively to 8.270 and 8.580, adding unity to the former, and dividing 9.27 the sum by 1.04 we have 8.913. Deducting 8.58 from this quotient, and multiplying .333, the remainder, into 1000, we have £333 for the value in *one* payment;

which being divided by 9.27 (found above) gives £35.92 for the same value in *annual* payments.

EXAMPLE II. If the assurance were made during the longest of two lives of the same age, and for the same term, the value of an annuity on those lives for 14 and 15 years, by Cor. 1. Prob. II. being equal to 10.366 and 10.872 respectively, the value in one payment will be $\left(\frac{11.366}{1.04}=\right)$ 10.929 lessened by 10.872, (or .057) multiplied into 1000, which is equal to £57; and 57 divided by 11.366 (the value of the longest of the two lives for 14 years, with unity added) gives £5.015, for the answer in annual payments to be made at the *beginning* of each year of the term, and subject to failure should *both* the lives fail in 14 years.

COROLLARY II. Supposing, instead of a *sum*, that the reversion of an *estate* depended on the contingency of A.'s dying in a given time, the value of such reversion may be found by the following rule:* "Subtract the value of A.'s life for the "given term from the value of an annuity certain "for the same term, and reserve the remainder. "Multiply the perpetuity into the probability that "A. *dies* in the given term, and also into £1 dis-"counted for that term. Add this product to "the reserved remainder, and the sum will be the

* See Dr. Price's 14th question in his Treatise on Reversionary Payments. p 51

"value of the reversion in years* purchase." Thus the value of the life of A. aged 30, for 20 years at £3 per cent., is 12.436. The value of an annuity certain for 20 years, by Tab. , is 14.877, the remainder to be reserved therefore is 2.441. The perpetuity is 33.333, the probability that A. dies in 20 years is $\frac{1528}{4385}$; the value of £1 payable at the end of 20 years by Tab. 18, is .5537. These three quantities multiplied into each other produce 6.431, which being added to 2.441, *the reserved remainder*, gives 8.87 for the number of years purchase required. By proceeding in the same manner the value of this contingent reversion after the *joint* lives, or the longest of two or three lives, may be obtained.

## Problem XXV.

To find the value of a given *sum* payable on the contingency of B. surviving A.

Solution. Let P. denote a life one year older than B., and H. denote a life one year older than

* This rule requires no demonstration, as it is self-evident. It may not be improper to observe, that the value in this case may likewise be derived from the rule in the problem.

"Add unity to the value of the life for one year less than the "given term. Multiply the value of the life for the given term "by £1 increased by its interest for a year. Subtract this pro-"duct from the former sum, divide the remainder by the interest "of £1 for a year, and the quotient will be the answer." Hence the value in the example to this corollary will be $\overline{1+12.075-12.436\times 1.03}$ (=.266) divided by .03, or 8.87.

A., multiply the value of the joint lives of A. and B. into the interest of £1 for a year, deduct the product from unity and reserve the remainder. Multiply the value of the joint lives of A. and P. increased by unity into the probability that B. lives a year. Multiply also the value of the joint lives of B. and H, increased by unity into the probability that A. lives a year. Add the former of these values to the reserved remainder, and from this sum subtract the latter value. Divide *half* this new remainder* by £1 increased by its interest for a year, and the quotient multiplied into the given sum will be the value of the reversion. †

* If, instead of a *sum*, the value of an *estate* is required, this *half*-remainder must be divided by the interest of £1 for a year; thus, in the following example .2228 divided by .03 gives 7.427 for the number of years purchase an estate is worth, depending on the contingency of B. aged 56 surviving A. aged 35.

† Mr. *Simpson* in his Select Exercises has given the following rule (deduced from M. *De Moivre*'s hypothesis) for determining the value of this contingent reversion. "Find the value of an "annuity on two equal joint lives, whereof the common age is "equal to the age of the older of the two proposed lives A. and "B.; which value subtract from the perpetuity, and take half "the remainder; then say, as the expectation of the duration of "the younger of the two lives A. and B., is to that of the elder, "so is the said half-remainder to a fourth proportional, which "will be the number of years purchase required, when the life B. "in expectation is the older of the two. But if B. be the "younger, then add the value so found to that of the joint lives "A. and B., and let the sum be subtracted from the perpetuity, "and you will also have the answer in this case."

This

EXAMPLES. Suppose the age of A. to be 35 and that of B. 56, the sum £100, and the rate of interest £3 per cent. By Tab. 4. the value of the two joint lives of A. and B. is 8.9608, which being multiplied into .03, and the product subtracted from unity, the remainder to be *reserved* will be .73118. The value of the joint lives of A. and P. by Tab. 4. is 8.785, the probability that B. lives a year is $\frac{2284}{2366}$, and $\frac{9.785 \times 2284}{2366} = 9.4459$. By Tab. 4. the value of the joint lives of B. and H. is 8.9167, the probability that A. lives a year by Tab. 1. is $\frac{3935}{4010}$, and $\frac{9.9167 \times 3935}{4010}$ is equal to 9.7315. Hence 9.7315 deducted from the sum of .73118 and 9.4459 (or 10.1771) leaves .4456, of which the half is .2228, which being

This rule is sufficiently correct in the middle stages of life; but like all the other rules deduced from De *Moivre*'s hypothesis, gives very inaccurate results in the earlier and latter stages of life. It should also be observed that the above rule gives the value of an *estate*. If the value of a *sum* is required, the number of years purchase, found above, must be multiplied into that sum, and divided by the perpetuity increased by unity. Thus in the example given above, the value of an *estate* depending on the contingency of B. aged 56 surviving A. aged 35, is by Mr. *Simpson*'s rule equal to 7.604 years purchase, which being multiplied into 100, and then divided by 34.333 will give £22.15 for the value of £100 to be received on this contingency. The value in *annual* payments, according to either rule, will be the *quotient* arising from the division of the value in a single payment by the value of the two joint lives, or by the value of the joint lives increased by unity, according as those payments are to be made at the *end* or the *beginning* of each year. See the demonstration in Note XVII.

divided by 1.03, and the quotient multiplied into 100, gives £21.631 for the value required.

If the age of A. be 56, and that of B. 35, the reserved remainder, as above, will be .73118, the sum to be *added* to this remainder will be 9.7315, and the sum to be *deducted* will be 9.4459; so that the *new* remainder will be 1.0168. Half this sum divided by 1.03, and multiplied into 100 will produce £49.36 for the value in this case. The sum of these two values is £70.99, which is also the value of the reversion after the extinction of the two joint lives (by the Corollary to Prob. XXIII.), and therefore proves the truth of all the operations.

## Problem XXVI.

To find the value of a given sum payable on the decease of B., provided that should happen *after* the decease of A.

Solution. From twice the value of the single life B., deduct the value of the joint lives A. B., multiply the remainder into the interest of £1 for a year, subtract the product from unity and *reserve* this remainder. Let P. and H. denote the same quantities as in the preceding problem, then multiply the value of the joint lives of A. and P. increased by unity, into the probability that B. lives a year; multiply also the value of the joint lives B. H., increased by unity, into the probability that A. lives a year. Subtract this latter product from the sum of the former product added to the

*reserved* remainder. Divide *half* this *new* remainder * by £1 increased by its interest for a year, and the quotient multiplied into the given sum will produce the answer. †

EXAMPLES. Let the age of A. be 42, that of B. 63 years, the sum £100, and the rate of interest £3 per cent. By Tab. 3. the value of the life of B. is 8.91. By Tab. 4. the value of the joint lives of A. B. is 7.372; therefore 17.82 lessened by 7.372, and multiplied into .03 is .31344, which being subtracted from unity leaves .68656 for the *reserved* remainder. By Tab. 4. the value of the joint lives of A. P. is 7.1719, the probability that B. lives a year is $\frac{1712}{1793}$, and 8.1719 multiplied into this probability is 7.8027. By Tab. 4. the value of the joint lives B. H. is 7.3327. The probability that A. lives a year by Tab. 1. is $\frac{3404}{3482}$, and 8.3327 multiplied into this fraction is 8.1460. Adding 7.8027 to .68656, we have 8.48926; from which deducting 8.146, the *new* remainder will be .34326. Hence .17163 divided by 1.03, and multiplied into 100, gives £16.663 for the value of the reversion. If the ages be inverted, so that A. may be 63 and B. 42 years of age, the value of B.'s life in this case by Tab. 3. will be 14.3911, and the

* The value of an *estate*, as in the former problem, is found by dividing this *half*-remainder by the interest of £1 for a year.

† The value in one sum divided by the value of an annuity on the longest of the two lives increased by unity, will give the value of this reversion in *annual* payments, to be made at the *beginning* of each year. See the demonstration, Note XVIII.

reserved remainder by proceeding as above, will be found equal to .35769. The sum to be *added* in this case will be 8.146 making 8.50369, and the sum to be deducted 7.8027, leaving .70099; the *half* of which being divided by 1.03, and then multiplied into 100 will give 34.029 for the value of the reversion. Adding both the values, or 16.663 and 34.029 we have 50.692 for the value of the reversion after the extinction of both lives without any restriction as to survivorship, which also is the value by the direct rule, as given in the corollary to Prob. XXIII.*

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* Mr. *Simpson* in his Select Exercises has given the following solution of this problem, derived as in that of the foregoing problem, from *De Moivre*'s hypothesis.

"Find the value of an annuity upon the longest of two equal "lives, whereof the common age is that of the older of the lives "of A. and B., which value subtract from the perpetuity, and "take half the remainder. Then it will be as the expectation of "duration of the younger of the lives A. and B. is to that of the "older, so is the said half-remainder to the number of years "purchase required, when the life B. is the older of the two. "But if B. be the younger, then to the number thus found, add "the value of an annuity on the longest of the lives A and B., "and subtract the sum from the perpetuity for the answer in "this case."

In the example given above, the value of an *estate* on the decease of B. aged 63, provided he shall have survived A. aged 42, is equal to 5.747 years purchase by Mr. *Simpson*'s rule, and consequently the value of £100 will be 574.7 divided by 34.333 or £16.742.



This,

THE solution of the following problems, from the great number of contingencies on which the values of the reversions depend, are necessarily very complicated, and render it impossible, if strict accuracy is required, to deduce such general rules as shall not be attended with considerable labour and difficulty. Having no correct tables of the probabilities of human life, Mr. *Thomas Simpson* availed himself of *De Moivre's* hypothesis * to approximate to the values of those reversions in a few cases, observing very wisely that it would be improper to aim at absolute exactness, while the precise law of the decrements of life was unknown. In the former edition of this work the solutions of a great number of the problems involving three lives, were derived from one of Mr. *Simpson's* approximations, and in those cases, where none of the lives are either very young or very old, the rules are tolerably correct. But it seldom happens in survivorships of this kind that all the lives are nearly of equal age: on the contrary, one of them is generally either very young or very old, and these rules are in consequence rendered so far inaccurate. Being possessed, by means of Dr *Price's* Treatise on Reversionary Payments, of those tables of the real probabilities of life, of which

* This, like the preceding rule, being derived from an hypothesis not universally true, is equally incorrect in the earlier and latter stages of life.

Mr. *Simpson* regreted the want; I was induced, about 30 years ago, to attempt the solution of the different problems involving two and three lives in the survivorship, on principles strictly true according to any table of observations. These were communicated to the Royal Society at different times, and published in the Philosophical Transactions.* Since the publication of those papers I have revised the whole of them, and with the exception of a few typographical errors, have had the satisfaction of finding the solutions perfectly correct. By altering some of the symbols, the general rules have been rendered less prolix, but when expressed in words at length, they are still rather complicated and laborious; nor should I have given them in this form, had I not determined to avoid all algebraical characters in the practical part of the work. The mathematical reader however will find less difficulty, by having recourse to the notes, which contain the demonstrations of these problems, and give the rules in an algebraical form.

## PROBLEM XXVII.

To determine the value of a given sum payable on the contingency of C. surviving B., provided the life of A. shall be then extinct.

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* See the Phil. Trans. for the years 1788, 1789. 1791. 1794, and 1796.

Solution. Let H. P. and T. represent lives one year older than A. B. and C. respectively. Multiply the value of the three joint lives A. B. C. (found by Prob. iv.) into the interest of £1 for a year; deduct the product from unity, and dividing the remainder by 3, reserve the quotient. Multiply the value of the three joint lives H. B. C. increased by unity into the probability that A. lives a year. Multiply the value of the three joint lives A. B. T. increased by unity into the probability that C. lives a year; divide the sum of these two products by 6, and reserve this second quotient. Multiply the value of the three joint lives H. B. T. increased by unity into the probability that A. and C. *both* live a year; add one-third of the product to the two quotients reserved above, and let the *sum* be reserved. Multiply the value of the three joint lives H. P. C. increased by unity into the probability that A. and B. *both* live a year. Multiply the value of the three joint lives A. P. T. increased by unity into the probability that B. and C. *both* live a year. Divide the sum of these two products by 6, and reserve the quotient. Multiply the value of the three joint lives A. C. P. increased by unity into the probability that B. lives a year. Add $\frac{1}{3}$ of the product to the *quotient* last reserved. Deduct the amount from the *sum* reserved above. Divide the remainder by £1 increased by its interest for a year, and let this quotient again be reserved.

Find by Prob. XXV. the value of the given sum depending on the contingency of C. surviving B., from which deducting the quotient just reserved, the remainder will be the value required. *

EXAMPLE. Let the ages of C. B. and A. respectively be 69, 44, and 14, the rate of interest £3 per cent., and the given sum £100. By Prob. IV. the value of the joint lives A. B. C. is 5.6417†, which being multiplied into .03 (the interest of £1 for a year) is equal to .169251; subtracting this from unity, and dividing the remainder by 3, we have .276916. The value of the three joint lives

* See the demonstration, Note XIX.

† Particular care should be taken to find the value of the three joint lives with all possible exactness; thus the value of the two joint lives B. C. being 6.0087 is equal to the value of a single life D. aged 72.$\frac{1}{4}$, and the value of the two joint lives A. D. by taking a mean between the values of two joint lives aged 14 and 72, and of two joint lives aged 14 and 73, is 5.6417. Again, the value of the two joint lives H. D. is a mean between the values of two joint lives aged 15 and 72, and two joint lives aged 15 and 73. In like manner the value of the two joint lives B. T. is 5.7758, which is equal to the value of a life D. aged 73 exactly; therefore the value of the three joint lives A. B. T. is equal to the value of two joint lives aged 14 and 73, or 5.4584. By proceeding as above the values of the joint lives H. P. C. may be found equal to two joint lives aged 15 and 72.$\frac{3}{5}$ or 5.5498, the value of the joint lives A. P. T. equal to the value of two joint lives aged 14 and 73.$\frac{1}{10}$, or 5.4314, and so on.

H. B. C. increased by unity is 6.6227, the probability that A. lives a year is $\frac{5423}{5473}$, which, being multiplied by 6.6227, produces 6.5622. The value of the three joint lives A. B. T. increased by unity is 6.4584, the probability that C. lives a year is $\frac{1232}{1312}$, therefore 6.4584 multiplied into this fraction is equal to 6.0646; the sum of these two products, or 12.6268, divided by 6, quotes 2.104467. The value of the three joint lives H. B. T. increased by unity is 6.4404, the probability that A. and C. *both* live a year is $\frac{5423 \times 1232}{5473 \times 1312}$, which multiplied by 6.4404 produces 5.99244. This divided by 3 quotes 1.99781. The *sum* therefore to be *reserved* is composed of the three quotients .276916, 2.104467, and 1.99781, and amounts to 4.379193. Again, the value of the three joint lives H. P. C. increased by unity is 6.5498. The probability that A. and B. both live a year is $\frac{5423 \times 3248}{5473 \times 3326}$; this fraction multiplied by 6.5498 produces 6.33776. The value of the three joint lives A. P. T. increased by unity is 6.4314, the probability that B. and C. both live a year $\left(\text{or } \frac{3248 \times 1232}{3326 \times 1312}\right)$ multiplied into this value produces 5.89761; these two products, (or 12.23537) divided by 6, quote 2.03923 to be reserved. The value of the three joint lives A. P. C. increased by unity, is 6.5684. The probability that B. lives a year is $\frac{3248}{3326}$, which being multiplied into 6.5684, and the product divided by 3, we have 2.13812. Adding this to 2.039231 (the quo-

tient just reserved) and deducting the amount, or 4.177351 from 4.379193, the *sum* reserved above, there remains .201842, which being divided by 1.03 (or £1 increased by its interest for a year) quotes .19596 to be reserved, or £19.596, supposing the given sum to be £100. By Prob. xxv. the value of £100 payable on the contingency of C. surviving B. is 21.359, deducting therefore 19.596 from this sum, the remainder or £1.763 will be the value required.

Scholium. As the solutions of the 24 following problems are all derived from the same joint lives increased by unity, and multiplied into the same probabilities respectively, perhaps the general rules may be more simply expressed by adopting the following symbols:

*H.* B. C. the value of the three joint lives increased by unity and multiplied into the probability that A. lives a year.

A. B. *T.* the value of the three joint lives increased by unity and multiplied into the probability that C. lives a year.

B. *H.* *T.* the value of the joint lives increased by unity and multiplied into the probability that A. and C. live a year.

*H.* *P.* C. the value of the joint lives increased by unity and multiplied into the probability that A. and B. live a year.

A. *P.* *T.* the value of the joint lives increased

by unity and multiplied into the probability that B. and C. live a year.

A. C. *P.* the value of the joint lives increased by unity and multiplied into the probability that B. lives a year.

The preceding general rule in this case will be expressed as follows:

" Multiply the value of the joint lives A. B. C. " into the interest of £1 for a year; deduct the " product from unity, and dividing the remainder " by 3, reserve the quotient. Add $\frac{1}{6}$ of the sum " of *H.* B. C., and A. B. *T.* to $\frac{1}{3}$ of B. *H.* *T.* and " also to the quotient just mentioned, and *reserve* " the *sum.* Add $\frac{1}{6}$ of *H.* *P.* C. and A. *P.* *T.* to $\frac{1}{3}$ " of A. C. *P.*, subtract the amount from the *re-* " *served sum,* divide the remainder by £1 increased " by its interest for a year; let this quotient be " deducted from the value of the given sum on " the contingency of C. surviving B., and the re- " mainder will be the value required."

As this problem may be applied to the solution of several of the succeeding problems, I shall insert the following approximation of Mr. *Simpson's*, which was given in the former edition of this work as the only solution then published; and had its correctness in all cases been equal to its simplicity, it would have been better adapted for practice than the preceding solution.

" CASE I.—*If the life C. be the oldest of the three.*

"From the value of an annuity on the life C. take "the value of the two joint lives B. and C., multiply the remainder by the given sum, and divide "the product by twice A.'s expectation, the result "will be the value sought."

"CASE II.—*If the life B. be the oldest of the three.* "Then from the value of an annuity for as many "years of C.'s life as are expressed by the double "of B.'s expectation, subtract the value of the "two joint lives B. and C.; multiply the remainder "by the given sum, and divide the product by "twice the expectation of A., as in the preceding "case."

"CASE III.—*If the life A. be the oldest of the "three:* Then find the value of the life of C., if "older than B., otherwise find the value of as "many years thereof as are expressed by the "double of B.'s expectation: And from the value "thus found, let the value of the joint lives A. "and C. be subtracted; multiply the remainder "by the given sum, then the product divided by "twice B.'s expectation will be the answer in this "case."

The following examples, computed at £3 per *cent.* from the Northampton Table, will shew how far the above approximation agrees with the correct rule.

| AGES. | | | Value of £100 by the rule, when C. is the oldest. | Value by the appro mation, when C. the oldest. |
|---|---|---|---|---|
| C. | B. | A. | | |
| 80 | 70 | 40 | 1.980 | 2.04 |
| 75 | 65 | 24 | 2.394 | 1.98 |
| 65 | 50 | 15 | 2.980 | 2.82 |
| 78 | 78 | 20 | 2.672 | 2.725 |
| 60 | 45 | 12 | 2.733 | 2.591 |
| 69 | 44 | 14 | 1.763 | 1.40 |
| 75 | 24 | 65 | 5.899 | 6.930 |

| AGES. | | | Value of £100 by the rule, when B. is the oldest. | Value of £100 by t approximation, wh B. is the oldest. |
|---|---|---|---|---|
| C. | B. | A. | | |
| 70 | 80 | 40 | 4.646 | 5.870 |
| 50 | 65 | 15 | 5.816 | 6.326 |
| 65 | 75 | 24 | 3.057 | 5.180 |
| 44 | 69 | 14 | 4.923 | 6.555 |
| 14 | 69 | 44 | 16.092 | 13.800 |

| AGES. | | | Value of £100 by the rule, when A. is the oldest. | Value of £100 by t approximation, wh A. is the oldest. |
|---|---|---|---|---|
| C. | B. | A. | | |
| 24 | 65 | 75 | 36.768 | 39.366 |
| 65 | 24 | 75 | 7.878 | 6.925 |
| 20 | 78 | 78 | 36.022 | 39.636 |
| 14 | 44 | 69 | 24.986 | 20.490 |
| 44 | 14 | 69 | 9.542 | 10.655 |

| AGES. | Value of £100 by the rule, when the three lives are equal. | Value of £100 by the approximation, when the three lives are equal. |
|---|---|---|
| 70 each | 12.90 | 14.41 |
| 75 each | 13.80 | 15.93 |
| 80 each | 14.51 | 17.47 |
| 85 each | 15.09 | 19.45 |
| 7 each | 5.678 | 5.445 |

In those cases where C. is the youngest, and A. the oldest of the three lives (and which are by far the most common) the approximation appears to differ most from the truth: But when C. is the oldest and A. the youngest, a case however which seldom occurs, the values are in reality so small and the difference between the rule and approximation so inconsiderable, that it is of very little consequence which of them is adopted.

## Problem XXVIII.

To find the value of a given sum payable on the decease of A. if his life should be the *first* that fails of the three lives, A. B. and C.

Solution. Retaining the same symbols as in the scholium to the preceding problem, multiply the value of the joint lives A. B. C. into the interest of £1 for a year, deduct the product from unity, and dividing the remainder by 3, reserve the quotient. Add $\frac{1}{6}$ of the sum of A. C. *P.* * and

* See Schol. to Prob. XXVII.

A. B. *T.* to $\frac{1}{3}$ of A. *P. T.* and also to the quotient just mentioned, and reserve the *sum.* Add $\frac{1}{3}$ of *H.* B. C. to $\frac{1}{6}$ of B. *H. T.* and *H. P.* C., subtract the amount from the reserved *sum,* divide the remainder by £1 increased by its interest for a year, multiply this quotient into the given sum, and the product will be the value required. *

EXAMPLE. Let the age of A. be 24, that of B. 65, and that of C. 75 years. The sum £1000, and the rate of interest three per *cent.* The value of the three joint lives A. B. C. by Prob. IV. is 3.7315, which being multiplied into .03, then the product subtracted from unity and the remainder divided by 3, the quotient to be reserved will be 2.9602. The value of A. C. *P.* is 4.43214, which being divided by 6 gives .73869. The value of A. B. *T.* is 4.11612, which being divided by 6, quotes .68602. In like manner the value of A. *P. T.* divided by 3 is 1.29455. These four quantities added together amount to 3.01528 for the *sum* to be reserved. The value of *H.* B. C. (or 4.65426) divided by 3 is 1.55142. The value of *H.* B. *T.* (or 4.049238) divided by 6 is .674873. And the value of *H. P.* C. (or 4.359834) divided by 6 is .726639. Adding these three last mentioned quotients we have 2.95293, which being subtracted from 3.01528, the reserved *sum,* leaves

* See the demonstration in Note XX.

.06235 for the remainder. Dividing this remainder by 1.03, and multiplying .06053, the quotient, by 1000, the product, or £60.53, will be the value required.

The solution of this problem may also be derived from the solutions of the preceding and of the 25th problems.—" Find by the preceding problem " the value of the given sum on the decease of A. " provided B. (the younger of the two lives of B. " and C.) should be then living, and C. dead; " find next by the 25th problem the value of the " given sum payable if B. is living at the decease " of A., whether C. be then living or not, subtract " the former value from the latter, and the re- " mainder will be the answer." Thus, in the present case, the value of £1000 payable on the contingency of B. surviving A. *after* C. is 78.78 by Prob. XXVII. And the value of £1000 payable on the contingency of B. surviving A. without any other restriction, is by Prob. XXV. = 139.31. Deducting 78.78 from this sum, we have £60.53 for the answer as before.

## Problem XXIX.

To find the value of a given sum payable on the death of A. if his life should be the *second* that fails of the three lives A. B. and C.

Solution. Find by Prob. XXV. the value of the given sum payable on the contingency of B. surviving A. Find by the same problem the value of

the given sum payable on the contingency of C. surviving A. Find by the preceding problem the value of the given sum payable on the death of A. if his life should be the *first* that fails of the three lives. From the sum of the two former values subtract twice the value just found, and the remainder will be the answer. *

Example. Let the ages, as in the preceding problem, of A. B. and C. respectively, be 24, 65, and 75, the sum £1000, and the rate of interest £3 per cent. By Prob. xxv. the value of £1000 payable on the contingency of B. surviving A. is 139.31, and the value of the same sum payable on the contingency of C. surviving A. is 87.08. By Prob. xxviii. the value of £1000 payable if A. should be the *first* that fails is 60.53, the double of which is 121.06. Deducting this from 226.39, the sum of the two former values, we have £105.33 for the value required.

## Problem XXX.

To determine the value of a given sum payable on the death of A. if his life should be the *last* that fails of the three lives A. B. and C.

Solution. Find by Problem xxiii. the value of the given sum payable on the death of A. Find by Problem xxviii. the value of the given sum payable on the death of A., if his life should be the

* See the demonstration, Note XXI.

the *first* that fails of the three lives A. B. and C. Find next by Prob. xxv. the value of the given sum payable if B. should survive A., and also the value provided C. should survive A. The sum of these two last mentioned values being subtracted from the sum of the two former ones, the remainder will be the value required. *

Example. Let the ages of A. B. and C. respectively be 14, 44, and 69, the sum £1000, and the rate of interest £3 per cent. By Prob. xxiii. the value of £1000 payable on the death of A. is 392.07. By Prob. xxviii. the value of £1000 payable on the death of A. if his life should be the *first* that fails may be found equal to 77.83, the sum of these two values is 469.9. By Prob. xxv. the values of £1000 payable if B. survives A., and also if C. survives A. may be found respectively equal to 173.25 and 107.95. The sum of these, or 281.2, being subtracted from 469.9 reserved above, leaves £188.7 for the value required.

## Problem XXXI.

To find the value of a given sum payable on the decease of A. and B., should they be the *first* that fail of the three lives A. B. and C.

Solution. Find by Prob. xxv. the value of the given sum payable on the contingency of C. surviving A., and also the value of the same on the

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* See the demonstration, Note XXII.

contingency of C. surviving B. Let the sum of these two values be *reserved.* Multiply the value of the three joint lives A. B. C. into the interest of £1 for a year, deduct the product from unity, and reserve *two thirds* of the remainder. To one sixth of the sum of *H. T.* B. and A. *P. T.* * add one third of A. B. *T.*, let their amount be added to the remainder just mentioned, and then let this *sum* be reserved; to $\frac{1}{3}$ of *H. P.* C., add $\frac{1}{6}$ of *H. B.* C. and of A. C. *P.* From the amount let the reserved *sum* be deducted, and the remainder be divided by £1 increased by its interest for a year, this quotient multiplied into the given sum, and deducted from the sum of the two values first *reserved* will give the value required.† The value of this reversion may be obtained, though not so expeditiously, from the solution of the 27th problem, by finding the value of the given sum on the contingency of C. surviving A. *after* B., and also on the contingency of C. surviving B. *after* A. The sum of these two values will be the value required. ‡

EXAMPLE. Let the ages of A. B. and C. respectively be 45, 12, and 60 the sum £1000, and the rate of interest £3 per cent. By Problem XXV. the value of the given sum depending on the con-

* See Schol. to Prob. XXVII.

† See the demonstration, Note XXII.

‡ See the demonstration, Note XXIII.

tingency of C. surviving B. is 249.66, and the value on the contingeney of C. surviving A. is 113.70, the sum of these two values is 363.36. The value of the joint lives A. B. C. or 7.238, by Prob. IV. multiplied into .03, and the product .21714 deducted from unity leaves .78286, which being multiplied into $\frac{2}{3}$ produces .52190. The quantities *H. T.* B. and A. *P. T.** are respectively equal to 7.63967 and 7.48062; their sum divided by 6 gives 2.52005 which being added to $\frac{1}{3}$ of A. B. *T.*, $\left(\text{or } \frac{7.7338}{3}\right)$ amounts, including the reserved product .52190, to 5.61989. *H. P.* C. may be found = 7.89452 ... A. C. *P.* = 7.98649, and *H.* B. C. = 8.14328; adding $\frac{1}{3}$ of the former to $\frac{1}{6}$ of the sum of the two latter values we have 5.31979, which being subtracted from 5.61989, reserved above, leaves .3001. This remainder being multiplied into 1000 and divided by 1.03 gives 291.36, which being deducted from 363.36, first reserved, we have £72 for the value required.

Had the value been computed from the 27th Problem, the result would have been exactly the same; for the value of the given sum by that Problem, on the contingency of C. surviving B. *after* A. is 27.3, and the value on the contingency of C. surviving A. *after* B. is 44.7, making together £72. Adopting Mr. Simpson's approximation, instead of the 27th Problem, the value by that rule will be only £47.63.

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* See Schol. to Prob. XXVII.

## Problem XXXII.

To find the value of a given sum on the death of A. provided B. should survive him in any given number of years.

Solution. Let the sum of the decrements of life from the age of A. during the term, be divided by the number of years in that term; let this again be divided by twice the number of persons living at the age of A., and *reserve* the quotient. Find the value of an annuity on the life of B. for one year less than the term. Add unity to such value and divide the sum by £1 increased by its interest for a year. To this quotient add the value of an annuity on the life of B. for the whole term: Multiply the sum into the *reserved* quotient, and this product being again multiplied into the given sum will be the value required.

Example. Let the ages of B. and A. respectively be 65 and 30, the sum £100, the rate of interest £4 per cent., and the term 10 years. By the Northampton Table the sum of the decrements of life for 10 years from the age of A. is 750. This being divided by 10, and the quotient (75) being again divided by 8770 (twice the number of persons living at the age of A.) will give .00855 for the quotient to be *reserved*. The value of an annuity on the life of B. for 9 years may be found by Prob. II. equal to 5.708. Hence 6.708 divided by 1.04 will give 6.45. Adding this quotient to 6.052, the value of an annuity on the life of B. for

10 years, we have 12.502; which being multiplied into .00855 the *reserved* quotient, and also into 100 the product £10.69 will be the answer. If the respective ages of B. and C. are 30 and 65, the value of £100 may, by pursuing the same steps, be found equal to 36.56. The sum of these two values is 47.25, which may be found to be the value of £100 payable on the extinction of the two joint lives in ten years, and therefore affording a proof of the accuracy of the rule.*

## Problem XXXIII.

To find the value of a given sum payable on the death of B. after A., should that happen in any given number of years.

Solution. Divide the sum of the decrements of life from the age of B. during the given term by the number of years in that term, and let this quotient again be divided by twice the number of persons living at the age of B. *reserving* this second quotient. From the value of an annuity certain, subtract the value of an annuity on the life of A., both for one year less than the given term; divide the difference by £1 increased by its interest for a year. To this quotient add the difference between the value of an annuity certain, and the value of an annuity on the life of A., both for the given term. Multiply

* See the demonstration in Note XVII.

the sum into the *reserved* quotient, and also into the given sum, and the product will be the answer.

EXAMPLE. Let the ages of B. and A. respectively be 60 and 35, the sum £100, the rate of interest £4 per cent., and the term 12 years. The decrements of life by Tab. 1. from the age of B. divided by 12 will be 80.5, and this again divided by 4076, or twice the number of persons living at the age of B., will give .01975 for the quotient to be *reserved.* The value of an annuity on the life of A. for 11 years by Prob. II. is 7.493, which being subtracted from 8.760, the value of an annuity certain for the same time by Tab. 19, and the remainder divided by 1.04, we have 1.218. The value of an annuity on the life of A. for 12 years by Prob. II. is 7.971, which being deducted from 9.385, the value of an annuity certain for the same time by Tab. 19., the difference will be 1.414. Adding this quantity to 1.218, found above, and multiplying 2.632, the sum, into .01975 the *reserved* quotient, the product will be .05198, and consequently the value of £100, the given sum, will be £5.198. If the ages of B. and A. are 35 and 60, the value may be found equal to £4.01. The sum of these two values is £9.208, which is also very nearly the value of the reversion after the extinction of both lives, should that happen in 12 years.

## Problem XXXIV.

To find the value of a given sum payable on the decease of A., if his life should be the *first*, or *second* that fails of the three lives A. B. and C.

Solution. Find by Prob. xxv. the value of the given sum payable on the contingency of C. surviving A. Find also by the same problem the value of the given sum payable on the contingency of B. surviving A. Lastly, find by Prob. xxviii. the value of the like sum payable on the death of A., if his life should be the *first* that fails. From the sum of the two former values deduct the latter, and the remainder will be the value required.

Example. Let the ages of A. B. and C. as in the Problem xxxi. be respectively 45, 12, and 60, the sum of £1000, and the rate of interest £3 per cent. The value by Prob. xxv. on the contingency of C. surviving A. is 249.66. The value by the same problem on the contingency of B. surviving A. is 463.44; the sum of these two values is 713.10. By Prob. xxviii. the value of the given sum payable on the contingency of A. being the *first* that fails of the three lives is 222.33, which being deducted from 713.10 reserved above we have £490.8 for the required value of the reversion.

Remark. The value of this reversion may also be obtained from the 28th and 29th Problems;

it being evidently equal to the sum of the values of the given sum payable if A. should be the *first*, and also payable if A. should be the *second* that fails of the three lives. Thus, in the present case, the value of the given sum payable if A. should be the *first* that fails appears to be 222.33, and by Problem XXIX. the value payable if A. should be the *second* that fails may be found equal to 268.48 the sum of these, or £490.8 is, as before, the value required.

## Problem XXXV.

To find the value of a given sum payable on the decease of *A*. if his life should be the *second* or *third* that fails of the three lives A. B. and C.

Solution. From the value of the reversion of the given sum after the decease of A., found by Prob. XXIII., deduct the value of the same payable if A. should be the *first* that fails, found by Prob. XXVIII., and the remainder will be the value required. *

Example. Let the ages of A. B. and C. respectively be 24, 65, and 75, the sum £1000, and the rate of interest £3 per cent. By Prob. XXIII. the value of the reversion after the decease of A. is 447.10. By Prob. XXVIII. the value of £1000 payable on the decease of A. if his life should be

* See the demonstration in Note XXV.

the *first* that fails is 60.53; the difference therefore or £386.57 is the value sought.

## Problem XXXVI.

To find the value of a given sum payable on the death of A., if his life should be the *first* or the *last* that fails of the three lives A. B. and C.

Solution. Find by Problem xxv. the value of the given sum payable on the contingency of B. surviving A., and also the value on the contingency of C. surviving A. Find by Problem xxiii. the value of the given sum on the decease of A., and let it be added to *twice* the value on the decease of A. if his life should be the *first* that fails, found by Problem xxviii. From the sum of these latter values, subtract the sum of the two former values, and the remainder will be the answer. *

Or more briefly; from the value of the absolute reversion after the decease of A. found by Prob. xxiii. deduct the value of the reversion depending on the contingency of A.'s being *second* that fails, found by Prob. xxix., and the remainder will be the value required. †

Example. Let the ages of A. B. and C. respectively be 24, 65, and 75, the sum £1000, and the

* See the demonstration, Note XXVI.

† Though the wording of this rule is shorter than that of the preceding, the operation in both is the same.

rate of interest £3 per cent. By Prob. XXIII. the value of the given sum after the decease of A. is 447.10. By Prob. XXIX. the value of the same on the contingency of A.'s being the *second* that fails is 105.33. Deducting this sum from 447.1 we have £341.77 for the value required.

*N. B.* The same results will follow if the solution be derived from the 28th and 30th Problems; or in other words, if the value of the reversion on the contingency of A.'s being the *first* that fails, be added to the value of the reversion on the contingency of A.'s being the *last* that fails.

## PROBLEM XXXVII.

To find the value of a given sum payable on the decease of A. or B., should *either* of them be the *first* that fails of the three lives A. B. and C.

SOLUTION. Multiply the value of the three joint lives A. B. C. by the interest of £1 for a year, deduct the product from unity and reserve two-thirds of the remainder. To one-third of A. B. *T.* * add one-sixth of *H. T.* B., and one-sixth of A. *P. T.* Let these be added to the reserved remainder, and the *sum* of the whole reserved. To one-third of *H. P.* C. add one-sixth of *H.* B. C., and one-sixth of A. C. *P.*, let these be deducted from the *sum* reserved above; multiply

* See Schol. to Prob. XXVII.

the remainder into the given sum, and the product divided by £1 increased by its interest for a year will be the value of the reversion. *

EXAMPLE. Let the ages of A. B. and C. respectively be 45, 12, and 60, the sum £1000, and the rate of interest £3 per cent. The value of the joint lives A. B. C. by Tab. 4. is 7.238 which being multiplied by .03, and the product subtracted from unity, the remainder will be .78286, two-thirds of which, or .52191 is the remainder to be reserved. A. B. *T.* may be found = 7.7338 of which one-third is 2.57794, *H.* *T.* B. and A. *P.* *T.* may be found respectively equal to 7.48062 and 7.63967, one-sixth of the sum of these two numbers is 2.52005, which being added to .52191 and 2.57794 gives 5.61990. Again, *H.* *P.* C. is equal to 7.89452, and *H.* B. C. and A. C. *P.* are respectively equal to 7.98649 and 8.14328, one-third of the former is 2.63150, and one-sixth of the sum of the two latter is 2.68829, these being added together give 5.31979, which being subtracted from 5.61990 will leave .30011. Multiplying this remainder by 1000, and then dividing the product by 1.03 the quotient or £291.37 will be the value required.

REMARK. The solution of this problem may also be derived from that of the 28th Problem, and from the Corollary to the 23d Problem; that is,

* See the demonstration, Note XXVII.

by subtracting from the value of the reversion after the three joint lives the value of the given sum payable if C. should be the *first* that fails; but the operation is not reduced by this means.

## Problem XXXVIII.

To determine the value of a given sum payable on the decease of A. or B. should *either* of them be the *second* that fails of the three lives A. B. and C.

Solution. Find by the Corollary to Prob. xxiii. the value of the given sum payable on the extinction of the joint lives A. and B. Find by Prob. xxv. the value on the contingency of C. surviving A. and also on the contingency of C. surviving B. Lastly, find by the preceding problem the value of the given sum payable on the contingency of A. *or* B. being either of them the *first* that fails of the three lives. From the sum of the three former values, deduct twice the value last mentioned, and the remainder will be the value required. *

Example. Let the ages of A. B. and C. as in the preceding problem, be 45, 12, and 60. The sum £1000, and the rate of interest £3 per cent. By Prob. xxiii. Cor. the value of the given sum after the extinction of the joint lives A. and B.

* See the demonstration, Note XXVIII.

is 625.39. By Prob. xxv. the value on the contingency of C. surviving A. is 249.66, and the value on the contingency of C. surviving B. is 113.70. By the preceding Problem the value of the given sum depending on the contingency of *either* A. or B. being the first that fails of the three lives is 291.37, of which the double is 582.74. Deducting this from 988.75, (the sum of the three former values) the remainder will be £406.01, which is the value of the reversion.

## PROBLEM XXXIX.

To determine the value of a given sum payable on the decease of A. or B., should *either* of them be the *last* that fails of the three lives A. B. and C.

SOLUTION. Find by Cor. to Prob. XXIII. the value of the given sum payable on the extinction of the longest of the two lives A. and B. Find by Problem XXXVII. the value on the contingency of either A. or B. being the first that fails of the three lives. Find by Prob. xxv. the value on the contingency of C. surviving A., and also the value on the contingency of C. surviving B.; the sum of these two latter values subtracted from the sum of the two former will give the value sought.* Or, find by Corollary to Prob. XXIII. the value of

* See the demonstration, Note XXIX.

the given sum payable on the extinction of the longest of the three lives; from which deduct the value of the same payable on the contingency of C. being the *last* that fails of the three lives (found by Prob. xxx.) and the remainder will be the answer.

EXAMPLE. Let the ages of A. B. and C. be 14, 44, and 69, the sum £1000, and the rate of interest £3 per cent. By the corollary to Prob. XXIII. the value of the given sum payable on the extinction of the two lives A. and B. is 333.5. By Prob. XXXVII. the value of the same on the contingency of either A. or B. being the first that fails is 273.48. The sum of these two values is 606.98. By Prob. XXV. the value of the given sum on the contingency of C. surviving A. is 85.96, and the value on the contingency of C. surviving B. is 213.59. Their sum is 299.55; which being subtracted from 606.98, the remainder, or £307.43 will be the value sought. The same result may be obtained from the second rule; for the value of the reversion after the extinction of the three lives by Corollary to Prob, XXIII. being £330, and the value of the same depending on the contingency of C. being the *last* that fails by Prob. xxx. being 22.57, the difference, or £307.43 will be the value as before.

## SECTION II.

IN the preceding section I have given general rules for determining the values of reversions depending upon one, two, and three lives in every case, which, as far as I can discover, will admit of an *exact* solution. But there are several other reversions depending on three lives which involve a contingency that does not appear to admit of such a general expression for determining their *exact* values as shall not render the rules much too complicated and laborious. The contingency to which I refer, is that of *one life's failing after another in any given time.* The fractions expressing the probability of this event are every year increasing, so that the value of the reversion must be represented by as many series at least as are equal to the difference between the age of one of the lives, and that of the oldest life in the table of observations. The 34th, 35th, and 36th problems in Mr. *Simpson*'s Select Exercises, involve this contingency, and by the assistance of M. *De Moivre*'s hypothesis admit of an easy solution. But, exclusive of the fallacy of this hypothesis, the supposition that it is an *equal* chance in all cases that one life shall die either *before* or *after* another, whatever be the disparity of their ages, or the number of years in which the event is to happen, must often lead to very erroneous conclusions. In the short term of a single year the chances are indeed so nearly equal, that it would

be wrong to perplex the solution by attempting greater accuracy. But when the number of years, and the difference between the two ages are considerable, those chances must vary in proportion; and therefore unless the contingency is blended with another which shall very much diminish the probability of the event, it will always be best to have recourse to a correct, though perhaps more laborious method of solution. By supposing these chances to be equal, and adopting *De Moivre's* hypothesis, the rule in Mr. *Simpson's* 36th problem has been rendered entirely absurd; for although the three lives of A. B. and C. are involved in the question, only two of them are comprehended in this rule; that is, the value of the reversion when C. is the oldest, is made equal to half the difference between the life of C. and the joint lives A. C., and therefore must always be the same, whether B., upon whose life it also depends, be 10 or 20 years younger than C., or indeed whether he does or does not exist at all. In all the different problems contained in this section, either the contingency of *one life surviving another*, or *of one life dying after another*, is involved in the computation. I have therefore in another part of this work * given a general method for determining each of those contingencies, and also for computing tables of the values of them at all ages

* See the two lemmas in Note XXX.; and also the 9th and 10th Tables.

without much labour or difficulty. By the assistance of these tables, though only partially applied to the solution of the following problems, the values appear to approximate sufficiently near the truth, to answer every useful purpose.

## Problem XL.

To find the value of an annuity on the life of C. after A. on the particular condition that A.'s life when it fails, shall fail *before* the life of B.

Solution. *When C. is the oldest of the three lives.* From the sum of the values of the single life C. and the joint lives B. C., subtract the sum of the values of the joint lives A. C., and of the three joint lives A. B. C., then will *half* the remainder be the value required. *When A. is the oldest of the three lives.* Find the value of an annuity on the life of C. and on the joint lives B. C. for as many years as are equal to the difference between the age of A. and the oldest life in the table (by Prob. II. and Corollary). To half the sum of these two values add the value of an annuity on the life of C. *after* the above term * multiplied into the probability that B. survives A. found in Tab. 9., and let the *sum* be *reserved.* To the value of an annuity on the joint lives A. and C., add the value of an annuity on the three joint lives A. B. C.,

* This is equal to the difference between the value of an annuity on the whole life of C. and of its value during the given term.

subtract half the sum of these two values from the *reserved* sum, and the remainder will be the value sought. *If B. is the oldest of the three lives:* Find the value of an annuity on the life of C., and on the joint lives A. and C., for as many years as are equal to the difference between the age of B. and the oldest life in the table; deduct the latter from the former, and to half the remainder add the value of an annuity on the life of C. after the above term multiplied into the probability that B. dies *after* A. (by Tab. 10.) and let the sum be *reserved.* From the value of an annuity on the joint lives B. C. deduct the value of an annuity on the three joint lives A. B. and C., add half the remainder to the *reserved* sum, and the amount will be the answer. *

EXAMPLE I. Let the ages of C. A. and B. be 60, 25, and 45 years, and the rate of interest £4 per cent. By Tab. 3. the value of an annuity on the life of C. is 9.039, and by Tab. 4. the value of an annuity on the joint lives B. C. is 7.274, the sum of these two values is 16.313. By Tab. 4. the value of an annuity on the joint lives A. C. is 7.906, and by Prob. IV. the value of an annuity on the three joint lives is 6.477. The sum of these two latter values, or 14.383, being deducted from 16.313 leaves 1.930, the half of which, or .965, is the answer.

* See the demonstration, Note XXXI.

Example II. The rate of interest being the same, let the respective ages of C. A. and B. be 45, 60, and 25 years. The difference between the age of A. and 96, (the age of the oldest life in the 1st table) is 36. The value of an annuity on the life of C. for 36 years by Prob. II. is 12.180, and the value of an annuity on the joint lives B. C. for the same number of years by Corollary to Prob. II. is 10.158. The value of an annuity on the life of C. after 36 years * is .103. The probability that B. survives A. by Tab. 9. is .788. These two sums multiplied into each other produce .081; which being added to 11.169 (or half the sum of the single and joint lives first found) gives 11.250 for the sum to be reserved. By Tab. 4. the value of the joint lives A. C. is 7.274. By Prob. IV. the value of the three joint lives is 6.477. Half their sum, or 6.875, being deducted from 11.250 reserved above, leaves 4.375 for the value required.

Example III. Suppose the ages of C. A. and B. respectively to be 25, 45, and 65, and the rate of interest still at £4 per cent. In this case B. is the eldest of the three lives, and the difference between his age and that of the oldest person in Tab. 1. is 31. The values of an annuity on the life of C. and on the joint lives A. C. for 31 years are 13.968 and 10.025, and half their difference is

* See Note, page 111.

1.9715. The value of an annuity on the life of C after 31 years is 1.470. The probability that B dies after A. by Tab. 10. is .2685. These tw numbers multiplied into each other produce .3947 which being added to 1.9715 gives 2.366 to be re served. By Table 4. the value of an annuity o the joint lives B. C. is 6.920. By Prob. IV. th value of an annuity on the three joint lives is 5.78( Half the difference of these two values is .57( which being added to 2.366, the reserved sum we have 2.936 for the number of years purchas required.

PROBLEM XLI.

To find the value of an annuity on the life ( C. after the decease of A., on the particular con dition that A.'s life when it fails, shall fail *afte* the life of B.

SOLUTION. *When C. is the oldest of the three lives.* From *half* the difference between the value of a annuity on the life of C. and of an annuity on th joint lives A. C. subtract *half* the difference b tween the value of an annuity on the joint lives I C. and on the three joint lives, and the remaind will be the answer. *When A. is the oldest of t three lives.* Find the value of an annuity on th life of C. and on the joint lives B. C. for as man years as are equal to the difference between the ag of A. and the oldest life in the table by Prob. II. an its Corollary. To half the difference between thes two values add the value of an annuity on the li

of C. after the above term * multiplied into the probability that A. dies after B. by Tab. 10., and reserve the sum. Find the difference between the value of an annuity on the joint lives A. C. and the value of an annuity on the three joint lives; subtract *half* this difference from the reserved sum and the remainder will be the value required. *If B. is the oldest of the three lives.* Find the value of an annuity on the life of C. and on the joint lives A. C. for a number of years equal to the difference between the age of B. and the oldest person in the table, and subtract the latter from the former value. From the value of an annuity on the joint lives B. C. subtract the value of an annuity on the three joint lives, add this to the preceding remainder, and let *half* the sum be reserved. From the value of an annuity on the life of C. subtract the value of an annuity on the joint lives A. C., and from the remainder subtract the product arising from the multiplication of the value of an annuity on the life of C. after the above mentioned term into the probability that A. dies *after* B. by Tab. 10. Deducting the *reserved* sum from this remainder, we have the value required. †

Example I. Let the ages of C. A. and B. respectively be 65, 35, and 15, the annuity as in

* See Note, page 111.

† See the demonstration, Note XXXII.

the examples to the preceding problem £1, and the rate of interest £4 per *cent.* By Tab. 3. the value of an annuity on the life of C. is 7.761, and by Tab. 4. the value of the joint lives A. C. is 6.747.; half their difference is .507. By Tab. 4. the value of an annuity on the joint lives B. C. is 7.127, and by Prob. IV. the value of an annuity on the three joint lives may be found equal to 6.263. Half the difference between these two values, or .432, being subtracted from .507 leaves .075 for the value required.

EXAMPLE II. Supposing the ages of C. A. and B. respectively to be 35, 65, and 15, and the rate of interest to be £4 per cent. The difference between the age of A. and that of the oldest person in the 1st table being 31 years, the value of an annuity on the single life C. for that term by Prob. II. will be 13.180, and the value of an annuity on the joint lives B. C. for the same term by Corollary to Prob. II. will be 11.369, and half their difference will be .906. The value of an annuity on the life of C. after 31 years is .859.* The probability that A. survives B. is .136 (by Tab. 9.) Hence 859 multiplied into .136 will produce .117, which being added to .906 will give 1.023 for the sum to be reserved. The value of an annuity on the joint lives A. C. (by Tab. 4.) is

* See Note, page 111.

6.747. The value of an annuity on the three joint lives by Prob. IV. is 6.263. Half their difference, or .242, subtracted from 1.023, the sum reserved, will leave .781 for the answer.

EXAMPLE III. If the ages of C. A. B. respectively be 15, 35, and 65, and the rate of interest, as above, £4 per cent., the value of an annuity on the life of C. by Tab. 3. will be 16.791, and on the joint lives A. C. by Tab. 4. = 11.787, and their difference will be 5.004. The value of an annuity on the life of C. for 31 years as in the preceding example, will be 14.696, and on the joint lives A. C. it will be 11.369. Half the difference of these two values is 1.6635. The value of an annuity on the joint lives B. C. is 7.127, and the value on the three joint lives by Prob. IV. is 6.267. Half their difference, or .432, added to 1.6635, gives 2.0955 for the sum to be reserved. The value of an annuity on the life of C. after 31 years is 2.095 * which being multiplied into .4037 (the probability by Tab. 10. that A. dies after B.) produces .846. Deducting this from 5.004, found above, we have 4.158; and 2.096, the reserved sum, being again deducted from 4.158, the remainder, or 2.062, will be the value required.

## PROBLEM XLII.

To find the value of an annuity on the lives of

* See Note, page 111.

B. and C. after the death of A. provided BOTH *of them should survive A.*

SOLUTION. *When A. is the youngest of the three lives.* From half the sum of the values of the single lives B. and C. subtract half the sum of the values of the joint lives A. C. and B. C., and the remainder will be the value required. *If B. and C. are* BOTH *younger than A:* To the value found as above, add half the sum of the values of the single lives of B. and C. after the expiration of as many years as are equal to the difference between the age of A. and that of the oldest life in the table, and from the sum subtract the value of the joint lives B. C. after the expiration of the same term,* then will the remainder be the required value. *If one life (B.) be older, and the other (C.) be younger then A.* Find the value of an annuity on the life of C. and on the joint lives A. C. for as many years as are equal to the difference between the age of B. and of the oldest life in the table, subtract the latter from the former value and *reserve* half the remainder. Find the value of the life of C. *after* the above term,* and multiply it into the probability (found by Tab. 10.) that B. dies after A. To the product add half the difference between the value of the life of B. and of the joint lives A. B., then will this sum added to the *reserved* remainder be the value sought. †

* See Note, pag. 111.

† See the demonstration, Note XXXIII.

Example I. Let the ages of A. B. and C. respectively be 20, 35, and 55, the annuity £1 and the rate of interest £4 per cent. By Tab. 3. the value of an annuity on the life of B. is 14.039, and the value of an annuity on the life of C. is 10.201. By Tab. 4. the value of an annuity on the joint lives of A. C. is 8.869, and of A. B. 11.445. Half the sum of the two former values is 12.120., half the sum of the two latter is 10.157; their difference, or 1.963, is the value required.

Example II. Supposing the ages of A. B. and C. to be 60, 50, and 30, respectively, the annuity £1, and the rate of interest £4 per cent. By Tab. 3. the values of the single lives of B. and C. being 11.264, and 14.781, half their sum will be 13.022; and by Tab. 4. the values of the joint lives A. C. and B. A. being 7.802 and 6.989, half their sum will be 7.395, which subtracted from 13.022 leaves 5.627. The difference between the age of A. and that of the oldest life in the first table being 35, the value of an annuity on the life of B. after 35 years by Corollary 3. Prob. ii. is .042, and the value of an annuity on the life of C. after the same term is .732. Half their sum, or .387, added to 5.627 gives 6.014. The value of the joint lives B. C. after 35 years is .013, which being deducted from 6.014, leaves 6.001 for the required value.

Example III. Supposing the ages of A. B. and C. respectively to be 50, 60, and 30, the annuity

and the rate of interest to be the same as in the preceding examples. The value of an annuity on the life of C. for 35 years by Prob. II. is 14.049; the value of an annuity on the joint lives A. C. for the same term by Corollary 1. Prob. II. is 7.789; half their difference therefore is 3.160, which is to be *reserved*. The value of an annuity on the life of C. after 35 years by Corollary 3. Prob. II. is .732. The probability that B. dies *after* A. is by Tab. 10 .375. These fractions multiplied into each other produce .2745. The value of an annuity on the life of B. by Tab. 3. is 9.039. The value of the joint lives A. and B. by Tab. 4. is 6.989. Half the difference of these two values, or 1.025 added to .2745 and also to 3.16, *reserved* above, gives 4.4595 for the answer.

## PROBLEM XLIII.

To find the value of a given sum S. payable on the extinction of the lives of A. and C., provided B. should survive one life in particular (A.)

SOLUTION. Find by Problem XXV. the value of the given sum payable on the contingency of B. surviving A. Find by Prob. XL., the value of an annuity of £1 during the life of C. after A., provided A. should die before B.; multiply this into the interest of S. for a year, and divide the product by £1 increased by its interest for a year; subtract the quotient from the value of S. by the 25th pro-

blem, and the remainder will be the value required.*

EXAMPLE. Supposing the respective ages of A. B. and C. to be 25, 45, and 60, the sum £1000, and the rate of interest £4 per *cent.* By the 25th prob. the value of £1000 payable on the contingency of B. surviving A. may be found = 204.37. By the first example to Prob. XL. the value of an annuity on the life of C. after A., provided A. dies before B., is .965, which multiplied into 40, (the interest of £1000), and divided by 1.04, gives 37.12. £204.37 lessened by this sum leaves £167.25† for the value of the reversion.

## PROBLEM XLIV.

To find the value of a given sum payable on the extinction of the lives of A. and C. provided B. should die before one life in particular (A.)

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* The solutions of this and the two following problems have been given in Notes 34, 35, and 36, independent of any other problems; but those theorems cannot well be expressed in *words* without being rendered too long and complicated. I have therefore in this part of the work preferred the rules which may be derived from other problems as being more concisely *described*, though perhaps the operations may not be much shorter than by those which are derived from the algebraical symbols in the notes above mentioned.

† By the algebraical rule the value is 204.37 — 37.89 = 166.48, which proves the correctness of both rules, as the small difference arises solely from being obliged to have recourse to the *approximated* values of three joint lives instead of the *real* ones.

Solution. Find by Prob. xxvi. the value of the given sum payable on the decease of A. and B. provided B. dies *before* A. Find by Prob. xli. the value of an annuity on the life of C. after A., provided A. should survive B. Multiply this into the interest of the given sum for a year, and divide the product by £1 increased by its interest for a year. Subtract this quotient from the value found by Prob. xxvi., and the remainder will be the value sought.

Example. Let the ages of A. B. and C. respectively be 35, 15, and 65 years, the sum £1000, and the rate of interest £4 per cent. By the 26th prob. the value of £1000 payable on the death of A. and B., provided B. dies before A., may be found equal to 87.018. By the 1st example to Prob. xli. the value of an annuity on the life of C. after A., provided B. dies before A. appears to be .075, which being multiplied into 40 and then divided by 1.04 gives 2.884. The difference therefore between 87.018, and 2.884, or £84.134, is the value of this contingent reversion, which agrees very nearly with the value derived from the algebraical theorem.

## Problem XLV.

To find the value of a given sum payable on the extinction of the lives of B. and C. provided *both* of them shall survive A.

SOLUTION. Find by Prob. XXVIII. the value of the given sum payable on the death of A. if his life should be the *first* that fails of the three lives. Find by Prob. XLII. the value of an annuity on the lives of B. and C. after A. provided both of them should survive him. Multiply this value into the interest of the given sum for a year, divide the product by £1 increased by its interest for a year, and subtract the quotient from the value found above by Prob. XXVIII.; then will the remainder be the value required.

EXAMPLE. Let the respective ages of A. B. and C. be 24, 65, and 75 years; the sum £1000, and the rate of interest £3 per cent. From the example to Prob. XXVIII. the value of the given sum payable on the death of A. if his life should be the *first* that fails appears to be 60.53. The value of an annuity on the lives of B. and C. after A. if both of them should survive him, may be found by Prob. XLII. equal to .6191, which being multiplied into 30, the interest of £1000 at £3 per cent., and then divided by 1.03, gives 18.03. Subtracting this sum from 60.53 found above, we have £42.5 for the value of the reversion, which is rather less than the value found by the algebraical theorem. *

## PROBLEM XLVI.

To find the value of a given sum payable on the

* See Note to Prob. XLIII. page 121.

death of C. if A. should be the first, B. the second, and C. the third that fails of the three lives.

SOLUTION. *When C. is the oldest of the three lives.* To one third of A. *P.* C.* add one-sixth of *H. P.* C., one-sixth of *H. T.* B., and half the value of the joint lives B. C. Multiply half the value of the two joint lives A. T. increased by unity into the probability that C. lives a year, and also into £1 payable at the end of a year. Subtract the value of the three joint lives A. B. C. multiplied into the interest of £1 for a year, from unity, divide $\frac{1}{6}$th of the remainder by £1 increased by its interest for a year, add the quotient to the five preceding values, and let the *sum be reserved.* To $\frac{1}{6}$th of A. B. *T.* add $\frac{1}{3}$ of A. *P. T.*, $\frac{1}{6}$th of *H.* B. C. and $\frac{1}{2}$ the value of the two joint lives A. C. divided by £1 increased by its interest for a year. Multiply $\frac{1}{2}$ the value of the two joint lives C. P. increased by unity into the probability that B. lives a year, and also into £1 payable at the end of a year. Multiply the value of the single life C. into the interest of £1 for a year, divide half the remainder by £1 increased by its interest for a year. Add this quotient to the five values just found, and let the whole be subtracted from the *reserved sum*, then will the remainder multiplied into the given sum be the value required. *When B. is the*

* See Schol. to Prob. XXVII.

*oldest of the three lives.* First find the value as in the preceding case, taking the values of the life of C. and of the joint lives A. C. only for as many years as are equal to the difference between the age of B. and that of the oldest life in the table, and taking the value of the joint lives A. T. for one year less than that difference. Find next the value of a life as many years older than C. as are equal to the said difference, which being subtracted from the perpetuity, let the remainder be multiplied into the interest of £1 for a year, also into the probability that C. lives to the end of the above term, and into the probability that B. dies *after* A. Again, multiply this product into the value of £1 payable at the end of a term exceeding the above difference by one year, and finally into the given sum. This product added to the value found above, will be the answer in this case. *When A. is the oldest of the three lives.* Find the value as in the first case, only taking the value of the life of C. for as many years as are equal to the difference between the age of A. and that of the oldest person in the table. Multiply the value of a life as many years older than C. as are equal to this difference into the interest of £1 for a year, also into the probability that B. and C. *both* live to the end of that term, and lastly into the value of £1 payable at the end of one year exceeding that term, *reserving* the product. Subtract the same value of the life of C. from the perpetuity, multiply the

remainder into the interest of £1 for a year, and also into the probability that B. dies *after* A., and into the probability that C. lives to the end of the term above-mentioned. Multiply this product again into the value of £1 payable at the end of one year exceeding that term. From this subtract the product just *reserved*, and multiply the remainder into the given sum. Add this product to the value found above, and the sum will be the value sought.*

EXAMPLE I. Let the ages of A. B. and C. respectively be 18, 39, and 68 years, the sum £1000; and the rate of interest £3 per cent. The value of $\frac{1}{3}$ of A. *P.* C. is $\frac{6.4879}{3} = 2.1626$ ... the value of $\frac{1}{6}$ of *H. P.* C. is $\frac{6.4181}{6} = 1.0697$ ... of $\frac{1}{6}$ of *H. T.* B. $\frac{6.0039}{6} = 1.0007$, of $\frac{1}{2}$ of B. C. is $\frac{6.3766}{2} = 3.1883$, and of the joint lives A. T. increased by unity and multiplied into the fractions directed in this rule is 3.4035. The value of the three joint lives, A. B. C., or 5.8902, multiplied into .03, and deducted from unity is .823294, which being divided by 6, and multiplied into £1 payable at the end of a year produces .1332. These six quantities added together make 10.9580 for the sum to be *reserved*.

* The latter part of this rule is not *exactly* the same with the algebraical theorem; but it is sufficiently correct for any useful purpose. See the demonstration in Note XXXVII.

The value of $\frac{1}{6}$ of A. B. *T.* is $\frac{6.0930}{6} = 1.0155$ ... of $\frac{1}{3}$ of A. *P. T.* is $\frac{5.9457}{3} = 1.9819$ ... of $\frac{1}{6}$ of *H.* B. C. is $\frac{6.5762}{6} = 1.0960$ ... of $\frac{1}{2}$ the joint lives of A. C. divided by £1 increased by its interest for a year is $\frac{6.4510}{2} = 3.2255$, and of half the joint lives of C. P. increased by unity and multiplied into the fractions directed in the rule, is $\frac{6.9748}{2} = 3.4874$. The value of the life of C. (7.3673) multiplied into .03 is .221019, and $\frac{.221019}{2}$ divided by 1.03 is $= .1073$; which being added to the five quantities found above, makes 10.9136. Deducting this from 10.9580, and multiplying the remainder (.0444) by 1000, we have £44.4 for the value of the reversion.

Example II. Supposing the ages of A. B. and C. respectively to be 18, 68, and 39, the sum £1000, and the rate of interest, as in the foregoing example, £3 per cent. Pursuing the same steps as above, the values of $\frac{1}{3}$ of A. *P.* C., $\frac{1}{6}$ of *H. P.* C., and $\frac{1}{6}$ of *H. T.* B., will be $\frac{6.0039}{3} = 2.0310$; $\frac{6.0039}{6} = 1.0007$, and $\frac{6.4181}{6} = 1.0697$. — the value of half the joint lives B. C. $= 3.1883$, and the value of the joint lives A. T. for 26 years (being one year less than the difference between the age of B. and that of the oldest person in the 1st Table) may be found $= 11.3867$. Half of this value multiplied into $\left(\frac{1312}{1392} =\right)$ the probability that B. lives a year, and then divided by 1.03 gives 5.8775. The three joint lives A. B. C. multiplied into .03, then deducted from unity, and $\frac{1}{6}$ of the remainder

multiplied into £1 payable at the end of a year produces as in the preceding example .1332; which being added to the five preceding values gives 13.3004 for the *sum to be reserved.*

Again; $\frac{1}{6}$ of A. B. *T.*, $\frac{1}{3}$ of A. *P. T.*, and $\frac{1}{6}$ of *H.* B. C., are severally equal to $\frac{6.4879}{6} = 1.0813$ ... $\frac{5.9457}{3} = 1.9819$ ... and $\frac{6.576}{6} = 1.0960$. The value of the joint lives A. C. for 27 years is 11.5929, therefore $\frac{11.5929}{2}$ divided by 1.03 is 5.6276. The value of the joint lives C. P. (6.1374) increased by unity, and half the sum multiplied into the probability that B. lives a year, and also into £1 payable at the end of a year, is 3.2558. The value of the life of C. for 27 years (13.5517) multiplied into .03, and half the product divided by 1.03 gives .1973. Adding these six values we have 13.2399; which being subtracted from 13.3004 (the reserved sum), and the remainder being multiplied into 1000, we have £60.5 for the first part of the answer. The value of a life of 66 (being 27 years older than that of C.) is 7.9937, which being deducted from 33.3333 (the perpetuity), and the remainder multiplied into .03, we have .760158. This product multiplied into $\frac{1552}{3710}$ (the probability that C. lives 27 years) also into .43708 (the value of £1 at the end of 28 years) and into .1311 (the probability that B. dies after A.) and into 1000 (the given sum) produces 18.1, which being added to £60.5 gives £78.6 for the value required.

EXAMPLE III. If the respective ages of A. B. and C. are 68, 39, and 18, let the sum and rate of interest be the same as in the two preceding examples. The values of $\frac{1}{3}$ of A. *P*. C., $\frac{1}{6}$ of *H*. *P*. C., and $\frac{1}{6}$ of *H*. *T*. B. are 2.1626, .9910, and 1.0007 respectively, and half the value of the joint lives B. C. is 6.1825. The value of the joint lives A. T. increased by unity is 7.6985. Half this sum multiplied into $\frac{5199}{5262}$ (the probability that C. lives a year) and also into .97087 (the value of £1 payable at the end of a year) produces 3.6839. The value of the three joint lives A. B. C. multiplied into the interest of £1 for a year, &c. is the same as in the two preceding examples, or .1332. These six values added together make 14.1539 for the *sum to be reserved*.

Again, $\frac{1}{6}$ of A. B. *T*., $\frac{1}{3}$ of A. *P*. *T*., and $\frac{1}{6}$ of *H*. B. C. are respectively equal to 1.0960, 2.1394, and 1.0155. Half the value of the joint lives A. C. (or 3.36075) divided by 1.03 is equal to 3.2629. The value of the joint lives C. P. (or 12.2224) increased by unity, and multiplied into the probability that B. lives a year, and also into the value of £1 payable at the end of a year, is 12.5488, of which one half is 6.2744. The value of the life of C. for 27 years is 15.2083, which being multiplied into .03, and half the product divided by £1 increased by its interest for a year, we have .2215. These six quantities, amounting to 14.0097, being deducted from 14.1539 (the

*reserved sum*) and the remainder multiplied into 1000 produces £144.2 for the first part of the value.

The value of a life aged 45 years is 13.692, which being multiplied into .03 gives .41076. This number being again multiplied into $\frac{1552 \times 3248}{3710 \times 5262}$ (the probability that B. and C. both live 27 years), and also into .43708 (the value of £1 payable at the end of 28 years) we have .04636 the *product to be reserved*. Again, 13.692 subtracted from 33.333, the perpetuity, and the remainder multiplied into .03, produces .58923, which being again multiplied into .3945 (the probability that B. dies after A.), also into $\frac{3248}{5262}$ (the probability that C. lives 27 years), and into .43708 (the value of £1 at the end of 28 years) it produces .06271. Deducting .04636, the *reserved product* from this sum, and multiplying the remainder into 1000, the given sum, and then adding 16.4 the product to 144.2, found above, we have £160.6 for the value required.

## Problem XLVII.

To determine the value of a given sum payable on the death of A. or B. should *either* of them be the *first* or *second* that fails of the three lives A., B. and C.

Solution. In this case the payment of the given sum must certainly take place on the extinction of the joint lives of A. and B., inde-

pendent of C., and therefore the value of the reversion may be obtained from the corollary to Prob. XXIII.

## PROBLEM XLVIII.

To determine the value of a given sum payable on the decease of A. or B., should either of them be the *second* or *third* that shall fail of the three lives A. B. and C.

SOLUTION. Find by Prob. XLI. the value of an annuity of £1 on the life of A. after C. provided C. should die after B. Find by the same problem the value of an annuity of £1 on the life of B. after C., provided C. should die after A. Find by Prob. VI. the value of an annuity on two out of the three lives. From the perpetuity subtract the sum of these three values, and multiply the remainder into the interest of the given sum for a year, then will the product divided by £1 increased by its interest for a year be the value required. *

EXAMPLE I. Let the respective ages of A. B. and C. be 60, 25, and 45., the sum £100, and the rate of interest £3 per *cent.* By Prob. XLI. the value of an annuity on the life of A. after C., provided C. dies after B., is .2785. By the same Problem the value of an annuity on the life of B.

* See the demonstration, Note XXXVIII.

after C., provided C. dies after A., is 2.4730. By Prob. VI. the value of an annuity on two out of the three lives is 13.5280. The sum of these three values, or 16.2795, being subtracted from 33.3333, the perpetuity, leaves 17.0538; which being multiplied into 3, the interest of £100 for a year, and then divided by 1.03, gives £49.675 for the value sought.

Example II. Supposing the ages of A. B. and C. to be 25, 45, and 60 respectively, the sum and rate of interest, as in the preceding example. By Prob. XLI. the value of an annuity on the life of A. after C., provided C. dies after B., is 2.2029. By Prob. XLI. the value of an annuity on the life of B. after C., provided C. dies after A., is .8626. By Prob. VI. the value of an annuity on two out of the three joint lives is 13.5280. The sum of these three values is 16.5935, which being subtracted from 33.333, and the remainder multiplied into 3, we have 50.2194. This product divided by 1.03 gives 48.756 for the answer. These values by the algebraical rule are a little greater; a difference arising, as I have already observed, from being obliged to have recourse to approximated values of the three joint lives.

## Problem XLIX.

To determine the value of a given sum payable on the decease of A. or B., should either of them be the *first* or *last* that fails of the three lives A. B. and C.

Solution. The payment of the sum in this problem, like that in the preceding one, must ultimately take place; and it is only the single contingency of C.'s being the *first* that fails of the three lives which can *postpone* it after their joint continuance. In order therefore to determine the value of this reversion, find by Problem XLI. the value of an annuity on the life of A. after B., provided B. shall have died *after* C., and the value of an annuity on the life of B. after A., provided A. shall have died *after* C. To these add the value of the two joint lives A. B., and let the sum be subtracted from the perpetuity. Multiply the remainder into the interest of the given sum for a year; divide the product by £1 increased by its interest for a year, and the quotient will be the required value. *

Example I. Let the ages of A. B. and C. as in the 1st example of the foregoing problem be 60, 25, and 45, the sum £100, and the rate of interest £3 per cent. By Prob. XLI. the value of an annuity on the life of A. after B., provided B. shall have died after C., is .2285, and the value of an annuity on the life of B. after A., provided A. shall have died after C., is 2.6347. The sum of these (2.5932) added to 8.496 gives 11.0892, which subtracted from 33.333, and the remainder multiplied into 3, produces 66.7323, and 66.7323 divided by 1.03 quotes £64.79 for the answer.

* See the demonstration, Note XXXIX.

EXAMPLE II. The ages of A, B. and C. being 25, 45, and 60 respectively, and the sum and rate of interest the same as in the preceding example; the value of an annuity on the life of A, after B., provided B. survives C., by Prob. XLI. will be 2.4846, and the value of an annuity on the life of B. after A., provided A. survives C., will be .851, the value of an annuity on the joint lives A. B. is 11.164. These three values, or 14.4996, subtracted from 33.333, and the remainder multiplied into 3, produces 56.501; which being divided by 1.03, we have £54.854 for the value of the reversion in this case.

## PROBLEM L.

To determine the value of a given sum payable on the death of A., should his life be the *first* or *second* that fails, and should B.'s life, if it fails, become extinct before the life of C.

SOLUTION. The value of the reversion in this case depends simply on the contingency of C.'s surviving A., and is determined by the solution of the 25th problem.*

## PROBLEM LI.

To find the value of a given sum payable on the death of A., should his life be the *second* or *third*

* See the demonstration, Note XL.

that fails, and should B.'s life, when it fails, become extinct before the life of C.

Solution. *When A. is the oldest of the three lives.* Multiply the value of the three joint lives A. B. C. by the interest of £1 for a year, add 2 to the product, and divide the sum by 6; to this quotient add one-third of *H*. *P*. C.* one-sixth of *H*. B. C., and one-sixth of A. B. *T*.; and from the sum deduct the sum of one-*sixth* of *H*. *T*. B., one-sixth of A. *P*. *T*., and one-third of *A*. *P*. C., *reserving* the remainder. Multiply half the value of an annuity on the life of A. into the interest of £1 for a year; and also half the difference of the joint lives H. B. and H. C. into the probability that A. lives a year. Deduct the former from the latter product, if B. is younger than C., otherwise add them together. In the one case deduct half the difference between the joint lives of A. C. and A. B. from the *remainder*; in the other case *deduct* the *sum* of the two values from this half-difference. Lastly, let the last-mentioned sum or remainder, according as it is a positive or negative quantity, be added to or subtracted from the *remainder* first reserved, and if the sum or the difference be divided by £1 increased by its interest for a year, and the quotient multiplied into the given sum, the product will be the value required.

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* See Schol. Prob. XXVII.

*When B. is the oldest of the three lives.* Find the value as in the preceding case, taking the values of A., A. C., and H. C. for as many years as are equal to the difference between the age of B. and the oldest life in the table, and let this value be *reserved.* From the perpetuity subtract the value of a life as many years older than A., as are equal to this difference. Multiply the remainder into the interest of £1 for a year, also into the probability that A. lives to that age, and into the value of £1 payable at the end of the above mentioned term, and reserve the product. To the number expressing the probability that C. dies *after* B. by Tab. 10. add the fraction expressing the probability that C. lives as many years as are equal to the difference above mentioned. Multiply the sum into the reserved product: let this second product again be added to the value first reserved, then will their sum, divided by £1 increased by its interest for a year, and multiplied into the given sum, be the value in this case.

*When C. is the oldest of the three lives.* The first part of the value is found in every respect in the same manner as in the two preceding cases, only taking the lives A., A. B., and H. B., for as many years as are equal to the difference between the age of C. and of the oldest life in the table. By pursuing nearly the same steps, the second part also is obtained. From the perpetuity subtract the value of a life as many years older than A. as

are equal to the difference between the age of C. and that of the oldest person in the table. Let the remainder be multiplied into the probability that A. lives to that age, and into the interest of £1 for a year, and *reserve* the product. Add the fraction expressing the probability that B. lives as many years as are equal to the difference above mentioned, to the number expressing the probability that C. dies *after* B. by Tab. 10. Multiply the sum into the value of £1 payable at the end of the above term, and also into the reserved product. Add this to the reserved *sum* or *remainder*. Divide the whole by £1 increased by its interest for a year, and the quotient multiplied into the given sum will be the value required. *

EXAMPLE I. Suppose the respective ages of A. B. and C. to be 65, 40, and 15. The sum £100, and the rate of interest £3 per cent. The value of the three joint lives is 6.51, which being multiplied into .03 produces .1953, and therefore 2.1953 divided by 6, gives .3659. One-third of *H. P.* C. may be found equal to 2.263, one-sixth of *H.* B. C. equal to 1.1620, and one-sixth of A. B. *T.* equal to 1.2357. These four quantities are 5.0266. One-sixth of *H. T.* B. is 1.1505, one-sixth of A. *P. T.* is 1.206, and one-third of A. *P.* C. is 2.442. The sum of these three quantities, or 4.7985, deducted from 5.0266, leaves .2281 for the remainder to be *reserved.* Half the value of an annuity on

* See the demonstration, Note XLI.

the life of A. or 4.15035 multiplied into .03 produces .12457. The difference between 7.3385, the value of the joint lives H. C. and 6.8086, the value of the joint lives H. B. is .5299, the half of which being multiplied into $\frac{1552}{1632}$, the probability that A. lives a year, produces .2520, which in this case is to be *added* to .12457, and gives .3766 to be deducted. Half the difference between 7.5970 the value of the joint lives A. C., and 7.0299, the value of the joint lives A. B., is. 2836, which being less than .3766, the difference .093 will be a negative quantity. Subtracting this difference from .2281, the remainder *reserved* above, we have .1351, which being divided by 1.03, and the quotient multiplied into 100, produces £13.107 for the required value.

EXAMPLE II. Let the ages of A. B. and C. be 15, 65, and 40 respectively, the sum and rate of interest the same as in the preceding example. The first expression will of course be the same, or .3659. One-third of *H. P.* C. is 2.3010, one-sixth of *H.* B. C. is 1.2357, and one-sixth of A. B. *T.* is 1.2210. The sum of these four quantities is 5.1236. One-sixth of *H. T.* B. is 1.206. One-sixth of A. *P. T.* is 1.1315, and one-third of A. *P.* C. is 2.324. The sum of these three quantities, or 4.6615, deducted from 5.1236, leaves .4621 for the *remainder* to be *reserved.* The value of an annuity on the life of A. for 31 years, or a term equal to the difference between the age of B. and

the oldest life in the table, is 16.513, one half of which being multiplied into .03 produces .2477. The value of an annuity on the joint lives H. C. for 31 years is 11.9787, the value of an annuity on the joint lives H. B. is 7.5613. Half the difference between these two values multiplied into $\frac{5373}{5423}$, the probability that A. lives a year, produces 2.1885, which being added to .2477 the sum will be 2.4362. The value of the joint lives A. C. for 31 years is 12.0524. The value of the joint lives A. B. is 7.597. Half the difference between these two values is 2.2277. Deducting this difference from 2.4362 the remainder will be .2085, which being deducted from .4621, the *reserved remainder* leaves .2536 for the first part of the value in this case. The value of an annuity on a life of 46, or a life 31 years older than A., is 13.450, which being subtracted from 33.333 and the remainder multiplied into .03, gives .59649. This product multiplied into .3999, the value of £1 at the end of 31 years, and into $\frac{3170}{5423}$, the probability that A. lives so long, produces .1395. By Tab. 10. .4516 expresses the probability that C. dies after B., and the fraction $\frac{1152}{3635}$ expresses the probability that C. lives 31 years. The sum of these two expressions is .7686, and .1395 multiplied into .7686 produces .1072. Adding this product to .2536, the first part of the value, and dividing the sum by 1,03, the quotient will be .3503, and consequently the required value £35.03.

EXAMPLE III. Suppose the ages of A. B. and C. to be 40, 15, and 65, the sum and rate of interest as in the two preceding examples. The first expression as in those cases will be .3659. One third of *H. P.* C. is 2.411. One-sixth of *H.* B. C. is 1.221, and one-sixth of A. B. *T.* 1.1620. The sum of these four expressions is 5.1599. One-sixth of *H. T.* B. is 1.131. One-sixth of A. *P. T.* 1.1505, and one-third of A. *P.* C. 2.4714, deducting 4.7529, their sum, from 5.1599 we have .407 for the *remainder* to be *reserved.* The value of an annuity on the life of A. for 31 years is 14.0344, which being multiplied into .03, the moiety of the product will be .2105. The value of an annuity on the joint lives H. B. for 31 years is 11.9474, the value of the joint-lives H. C. is 6.9957; half the difference of these two values multiplied into $\frac{3559}{3685}$, the probability that A. lives a year is 2.419, from which subtracting .2105 we have 2.2085. The value of the joint lives A. B. for 31 years is 12.0524. The value of the joint lives A. C. is 7.0299; half their difference is 2.5112, which exceeding 2.2085 by .3027, this difference will be a negative quantity to be subtracted from .407, the reserved remainder, so that the first part of the value is .1043. The value of an annuity on a life of 71, being 31 years older than C., is 6.418. Deducting this from 33.333, and multiplying the remainder into .03, we have .80745; this being again multiplied into $\frac{1152}{3685}$, the probability that A. lives 31 years, pro-

duces .2559. By Tab. 10. the probability that C. dies after B. is .1359, which added to $\frac{3170}{5423}$, the probability that B. lives 31 years, is .7339. The value £1 payable at the end of 31 years is .3999. These three quantities multiplied into each other produce .0751. Adding this product to .1043, the first part of the value, and dividing the sum by 1.03, the quotient will be .1742, and the value of the reversion will in consequence be £17.420.

### Problem LII.

To find the value of a given sum payable on the death of A. should his life be the *first* or *last* that fails of the three lives, and should B.'s life, if it fail, become extinct before the life of C.

Solution. Multiply the sum of the values of the three joint lives, and of the life of A. into the interest of £1 for a year, deduct the product from unity, and take half the remainder. Multiply half the difference between the values of the joint lives H. B. and the three joint lives H. B. C. into the probability that A. lives a year. To these two quantities add one-half of A. *P*. *T*.* and reserve the *sum*. From the value of the joint lives A. T. subtract the value of the three joint lives A. B. T., multiply half the remainder into the probability that C. lives a year, and to the product add one half of B. *H*. *T*., and one-half the value of the

* See Schol. Prob. xxvii.

joint lives A. B. From the sum of these three quantities deduct the reserved sum and divide the remainder by £1 increased by its interest for a year. Let the quotient be subtracted from half the value of the joint lives A. C., then will the remainder multiplied into the given sum be the value required. The above rule gives the true value *when A. is the oldest;* but if *C. be the oldest of the three lives,* the annuities on the lives of A., A. B., and H. B. must be continued only for as many years as are equal to the difference between the age of C. and the oldest life in the table; this will give the first part of the value. Find next the value of a life as much older than A. as is equal to the above difference. Deduct this from the perpetuity, and multiply the remainder into the interest of £1 for a year, and into the value of £1 payable at the end of a term one year longer than the said difference. This product again being multiplied into the probability that A. lives as many years as are equal to the said difference, and into the probability that C. *dies* after B. will give another product, which is to be added to the first part of the value; and their sum being multiplied into the given sum will be the value sought. *If B. be the oldest of the three lives,* the values of the lives of A. and A. T. in the first part of the solution must be continued only for as many years as are equal to the difference between the age of B. and that of the oldest life in the table, and the

value of the joint lives A. C. for one year longer. From the perpetuity subtract the value of a life as many years older than A. as is equal to the above difference; multiply the remainder into the interest of £1 for a year, and also into the value of £1 payable at the end of a term one year longer than the said difference. Again multiply this expression into the probability that C. dies after B. and also into the probability that A. lives to the end of the term above mentioned. Add this second product to the *first* part of the value, and multiply the amount into the given sum for the value required.*

EXAMPLE I. Let the ages of A. B. and C. respectively be 70, 50, and 30, the sum £100, and the rate of interest £3 per cent. The value of the joint lives A. B. C. by Prob. IV. is 5.087, the value of the single life A. is 6.734, their sum, or 11.821, multiplied into .03, and the product subtracted from unity, leaves .6454; half of which is .3227. The value of the three joint lives H. B. C. or 4.888, subtracted from 5.361, the value of the two joint lives H. B., leaves .473, which being multiplied into $\frac{1152}{1232}$, the probability that A. lives a year, and the product divided by 2, gives .2212. One-half of A. *P. T.* is 2.891. These three quantities added together give 3.4349. The difference between 6.031, the value of the joint lives A. T., and 5.079, the value of the three joint lives A. B.

* See the demonstration, Note XLII.

T. is .952. Half of this sum multiplied into $\frac{4310}{4385}$, the probability that C. lives a year, produces .4679. One-half of B. *H. T.* is 2.6815, and half of 5.582, the value of the joint lives A. B. is 2.791. From 5.9404, the sum of these three values, deducting 3.4349 reserved above, we have 2.5055, which being divided by 1.03 quotes 2.4325. Half the sum of 6.043, the value of the joint lives A. C. is 3.0215. Deducting 2.4325 from 3.0215 and multiplying the remainder into the given sum, the product, or £58.90, is the value required.

EXAMPLE II. Supposing the ages of A. B. and C. respectively to be 30, 50, and 70, the sum and rate of interest the same as in the preceding example. The value of the joint lives A. B. C. is 5.087. the value of the life of A. for 26 years, being the difference between the age of C. and that of the oldest life in the table is 14.200; the sum of these two values multiplied into .03, and the product being subtracted from unity, the remainder will be .4214, of which one-half is .2107. The value of the joint lives H. B. for 26 years is 9.851, the value of the three joint lives H. B. C. is 5.079. Half the difference of these two values multiplied into $\frac{4310}{4385}$, the probability that A. lives a year, produces 2.345. One-half of A. *P. T.* is 2.663. The sum of these three quantities is 5.2187. The difference between 5.785 the value of the joint lives A. T., and 4.888 the value of the three joint lives A. B. T., is .897. The probability that C. lives a

year is $\frac{1152}{1232}$. Multiplying these expressions into each other, and dividing the product by 2, we have .4194. One half of B. H. T. is 2.6815. The value of the joint lives A. B. for 26 years is 9.888, of which the half is 4.944. Deducting 5.2187, the sum reserved above, from 8.0449, the sum of these last three quantities, and dividing the remainder by 1.03, the quotient will be 2.744. Half the value of the joint lives A. C. is 3.0125, and 2.744 subtracted from 3.0125 leaves .2775 for the first part of the solution. The value of an annuity on a life of 56, being 26 years older than the life of A. is 10.883 which being deducted from 33.333 and the remainder multiplied into .03, we have .6735. The value of £1 payable at the end of 27 years is .4502, the probability that A. lives 26 years is $\frac{2366}{4385}$, and the probability that C. dies *after* B. by Tab. 10. is .2455. These four quantities being multiplied into each other produce .041. Adding this product to .2775, the first part of the solution, and multiplying the sum into 100, we have £31.76 for the value of the reversion in this case.

Example III. If the respective ages of A. B. and C. be 50, 70, and 30, the sum and rate of interest as in the two foregoing examples; the value of the life of A. and of the joint lives A. T. respectively for 26 years will be 11.835 and 9.851, the value of the joint lives A. C. for 27 years will be 9.943, and the first part of the solution, by proceeding as above, will be found = .2575. The

value of a life of 76, or of a life 26 years olde than A. is 4.925. Deducting this from 33.33 and multiplying the remainder into .03 we hav .8522. The value of £1 payable at the end c 27 years is .4502. The probability that A. live 26 years is $\frac{752}{2857}$, and the probability that C. die after B. is .3122. Multiplying these four quanti ties into each other, the product becomes .0315 Adding this product to .2575, the first part of the solution, the sum becomes equal to .289, and con sequently the value of the reversion of £100 will be £28.900.

## Problem LIII.

To determine the value of a given sum payable on the extinction of the three lives A. B. C. provided C. shall die after one life in particular (A.)

Solution. Find by Prob. xli. the value of an annuity on the life of B. after C. provided A. shall have died before C. Multiply this value into the interest of the given sum; divide the product by £1 increased by its interest for a year, and reserve the quotient. Find next by Problem xxvi. the value of the given sum on the event of C.'s dying after A., subtract the reserved quotient from this value, and the remainder will be the value required.*

Example I. Let the respective ages of A. B. and C. be 25, 65, and 35, the sum £100, and

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* See the demonstration Note XLIII.

the rate of interest £3 per cent. By Prob. XLI. the value of an annuity on the life of B. after C. provided A. shall have died before C. is = .111, which being multiplied by 3, and then divided by 1.03, quotes .323. By Prob. XXVI. the value of £100 depending on the contingency of C. dying after A. is = 15.903. Deducting .323 from this value, the remainder, or £15.58, will be the value sought.

EXAMPLE II. Supposing the ages of A. B. and C. respectively to be 35, 25, and 65, the sum and rate of interest the same as in the preceding example. The value of the annuity on the life of B. after C. provided A. shall have died before C. may be found by the 2d case Prob. XLI. to be equal to 1.8273. This being multiplied into 3 and then divided by 1.03 gives 5.322 for the quotient to be reserved. By Prob. XXVI. the value of £100 payable on the death of C. after A. is 13.272. Deducting 5.322 the reserved quotient from this sum we have £7.95 for the value required.

The values deduced from the algebraical rule (Note 43.) agree very nearly with the above, and would agree exactly if we were possessed of tables giving correct values of annuities on three joint lives.

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# NOTES.

## NOTE I.

Let $r$ be £1 increased by its interest for a year, $b$ the number of the living in the table opposite to the present age of A. the given life, and $c$, $d$, $e$, &c. the numbers of the living at the end of 1, 2, 3, &c. years from his present age; then it follows, from what has been said in page 17, that the value of an annuity on this life will be $\frac{c}{br} + \frac{d}{br^2} + \frac{e}{br^3}$ + &c. = N. Let $a$ denote the number of the living in the table opposite to the age of a person one year younger than A., and the value of an annuity on the life of this younger person will be expressed by $\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3}$ &c. But this series is equal to $\frac{b}{ar} \times \overline{1 + \frac{c}{br} + \frac{d}{br^2} + \frac{e}{br^3} + \&c.}$ $= \frac{b}{ar} \times \overline{1 + N}$.

In the same manner may the theorem be obtained for computing a table of the values of two *joint* lives. Let $n$ be the number of the living in the table opposite to the age of B. the second life, $o$, $p$, $q$, &c. the numbers living at the end of 1, 2, 3, &c. years from his present age, and the value of an annuity on the two *joint* lives of A. and B. will be $\frac{oc}{nbr} + \frac{pd}{nbr^2} + \frac{qe}{nbr^3}$ + &c. = M. Let $m$ represent the number living opposite to the age of a person one year younger than B., and the value of an annuity on two *joint* lives, each one year younger than A. and B. will be

$\frac{nb}{amr} + \frac{oc}{amr^2} + \frac{pd}{amr^3} + \text{\&c.} = \frac{nb}{amr} \times \overline{1 + \frac{oc}{nbr} + \frac{pd}{nbr^2} + \frac{qe}{nbr^3} + \text{\&c.}} = \frac{nb}{amr} \times \overline{1 + M}.$

As the *expectations* are the same with the values of annuities on single and joint lives *without interest*, tables o those expectations may be computed from the preceding theorems, by omitting only the fraction $\frac{1}{r}$: that is, the expectation of a life one year younger than A. will be $\frac{b}{a} \times \overline{1 + E.}$, and the expectation of two joint lives each one younger than A. and B. will be $\frac{nb}{am} \times \overline{1 + F.}$; E. being the expectation of the life of A. and F. the expectation of the two joint lives A. and B.

## NOTE II.

In the case of the *youngest* life, it is evident that there is no other difference between the operations in this rule and the theorem in Note I., than that those in the former are begun at the last year and continued upwards to the first year of life, and that those in the latter are begun at the first year, and continued downwards to the last year of life. Let $a, b, c, d, e,$ --------- $z$ express the numbers of persons living at all ages in the table, $n$ the difference between the ages of the youngest and oldest life, and $r$ £1 increased by its interest for a year. Then will the value of the youngest life by the theorem be $\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3} + \frac{e}{ar^4}$, &c. ----- $+ \frac{z}{ar^n}$; and by the rule the sum of all the values of £1 payable if this life exists to the several ages of the oldest, 2d, 3d, 4th, &c. lives is $\frac{z}{ar^n} + \frac{y}{ar^{n-1}} + \frac{x}{ar^{n-2}} + \frac{u}{ar^{n-3}}$ ------- $+ \frac{b}{ar}$. The value of an

annuity found by the theorem is, in the present instance, to be multiplied by unity, and will, therefore, remain unaltered, and be just the same with the value found by the rule.

Again; the value of a life one year older by the theorem is $\frac{c}{br} + \frac{d}{br^2} + \frac{e}{br^3}$ -------- $+ \frac{z}{br^{n-1}}$ which being multiplied by the value of £1 payable if the youngest life should continue one year, or by $\frac{b}{ar}$, is equal to $\frac{c}{ar^2} + \frac{d}{ar^3} + \frac{e}{ar^4}$ -------- $+ \frac{z}{ar^n}$. Now the sum of the values of £1 payable if the youngest life should exist to the age of the oldest, and all the intermediate ages between it and a life one year older than the youngest, is expressed by the same fraction in a contrary order, or by $\frac{z}{ar^n} + \frac{y}{ar^{n-1}} + \frac{x}{ar^{n-2}}$ ------- $+ \frac{c}{ar^2}$. The value of a life two years older by the theorem is $\frac{d}{cr} + \frac{e}{cr^2} + \frac{f}{cr^3}$ ------ $+ \frac{z}{cr^{n-2}}$, which being multiplied by £1, payable if the youngest life exists two years, or by $\frac{c}{ar^2}$, is equal to $\frac{d}{ar^3} + \frac{e}{ar^4} + \frac{f}{ar^5}$ ------- $+ \frac{z}{ar^n}$; and this is easily determined to be the sum of all the values of £1 payable if the youngest life exists to the age of the oldest, and all the intermediate ages between it and a life two years older than the youngest. Lastly, the value of an annuity on the oldest life but one is equal to $\frac{z}{yr}$, which being multiplied by the value of £1 payable if the youngest life exists to this age, or by $\frac{y}{ar^{n-1}}$, is equal to $\frac{z}{ar^n}$; and this is evidently the value of £1 payable if the youngest life exists to the age of the oldest. Hence the truth of the rule is manifest.

In the same manner may the proof of the accuracy the operations in computing the tables of the values of t joint lives be demonstrated.

## NOTE III. (Prob. v.)

Let $a$, $b$, $c$, $d$, &c. be the number of persons living the table at the age of A. in the 1st, 2d, 3d, &c. yea $m$, $n$, $o$, $p$, &c. the numbers living at the age of B., a $s$, $t$, $u$, $w$, &c. the numbers living at the age of C. in t same years; then will $\frac{b}{a}$, $\frac{c}{a}$, $\frac{d}{a}$, &c. express the probabi ties that A. lives 1, 2, 3, &c. years, $\frac{n}{m}$, $\frac{o}{m}$, $\frac{p}{m}$, &c. the prob bilities that B. lives, and $\frac{t}{s}$, $\frac{u}{s}$, $\frac{w}{s}$, &c. the probabilities th C. lives to the end of those years. If these fractions severally deducted from unity, we shall have the probab lities that they will *die* in 1, 2, 3, &c. years, and cons quently $\overline{1-\frac{b}{a}} \times \overline{1-\frac{n}{m}} \times \overline{1-\frac{t}{s}}$ will be the probability th they will all die in the first year, $\overline{1-\frac{c}{a}} \times \overline{1-\frac{o}{m}}, \times \overline{1-}$ that they will all die in two years, $\overline{1-\frac{d}{a}} \times \overline{1-\frac{p}{m}} \times \overline{1-}$ the probability that they will all die in three years, and on. The first of these expressions is $1-\frac{b}{a}-\frac{n}{m}-\frac{t}{s}+\frac{nb}{am}$ $\frac{nt}{ms}+\frac{bt}{as}-\frac{bnt}{ams}$, the second $=1-\frac{c}{a}-\frac{o}{m}-\frac{u}{s}+\frac{oc}{am}+$ $+\frac{cu}{us}-\frac{cou}{ams}$, the third $=1-\frac{d}{a}-\frac{p}{m}-\frac{w}{s}+\frac{pd}{am}+\frac{pw}{ms}+$ $-\frac{dpw}{ams}$, &c. Deducting these expressions again from unity we shall have the probabilities that they will not *all* di that is, that one of them at least will live 1, 2, 3, &c years. If $\frac{1}{r}$, $\frac{1}{r^2}$, $\frac{1}{r^3}$, &c. be the values of £1 payable at

the end of 1, 2, 3, &c. years, and these fractions be respectively multiplied into the series expressing the probabilities in the 1st, 2d, 3d, &c. years, the value of the annuity in the 1st year will be $\frac{b}{ar} + \frac{n}{mr} + \frac{t}{sr} - \frac{nb}{amr} - \frac{nt}{msr} - \frac{bt}{asr} + \frac{bnt}{amsr}$, the value in the 2d year $= \frac{c}{ar^2} + \frac{o}{mr^2} + \frac{u}{sr^2} - \frac{oc}{amr^2} - \frac{ou}{msr^2} - \frac{cu}{asr^2} + \frac{cou}{amsr^2}$, the value in the 3d year $= \frac{d}{ar^3} + \frac{p}{mr^3} + \frac{w}{sr^3} - \frac{pd}{amr^3} - \frac{pw}{msr^3} - \frac{dw}{asr^3} + \frac{dpw}{amsr^3}$, &c. &c. But $\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3}$, &c. is the value of an annuity on the single life of A. $\frac{n}{mr} + \frac{o}{mr^2} + \frac{p}{mr^3}$, &c. the value of an annuity on the single life of B. $\frac{t}{sr} + \frac{u}{sr^2} + \frac{w}{sr^3}$, &c., the same value on the single life of C. $\frac{nb}{amr} + \frac{oc}{amr^2} + \frac{pd}{amr^3}$, &c., $\frac{nt}{msr} + \frac{ou}{nw^2} + \frac{pw}{msr^3}$, &c., $\frac{bt}{asr} + \frac{cu}{asr^2} + \frac{dw}{asr^3}$, &c. are respectively the values of an annuity on the joint lives of A and B., of B and C., and of A and C., and $\frac{bnt}{amsr} + \frac{cou}{amsr^2} + \frac{dpw}{amsr^3}$, &c. the value of an annuity on the three joint lives. These several values being denoted by A, B, C., AB., BC., AC., and ABC., the general rule will be = A. + B. + C. − AB. − BC. − AC. + ABC. Q. E. D.

## NOTE IV. (Prob. vi.)

This annuity will be enjoyed during the three joint lives, and also during the joint lives of A and B after C., during the joint lives of A and C after B., and during the joint lives of B and C after A. To find the value of an annuity during the joint lives of A and B after C., let the same symbols be retained as in the preceding note. In the first year the payment of the annuity will depend

on the contingency of A and B both living, and C dyin in that year, which by reasoning as in that note will b expressed by $\frac{nb}{am}\times\overline{1-\frac{t}{s}}\times\frac{1}{r}=\frac{nb}{amr}-\frac{nbt}{amsr}$. In the secon year the payment will depend on the contingency of A and B both living two years, and C dying in that tim and therefore the value will be $=\frac{oc}{am}\times\overline{1-\frac{u}{s}}\times\frac{1}{r^2}=\frac{oc}{amr^2}-\frac{ocu}{amsr}$ In the third year the value will be $=\frac{pd}{amr^3}-\frac{pdw}{amr^3}$, and s on. Hence the whole value will be equal to $\overline{AB-ABC}$ In like manner the value during the joint lives o AC after B will be AC — ABC., and the value durin the joint lives of BC after A will be equal to BC.—ABC. Adding these three values to the value of the annuity during the three joint lives first mentioned, and the sum will be AB. + AC. + BC. — 2 ABC., agreeable to the general rule.

## NOTE V. (Prob. VII.)

The amount of the annuity by any table of observa tions, supposing the number of persons living at the ag of the given life, and at every subsequent year to be de noted by $b$, $c$, $d$, $e$, &c. will be $\frac{c}{b}+\frac{dr}{b}+\frac{er^2}{b}+\frac{fr^3}{b}$, &c. $=Q$ Let $a$ denote the number of persons living at the age o a person one year younger than the given life, then wil the amount of the annuity during the continuance of suc younger life be $\frac{b}{a}+\frac{cr}{a}+\frac{dr^2}{a}+\frac{er^3}{a}$, &c. $=\frac{b}{a}\times\overline{1+\frac{cr}{b}+\frac{dr^2}{b}+\frac{er^3}{b},\text{ \&c.}}=\frac{b}{a}\times\overline{1+rQ}$. - - - Q. E. D.*

In the former edition of this work, the solution of the

* See Table 7th.

above problem was derived from M. *De Moivre's* hypothesis, and the general rule was expressed by $\frac{r^n - 1}{n.\overline{r-1}|^2} - \frac{1}{r-1}$, where $n$ denotes the complement or twice the expectation of the given life. In the middle stages of life this rule is sufficiently correct; but in the earlier and later stages it is unfit for use.

## NOTE VI. (Prob. VIII.)

Retaining the same symbols as in the preceding note, the value of the annuity on the given life will be expressed by the series $\frac{c}{br} + \frac{2d}{br^2} + \frac{3e}{br^3} + \frac{4f}{br^4} +$ &c. = R., and the value of the like annuity on a life one year younger will be $= \frac{b}{ar} + \frac{2c}{ar^2} + \frac{3d}{ar^3} +$ &c. $= \frac{b}{ar} \times \overline{1 + \frac{c}{br} + \frac{d}{br^2} + \frac{e}{br^3} + \&c.}$ $+ \frac{b}{ar} \times \overline{\frac{c}{br} + \frac{2d}{br^2} + \frac{3e}{br^3} + \&c.} = \frac{b}{ar} \times \overline{1 + R. + N.}$,* (N. being the value of an annuity on the given life.) The solution of this problem, like that of the preceding, was deduced in the former edition of this work from M. *De Moivre's* hypothesis, and the general rule was expressed by $n \times \overline{V - G.}$, $n$ being the *complement*, V. the value of an annuity on the given life, and G. the value of two equal joint lives of the same age. This theorem, like all those which are founded on an equal decrement of life, is correct only in the middle stages of it.

Were the annuity to increase £1 per annum for $n$ years, without being subject to any contingency, the series expressing the value of such annuity, $\left(\text{or } \frac{1}{r} + \frac{2}{r^2} + \frac{3}{r^3} + \frac{4}{r^4} \cdots + \frac{n}{r^{n-1}}\right)$ may be found equal to $\frac{r}{\overline{r-1}|^2} - \frac{n}{r^n.\overline{r-1}} - \frac{r}{r^n.\overline{r-1}|^2}$

* See Table 8th.

And the series expressing the *amount* of £1 in one year, £2 in two years, £3 in three years, &c. for $n$ years (or $1 + 2r + 3r^2 + 4r^3 - - - - - nr^{n-1}$) may be found equal to $\frac{\overline{n+1}.r^n - 1}{r-1} - \frac{\overline{r^n-1}.r}{\overline{r-1}|^2}$.

DEMONSTRATION of Cor. II. The value of this annuity will be $\frac{\alpha b}{ar} + \frac{\overline{\alpha-\beta}.c}{ar^2} + \frac{\overline{\alpha-2\beta}d}{ar^3} + \frac{\overline{\alpha-3\beta}.e}{ar^4}$, &c. $= \alpha \times \overline{\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3} + \&c.} - \beta \times \overline{\frac{c}{ar^2} + \frac{2d}{ar^3} + \frac{3e}{ar^4}}$, &c. $= \alpha$ A. $- \frac{\beta b}{ar} \times \overline{\frac{c}{br} + \frac{2d}{br^2} + \frac{3e}{br^3} + \&c.} = \alpha$ A. $- \frac{\beta b}{ar} \times$ R.

DEMONSTRATION of Cor. III. The probability that the life fails in the 1st year is $1 - \frac{b}{a} = \frac{a-b}{a}$, the probability that it fails in the 2d year, after having survived the 1st, is $\frac{b}{a} \times \overline{1 - \frac{c}{b}} = \frac{b-c}{a}$, the probability that it fails in the 3d after having survived the first two years is $\frac{c}{a} \times \overline{1 - \frac{d}{c}} = \frac{c-d}{a}$, &c., therefore the value of the given sum is $\frac{a-b}{ar} + \frac{\overline{2b-c}}{ar^2} + \frac{\overline{3c-d}}{ar^3} + \&c. = \frac{1}{r} + \frac{b}{ar^2} + \frac{c}{ar^3} + \frac{d}{ar^4}$, &c. $+ \frac{b}{ar^2} + \frac{2c}{ar^3} + \frac{3d}{ar^4} + \&c. - \frac{b}{ar} - \frac{2c}{ar^2} - \frac{3d}{ar^3} - \&c. = \frac{1+A}{r} + \frac{M}{r} - M = \frac{1+A-\overline{r-1}.M}{r.}$ Q. E. D. If F. be the value of an annuity certain for a number of years equal to the *complement* of the given life, the value of this reversion by De Moivre's hypothesis will be $= \overline{F - A}$.

## NOTE VII. (PROB. IX.)

The annuity in the first year depends on the contingency of A.'s living and B.'s dying in that year, therefore retaining

the same symbols as in the preceding notes, the value will be expressed by $\frac{b}{a} \times \overline{1 - \frac{n}{m}} \times \frac{1}{r} = \frac{b}{ar} - \frac{bn}{amr}$. The annuity in the second year depends on A.'s living two years and B.'s dying in that time, the value therefore for that year will be $\frac{c}{a} \times \overline{1 - \frac{o}{m}} \times \frac{1}{r^2} = \frac{c}{ar^2} - \frac{oc}{amr^2}$. In the same manner the value for the third year will be $\frac{d}{ar^3} - \frac{pd}{amr^3}$, and so on for the succeeding years. Hence the whole value will be $\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3} + \frac{e}{ar^4} +$, &c. $- \frac{bn}{amr} - \frac{co}{amr^2} - \frac{dp}{amr^3} -$, &c. But the first series expresses the value of an annuity on the life of A., and the second series expresses the value of an annuity on the *joint* lives of A. and B., consequently the truth of the solution is manifest.

## NOTE VIII. (Prob. x. xii. and xiii.)

The value of the reversion in this case is the *difference* between the values of an annuity on the longest of the three lives and of an annuity on the longest of the two lives in possession, B and C; that is, it will be equal to A. + B. + C. + ABC. − AB. − AC. − BC. − B − C. + BC. = A. + ABC. − AC. − BA. Q. E. D.—See page 51, and Note III.

Remark. By reasoning as in *Note* VII., the value of the annuity in the 12th *Problem* is $\frac{b}{ar} \times \overline{1 - \frac{tn}{ms}} + \frac{c}{ar^2} \times \overline{1 - \frac{uo}{ms}} + \frac{d}{ar^3} \times \overline{1 - \frac{wp}{ms}}$, &c. $= \frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3}$, &c. $- \frac{tnb}{amsr} - \frac{uoc}{amsr^2} - \frac{wpd}{ams} -$, &c. = A. − ABC.; and the value of the annuity in the 13th Problem is there proved to be $= \frac{bn}{amr} + \frac{co}{amr^2} + \frac{dp}{amr^3} +$ &c. $- \frac{bnt}{amr} - \frac{cou}{amr^2} - \frac{dpw}{amr^3} -$, &c. = AB. − ABC.

## NOTE IX. (Prob. xiv.)

Since the whole annuity is to be enjoyed during the joint lives, and one half of it either during the life of A. after B., or during the life of B. after A., if the first part be denoted by AB., the second part by Note VII. will be equal to $\frac{A.-AB.}{2} + \frac{B.-AB.}{2}$, consequently the required value will be AB. $+ \frac{A.-AB.}{2} + \frac{B.-AB.}{2} = \frac{A.+B.}{2}$.

## NOTE X. (Prob. xv.)

The values of A. and B.'s interest during the joint lives are each equal to $\frac{AB.}{2}$. The values of their reversionary interest by Prob. ix. are either A.—AB., or B.—AB.; adding therefore $\frac{AB.}{2}$ to each of these, the value of A.'s interest will be A. $- \frac{AB.}{2}$, and the value of B.'s interest will be B. $- \frac{AB.}{2}$.

## NOTE XI. (Prob. xvi.)

The value of A.'s interest in the annuity during the three joint lives is $\frac{ABC.}{3}$. The value of his interest during the joint lives of himself and B. after C. is $\frac{AB.-ABC.}{2}$, by Prob. xii. The value of his interest during the joint lives of himself and C. after B. is $\frac{AC.-ABC.}{2}$, by the same Problem; and the value of his interest during the remainder of his life after the decease of B and C., by Prob. x., is A. + ABC. — AC. — AB. Adding all these together we have A. $+ \frac{ABC.}{3} - \frac{AC.+AB.}{2}$.

## NOTE XII. (PROB. XVII.)

During the joint lives of BC. after the decease of A., the value of B.'s expectation will be $= \frac{\text{BC.} - \text{ABC.}}{2}$, by Prob. XII.; and during the remainder of his life after the decease of A. and C., its value by Prob. x. will be B. + ABC. − AB. − BC.; hence the whole value will be A. − AB. − $\frac{\text{BC.} - \text{ABC.}}{2}$.

## NOTE XIII. (PROB. XVIII.)

By the tenth Problem and Note VIII. it appears that the value of the annuity during the life of A. after B. and C. is A. + ABC. − AB. − AC.; that the value of the same annuity during the life of B. after A. and C. is B. + ABC. − BC. − AB.: and that its value during the life of C. after A. and B. is C. + ABC. − AC. − BC. Adding these three values together, we have A. + B. + C. + 3 ABC. − 2 AB. − 2 AC. − 2 BC. for the value required.

## NOTE XIV. (PROB. XIX.)

Let the probabilities that A. lives 1, 2, 3, &c. years be, as usual, denoted by $\frac{b}{a}$, $\frac{c}{a}$, $\frac{d}{a}$, &c., then will the probability that he dies in the first year be $1 - \frac{b}{a} = \frac{a-b}{a}$; the probability that he dies in the second year after having survived the first year will be $\frac{b}{a} \times \frac{b-c}{b} = \frac{b-c}{a}$; the probability that in like manner he dies in the third year will be $\frac{c-d}{a}$, and so on for the other years. Let B. be the value of an an-

nuity on the life to be nominated, then will the value of the reversion for the first year be $\frac{a-b}{a} \times$ B.; for the second year $\frac{b-c}{ar} \times$ B.; for the third year $\frac{c-d}{r^2} \times$ B., &c. &c.— $= \text{B.} + \text{B.} \times \overline{\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3},\ \&\text{c.}} - \text{B.}\ \overline{\frac{b}{a} + \frac{c}{ar} + \frac{d}{ar^2},\ \&\text{c.}}$ $= \text{B.} + \overline{\text{A.} - r\ \text{A.}} \times \text{B.}$ (A. being the value of the life of A). $= \text{B.} - \overline{r - 1.}\ \text{A.} \times \text{B.} = \overline{1 - \overline{r-1}.\ \text{A.}} \times \text{B.} =$ $\overline{\frac{1}{r-1}} - \text{A.}\,\overline{r-1} \times \text{B.}$ But $\frac{1}{r-1}$ is = P. (the perpetuity), therefore the general rule will be expressed by $\frac{\overline{\text{P.} - \text{A.}} \times \text{B.}}{\text{P.}}$.

## NOTE XV. (Prob. xxiii.)

By reasoning as above, the value of the assurance in one payment will be $= \text{S.} \times \overline{\frac{a-b}{ar} + \frac{b-c}{ar^2} + \frac{\overline{c-d}}{ar^3} +,\ \&\text{c.}} =$ $\text{S.} \times \frac{1}{r} + \frac{b}{ar^2} + \frac{c}{ar^3} + \frac{d}{ar^4},\ \&\text{c.} - \text{S.} \times \overline{\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3} +,\ \&\text{c.}}$ $= \text{S.}\ \frac{1 + \text{A.}}{r} - \text{S. A.} = \text{S.} \times \frac{\overline{\frac{1}{r-1}} - \text{A.}\,\overline{r-1}}{r}$. Let $p$ denote the perpetuity $\left(\text{or } \frac{1}{r-1}\right)$, then will $p + 1 = \frac{r}{r-1}$ and the above expression become $= \frac{\text{S.} \times \overline{\text{P.} - \text{A.}}}{\text{P.} + 1.}$.

The value of the reversion of an *estate* depending on the life's becoming extinct in 1, 2, 3, &c. years, is $\overline{\frac{1}{r} - \frac{b}{ar}}$ $+ \overline{\frac{1}{r^2} - \frac{c}{ar^2}} + \overline{\frac{1}{r^3} - \frac{d}{ar^3}}$ - - - - - ($z$) + $\frac{1}{r^z+1} + \frac{1}{r^z+2}$, &c. *ad infin.* But $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$, &c. + $\frac{1}{r^z+1} + \frac{1}{r^z+2}$, &c. is equal to P, therefore the reversion is = P − A.;—which is indeed so evident as hardly to require a demonstration.

## NOTE XVI. (Prob. xxiv.)

It appears, from the preceding note, that the value of the given sum for $n$ years will be $= S. \times \overline{\frac{1}{r} + \frac{b}{ar^2} + \frac{c}{ar^3} + \frac{d}{ar^4}}$ $\overline{- - - - - (n-1)} - S. \times \overline{\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3} + - - - - - (n)}$. Let $A''$ represent the value of the life of A for $\overline{n-1}$ years, and $A'$ the same value for $n$ years, then will the required value of the assurance be $S. \times \overline{\frac{1 + A''}{r}} - A' - - - - -$ Q. E. D.

If the reversion be an *estate* instead of a sum, the value derived from the above expression will be $= \frac{\overline{1 + A'' - r\,A'}}{r-1}$.

## NOTE XVII. (Prob. xxv.)

The value of S. in the first year will depend on either of two events: 1st, On the contingency of *both* lives failing in the year, B. having died *last*; 2dly, On the contingency of A. having died and B. having survived him in that year. As it is nearly an equal chance that either A. or B. dies first in any particular year, the first contingency may be fairly expressed by $\frac{a-b}{a} \times \frac{m-n}{m} \times \frac{1}{2}$; the second contingency is $= \frac{a-b}{a} \times \frac{n}{m}$. The value therefore of S. in this year will be $S \times \frac{m+n}{2n} \times \frac{a-b}{ar}$.—In the second year, the value will depend on *both* lives failing in that year, B. having died *last*, or on A's life only having failed in that year. These probabilities are expressed by the fractions $\frac{b-c}{a} \times \frac{n-o}{m} \times \frac{1}{2}$ and $\frac{b-c}{a} \times \frac{o}{m}$, so that the value of S. in the second year is $S \times \frac{n+o}{2m} \times \frac{b-c}{ar^2}$. In the same manner

the value in the third year will be $S \times \frac{o+p}{2m} \times \frac{c-d}{ar^3}$, and so on for the other years. The whole value, therefore will be $S$ into $\frac{m+n}{2m} \times \frac{a-b}{ar} + \frac{n+o}{2m} \times \frac{b-c}{ar^2} + \frac{o+p}{2m} \times \frac{c-d}{ar^3} + \frac{p+q}{2m} \times \frac{d-e}{ar^4} +$, &c. This series may be resolved into the four following series - - - - $\frac{ma}{2amr} + \frac{nb}{2amr^2} + \frac{oc}{2amr^3} + \frac{pd}{2amr^4} +$, &c.; - - - - $+ \frac{na}{2amr} + \frac{ob}{2amr^2} + \frac{pc}{2amr^3} + \frac{qd}{2amr^4} +$ &c.; and $-\frac{mb}{2amr} - \frac{nc}{2amr^2} - \frac{od}{2amr^3} -$, &c.; $-\frac{nb}{2amr} - \frac{oc}{2amr^2} - \frac{pd}{2amr^3} -$ &c. The first of these series is $= \frac{1}{2r} + \frac{AB}{2r}$; the second $= \frac{n}{2mr} \times \overline{1+AP}$ (P being a life one year older than B); the third $= -\frac{b}{2ar} \times \overline{1+BH}$ (H being a life one year older than A); the 4th $= -\frac{AB}{2}$. Hence the general rule becomes $\frac{S.}{2r}$ into $\left(\overline{1-\overline{r-1}AB} + \frac{n.\overline{1+AP}}{m} - \frac{b.\overline{1+BH}}{a}\right)$. When the lives are equal, the last two fractions destroy each other, and the rule is reduced to $\frac{S}{2r} \times \overline{1-\overline{r-1}.A}$ or $\frac{S.}{2r} \times \overline{r-1}.\overline{V-AB}$. *

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* As $\overline{a-b}$, $\overline{b-c}$, $\overline{c-d}$, &c. in the above demonstration are the decrements of life in each year at the age of A.; if these be nearly equal during any given term ($t$), their average may be assumed as a constant quantity $\alpha$, and the first mentioned series $\frac{\overline{m+n}.\overline{a-b}}{2mar} + \frac{\overline{n+o}.\overline{b-c}}{2mar^2} + \frac{\overline{o+p}.\overline{c-d}}{2mar^3}$, &c. will become $\frac{\overline{m+n}.\alpha}{2mar} + \frac{\overline{n+o}.\alpha}{2mar^2} + \frac{\overline{o+p}.\alpha}{2mar^3}$, &c. $(t) = \frac{\alpha}{2a} \times \overline{N + \frac{N'+1}{r}}$, N. being the value of an annuity on the life of A. for $t$ years, and N$'$ the same value for $\overline{t-1}$ years. Hence the value of the sum S. will be $\frac{S.\alpha}{2ar} \times \overline{N. + \frac{N'+1}{r}}$.——supposing it to depend on the contingency of B. surviving A. in $t$ years. See Prob. XXXII.

## NOTE XVIII. (Prob. xxvi.)

Retaining the same symbols as in the preceding note, the value of S in the 1st year depending on the contingency of *both* lives failing in that year, B having died last, will be $\frac{S.\overline{a-b}.\overline{m-n}}{2amr}$. The value of S. in the 2d year will depend on either of two events happening. First, that A. and B. both die in that year after having survived the first year, restrained to the contingency of B's having died last; 2dly, that B. dies in the second, and A. in the first year. The value therefore of S. in this year will be expressed by $\frac{S.\overline{b-c}.\overline{n-o}}{2amr^2} + \frac{S.\overline{a-b}.\overline{n-o}}{amr^2}$. Again, the value of S in the 3d year will depend on A. and B.'s both dying in that year, B having died last, or on B.'s dying in that year, A. having died in the 1st or 2d year. Hence the value of S in the 3d year will be $\frac{S.\overline{c-d}.\overline{o-p}}{2amr^3} + \frac{S.\overline{a-c}.\overline{o-p}}{amr^3}$. By proceeding in this manner for the other years, the whole value of the reversion will be found $= \frac{S}{2am} \times \left(\frac{\overline{a-b}.\overline{m-n}}{r} + \frac{\overline{b-c}.\overline{n-o}}{r^2} + \frac{\overline{c-d}.\overline{o-p}}{r^3}, \&c.\right) + \frac{S}{amr} \times \left(\frac{\overline{a-b}.\overline{n-o}}{r} + \frac{\overline{a-c}.\overline{o-p}}{r^2} + \frac{\overline{a-d}.\overline{p-q}}{r^3} + \&c.\right)$ These different series being expanded will become $\frac{S}{2am} \times \left(\frac{am}{r} + \frac{bn}{r^2} + \frac{co}{r^3} + \&c.\right) - \frac{S}{2am} \times \left(\frac{an}{r} + \frac{bo}{r^2} + \frac{cp}{r^3}, \&c.\right) - \frac{S}{2am} \times \left(\frac{bm}{r} + \frac{cn}{r^2} + \frac{do}{r^3} + \&c.\right) + \frac{S}{2am} \times \left(\frac{bn}{r} + \frac{co}{r^2} + \frac{dp}{r^3} + \&c.\right) + \frac{S}{amr} \times \left(\frac{an}{r} + \frac{ao}{r^2} + \frac{ap}{r^3} + \&c.\right) - \frac{S}{amr} \times \left(\frac{bn}{r} + \frac{co}{r^2} + \frac{dp}{r^3}, \&c.\right) - \frac{S}{amr} \times \left(\frac{ao}{r} + \frac{ap}{r^2} + \frac{aq}{r^3} + \&c.\right) + \frac{S}{amr} \times \left(\frac{bo}{r} + \frac{cp}{r^2} + \frac{dq}{r^3} + \&c.\right)$ The 1st and

6th of these series are $= \frac{S}{2r} \times \overline{1 - AB}$. The 4th $= \frac{S.AB.}{2}$, and consequently these three series are $= \frac{S}{2r} \times \overline{1 - \overline{r-1}.\ AB}$. The 2d and 8th series are $= S \times \overline{\frac{n.\ AP.}{2mr} - \frac{n}{2mr}}$. The 3d series is $= \frac{S. \times b.\ \overline{1 + HB}}{2ar}$. The 5th is $= \frac{S.\ B.}{r}$, and the 7th is $= S. \times \overline{\frac{n}{mr} - B}$. Adding these last two series, we have $S. \times \overline{\frac{n}{mr} - \frac{r-1.\ B}{r}}$, so that the whole value of the reversion will be $\frac{S}{2r} \times \left(\overline{1 - \overline{r-1}.\ \overline{2B. - AB.}} + \frac{n.\ \overline{1 + AP.}}{m} - \frac{b.\ \overline{1 + BH.}}{a}.\right)$ When the lives are equal, the last two fractions, as in the former case, destroy each other, and the value becomes $\frac{S. \times \overline{1 - \overline{r-1}.\ \overline{2B. - BB.}}}{2r}$, or $\frac{S}{2r} \times \overline{\overline{r-1}.\ V. - \overline{2B. - B.B.}}$ *

## NOTE XIX. (PROB. XXVII.)

Retaining the same symbols as in the IIId and following notes, the value of the given sum S. in the first year

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* By proceeding as in the foregoing note, and supposing the decrements of life at the age of B. for $t$ years to be nearly equal (or a constant quantity $\beta$) the value of S. will be expressed by the series $\frac{\beta.\overline{a-b}}{2amr} + \frac{\beta.\overline{b-c}}{2amr^2} + \frac{\beta.\overline{c-d}}{2amr^3}$ - - - - - $(t)$ $+ \frac{\beta.\overline{a-b}}{2amr^2} + \frac{\beta.\overline{a-c}}{2amr^3} + \frac{\beta.\overline{a-d}}{2amr^4}$ - - - - $(t-1) = \frac{S}{2b} \times \overline{N - A + \frac{N' - A'}{r}}$; N. denoting the value of an annuity certain for $t$ years, N′ the same value for $\overline{t-1}$ years, and A and A′ the value of an annuity on the life of A. for $t$ and $t-1$ years respectively. See Prob. XXXIII.

depending on the contingency of C. having survived A. and B. (A. having died before B.) or on the contingency of the three lives having become extinct, A. having died first, B. next, and C. last, will be expressed by $\frac{S}{amsr} \times \overline{\frac{\overline{a-b}.\overline{m-n}.t}{2} + \frac{\overline{a-b}.\overline{m-n}.\overline{s-t}}{6}} = \frac{S}{amsr} \times \overline{\frac{\overline{a-b}.ms}{6} - \frac{\overline{a-b}.ns}{6} + \frac{\overline{a-b}.mt}{3} - \frac{\overline{a-b}.nt}{3}}$. In the second year the value of S. will depend on either of four events:—1st, on the contingency of all the three lives dying in that year, A. having died first, B. next, and C. last, which probability is expressed by the fraction $\frac{\overline{b-c}.\overline{n-o}.\overline{t-u}}{6ams}$; 2dly, On the contingency of B.'s dying in that year, C. living to the end of it, and A.'s dying in the first year $= \frac{\overline{n-o}.u.\overline{a-b}}{ams}$; 3dly, on the contingency of B.'s dying after A. in the second year, and of C.'s living to the end of that year $= \frac{\overline{n-o}.\overline{b-c}.u}{2ams}$; 4thly, on the contingency of A.'s dying in the first year, and of B. and C.'s both dying in the second year, B. having died first $= \frac{\overline{a-b}.\overline{n-o}.\overline{t-u}}{2ams}$. Hence the whole value in the second year will be found $= \frac{S}{amsr^2} \times \left(\frac{\overline{b-c}.nt}{6} - \frac{\overline{b-c}.ot}{6} + \frac{\overline{b-c}.mu}{3} - \frac{\overline{b-c}.ou}{3} + \frac{\overline{a-b}.nt}{2} - \frac{\overline{a-b}.ot}{2} + \frac{\overline{a-b}.nu}{2} - \frac{\overline{a-b}.ou}{2}.\right)$ In like manner the value of S. in the third year will depend on the same number of events as in the second year; that is, it will depend on the contingency of all the three lives dying in that year, A. having died first, B. next, and C. last; 2dly, on the contingency of B.'s dying in that particular year, C.'s living to the end of it, and A.'s dying in the first or second year; 3dly, on the con-

tingency of B.'s dying after A. in the third year (both of them having survived the two preceding years), and of C.'s living to the end of that year; and 4thly, on the contingency of A's dying in the first or second year, and of B and C's both dying in the third year, C having died last. These several contingencies are expressed by the respective fractions $\frac{\overline{c-d}.\overline{o-p}.\overline{u-w}}{6ams}$ ---- $\frac{\overline{a-c}.\overline{o-p}.w}{ams}$ --- $\frac{\overline{c-d}.\overline{o-p}.w}{2ams}$ ---- and $\frac{\overline{a-c}.\overline{o-p}.\overline{u-w}}{2ams}$. Consequently the value for the third year will be $\frac{S}{amsr^3} \times \left(\frac{\overline{c-d}.ou}{6} - \frac{\overline{c-d}.pu}{6} + \frac{\overline{c-d}.ow}{3} - \frac{\overline{c-d}.pw}{3} + \frac{\overline{a-c}.ou}{2} - \frac{\overline{a-c}.pu}{2} + \frac{\overline{a-c}.ow}{2} - \frac{\overline{a-c}.pw}{2}\right)$. By reasoning in the same manner, the value of S in the fourth and following years may be found, and the whole value of the reversion will be $\frac{S}{6ams} \times \left(\frac{\overline{a-b}.ms}{r} + \frac{\overline{b-c}.nt}{r^2} + \frac{\overline{c-d}.ou}{r^3} + \&c.\right) + \frac{S}{2amsr} \times \left(\frac{\overline{a-b}.nt}{r} + \frac{\overline{a-c}.ou}{r^2} + \frac{\overline{a-d}.pw}{r^3} + \&c.\right) - \frac{S}{6ams} \times \left(\frac{\overline{a-b}.ns}{r} + \frac{\overline{b-c}.ot}{r^2} + \frac{\overline{c-d}.pu}{r^3} + \&c.\right) - \frac{S}{2amsr} \times \left(\frac{\overline{a-b}.ot}{r} + \frac{\overline{a-c}.pu}{r^2} + \frac{\overline{a-d}.qw}{r^3} + \&c.\right) + \frac{S}{3ams} \times \left(\frac{\overline{a-b}.mt}{r} + \frac{\overline{b-c}.nu}{r^2} + \frac{\overline{c-d}.ow}{r^3} + \&c.\right) + \frac{S}{2amsr} \times \left(\frac{\overline{a-b}.nu}{r} + \frac{\overline{a-c}.ow}{r^2} + \frac{\overline{a-d}.px}{r^3} + \&c.\right) - \frac{S}{3ams} \times \left(\frac{\overline{a-b}.nt}{r} + \frac{\overline{b-c}.ou}{r^2} + \frac{\overline{c-d}.pw}{r^3} + \&c.\right) - \frac{S}{2amsr} \times \left(\frac{\overline{a-b}.ou}{r} + \frac{\overline{a-c}.pw}{r^2} + \frac{\overline{a-d}.qx}{r^3} + \&c.\right)$. These eight series being expanded will become (first) $\frac{S}{6ams} \times \left(\frac{ams}{r} + \frac{bnt}{r^2} + \frac{cou}{r^3} + \&c.\right)$; (second) $- \frac{S}{6ams} \times \left(\frac{bms}{r} + \frac{cnt}{r^2} + \frac{dou}{r^3} + \&c.\right)$; (third) $- \frac{S}{6ams} \times \left(\frac{ans}{r} + \frac{bot}{r^2} + \frac{cpu}{r^3} + \&c.\right)$; (fourth) $+ \frac{S}{6ams} \times \left(\frac{bns}{r} + \frac{cot}{r^2} +\right.$

$\frac{dpu}{r^3}$ + &c.); (fifth) + $\frac{S}{3ams} \times \left(\frac{amt}{r} + \frac{bnu}{r^2} + \frac{cow}{r^3} + \&c.\right)$; (sixth) $- \frac{S}{3ams} \times \left(\frac{bmt}{r} + \frac{cnu}{r^2} + \frac{dow}{r^3} + \&c.\right)$; (seventh) $- \frac{S}{3ams} \times \left(\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3} + \&c.\right)$; (eighth) $+ \frac{S}{3ams} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} + \&c.\right)$; (ninth) $+ \frac{S}{2msr} \times \left(\frac{nt}{r} + \frac{ou}{r^2} + \frac{pw}{r^3} + \&c.\right)$; (tenth) $- \frac{S}{2amsr} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} + \&c.\right)$; (eleventh $- \frac{S}{2ms} \times \left(\frac{ot}{r} + \frac{pu}{r^2} + \frac{qw}{r^3} + \&c.\right)$; (twelfth) $+ \frac{S}{2amsr} \times \left(\frac{bot}{r} + \frac{cpu}{r^2} + \frac{dqw}{r^3} + \&c.\right)$; (thirteenth) $+ \frac{S}{2msr} \times \left(\frac{nu}{r} + \frac{qw}{r^2} + \frac{px}{r^3} + \&c.\right)$; (fourteenth) $- \frac{S}{2amsr} \times \left(\frac{bnu}{r} + \frac{cow}{r^2} + \frac{dpx}{r^3} + \&c.\right)$; (fifteenth) $- \frac{S}{2msr} \times \left(\frac{ou}{r} + \frac{pw}{r^2} + \frac{qx}{r^3} + \&c.\right)$; (sixteenth) $+ \frac{S}{2amsr} \times \left(\frac{bou}{r} + \frac{cpw}{r^2} + \frac{dqx}{r^3}\ \&c.\right)$. The first and [t]enth series are equal $\frac{1}{6r} - \frac{ABC.}{3r}$. The third and [t]welfth $= - \frac{n}{6mr} + \frac{n.APC.}{3.mr}$. The fifth and fourteenth $= \frac{t}{3sr} - \frac{t.ABT.}{6sr}$. The seventh and sixteenth $= - \frac{nt}{3msr} + \frac{t.APT.}{6msr}$. The second series $= - \frac{b}{6ar} \times \overline{1 + HBC}$. The [f]ourth $= \frac{bn}{6amr} \times \overline{1 + HPC}$. The sixth $= - \frac{bt.\overline{1+HBT}.}{3asr}$. The eighth series $= \frac{ABC}{3}$. The eleventh $= - \frac{n.\ CP}{2mr}$. The [f]ifteenth $- \frac{nt}{2msr} - \frac{BC}{2}$. The ninth $= \frac{BC}{2r}$. The thir[t]eenth $= \frac{t}{2sr} \times$ BT. (H, P, T, being lives one year [o]lder than A, B, and C.) Now since it appears from Note XVII. that $\frac{S}{2r} \times \left(\overline{1 - \overline{r-1}.\ BC.} + \frac{t}{2sr} \times \overline{1 + BT.} - \frac{n}{2mr} \times \overline{1 + PC.}\right)$ is the value of S. depending on the

contingency of C. surviving B.; let R. express that value, then will the value of S. in this Problem be $= R. - S. \times \left(\frac{\overline{1-\overline{r-1}}.\,ABC.}{3r} + \frac{b.\,\overline{1+HBC.}}{6ar} + \frac{t.\,\overline{1+ABT.}}{6sr} + \frac{bt.\,\overline{1+HBT.}}{3asr} - \frac{nb.\,\overline{1+HPC.}}{6amr} - \frac{nt.\,\overline{1+APT.}}{6msr} - \frac{n.\,\overline{1+ACP.}}{3mr}\right)$. When the three lives are of equal age, the value is expressed by $\frac{S}{2r} \times \overline{1-\overline{r-1}.\,CC.} - \frac{S}{3r} \times \overline{1-\overline{r-1}.\,CCC.} = \frac{S}{6r} \times \overline{1-\overline{r-1}.\,3\,CC-2\,CCC.}$, all the fractions after $\frac{\overline{1-\overline{r-1}}.\,ABC}{3r}$ destroying each other.

## NOTE XX. (PROB. XXVIII.)

The payment of the sum in the first year depends on one or other of four events:—1st. That the three lives fail, and that A. dies first; 2dly. That B. dies *after* A., and C. lives; 3dly. That C. dies *after* A., and that B. lives; 4thly. That A. only dies, and that B. and C. both live. These several contingencies are expressed by $\frac{\overline{a-b}.\,\overline{m-n}.\,\overline{s-t}}{3ams} + \frac{\overline{a-b}.\,\overline{m-n}.\,t}{2ams} + \frac{\overline{a-b}.\,\overline{s-t}.\,n}{2ams} + \frac{\overline{a-b}.\,tn}{ams}$. In the second and following years one or other of the same events must take place in order to receive the given sum; that is, the three lives must all fail, A. dying first; or only A. and B. must fail, A. dying first, and C. live; or only A. and C. must fail, A. dying first, and B. live; or only A. must fail, and B. and C. both live. The fractions expressing these contingencies in the second year will be $\frac{\overline{b-c}.\,\overline{n-o}.\,\overline{t-u}}{3ams} + \frac{\overline{b-c}.\,\overline{n-o}.\,u}{2ams} + \frac{\overline{b-c}.\,\overline{t-u}.\,n}{2ams} + \frac{\overline{b-c}.\,ou}{ams.}$; in the third year, $\frac{\overline{c-d}.\,\overline{o-p}\,\overline{u-w}}{3ams} + \frac{\overline{c-d}.\,\overline{o-p}.\,w}{2ams} + \frac{\overline{c-d}.\,\overline{u-w}.\,o}{2ams} + \frac{\overline{c-d}.\,pw}{ams}$, and so on. These fractions being expanded will

become $=\frac{ams}{3amsr}+\frac{bnt}{3amsr^2}+\frac{cou}{3amsr^3}+$, &c. $\left(=\frac{1}{3r}\times\overline{1+ABC.}\right)$
$-\frac{bms}{3amsr}-\frac{cnt}{3amsr^2}-\frac{dou}{3amsr^3}-$, &c. $\left(=-\frac{b}{3ar}\times\overline{1+HBC.}\right)$
$+\frac{ans}{6amsr}+\frac{bot}{6amsr^2}+\frac{cpu}{6amsr^3}+$, &c. $\left(=\frac{n}{6mr}\times\overline{1+APC.}\right)$
$-\frac{nbs}{6amsr}-\frac{oct}{6amsr^2}-\frac{pdu}{6amsr^3}-$, &c. $\left(=-\frac{nb}{6amr}\times\overline{1+HPC.}\right)$
$+\frac{amt}{6amsr}+\frac{bnu}{6amsr^2}+\frac{cow}{6amsr^3}+$, &c. $\left(=\frac{t}{6sr}\times\overline{1+ABT.}\right)$
$-\frac{mbt}{6amsr}-\frac{ncu}{6amsr^2}-\frac{odw}{6amsr^3}-$, &c. $\left(=-\frac{bt}{6asr}\times\overline{1+HBT.}\right)$
$+\frac{ant}{3amsr}+\frac{bou}{3amsr^2}+\frac{cpw}{3amsr^3}+$, &c. $\left(=\frac{nt}{3msr}\times\overline{1+APT.}\right)$
$-\frac{nbt}{3amsr}-\frac{cou}{3amsr^2}-\frac{dpw}{3amsr^3}-$ &c. $\left(=-\frac{ABC.}{3}\right)$. Hence the whole value will be $\frac{S.}{r}$ into $\left(\frac{\overline{1-r}-1.\ ABC.}{3}+\frac{n.\overline{1+APC.}}{6m}+\frac{t.\overline{1+ABT.}}{6s}+\frac{nt.\overline{1+APT.}}{3ms}-\frac{b.\overline{1+HBC.}}{3a}-\frac{bt.\overline{1+HBT.}}{6as}-\frac{nb.\overline{1+HPC.}}{6ma}\right)$. When the three lives are equal, the value becomes $\frac{S.\overline{1-r}-1.\ CCC.}{3r}$.

From the demonstration in the preceding Note (XIX.*), it appears, that if the value of the reversion, depending on the contingency of B. surviving A. *after* C, be deducted from the value depending on the contingency of B. surviving A, whether C. be then living or not, we shall have the value of the reversion in the present case.

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* It should be observed, that in that Note C is made to survive B after A; but in the present case, B is made to survive A after C. The symbols therefore in the one must all be changed to produce the proper expressions in the other: that is, B must be substituted in the present case for C, A for B, C for A, *m* for *s*, *a* for *m*, *s* for *a*, &c.

## NOTE XXI. (Prob. xxix.)

In the first year the value depends on the contingency of one or other of three events:—1st. That the three lives become extinct, A. being the second that fails; 2dly. That A. dies after B., and C. lives; 3dly. That A. dies after C., and B. lives. But in the second and following years, the value will depend on the contingency of either of seven events:—1st. That the three lives fail in the year, A. being the second that fails; 2dly. That A. dies after B. in the year, and C. lives to the end of it; 3dly. That A. dies after C. in the year, and B. lives to the end of it; 4thly. That A. only dies in the year, B. having died before the beginning, and C. living to the end of it; 5thly. That A. only dies in the year, C. having died before the beginning, and B. living to the end of it; 6thly. That A. and C. both die in the year (A. dying first), B. having died before the beginning of that year; and, 7thly. That A. and B. both die in the year (A. dying first), C. having died before the beginning of that year. The several expressions denoting these contingencies in the first year will be $\frac{\overline{a-b}\cdot\overline{m-n}\cdot\overline{s-t}}{3ams} + \frac{\overline{a-b}\cdot\overline{m-n}\cdot t}{2ams} + \frac{\overline{a-b}\cdot\overline{s-t}\cdot n}{2ams}$. In the second year $\frac{\overline{b-c}\cdot\overline{n-o}\cdot\overline{t-u}}{3ams} + \frac{\overline{b-c}\cdot\overline{n-o}\cdot u}{2ams} + \frac{\overline{b-c}\cdot\overline{t-u}\cdot o}{2ams} + \frac{\overline{b-c}\cdot\overline{m-n}\cdot u}{ams} + \frac{\overline{b-c}\cdot\overline{s-t}\cdot o}{ams} + \frac{\overline{b-c}\cdot\overline{t-u}\cdot\overline{m-n}}{2ams} + \frac{\overline{b-c}\cdot\overline{n-o}\cdot\overline{s-t}}{2ams}$.

In the third year, $\frac{\overline{c-d}\cdot\overline{o-p}\cdot\overline{u-w}}{3ams} + \frac{\overline{c-d}\cdot\overline{o-p}\cdot w}{2ams} + \frac{\overline{c-d}\cdot\overline{u-t}\cdot p}{2ams} + \frac{\overline{c-d}\cdot\overline{m-o}\cdot w}{ams} + \frac{\overline{c-d}\cdot\overline{s-u}\cdot p}{ams} + \frac{\overline{c-d}\cdot\overline{u-w}\cdot\overline{m-o}}{2ams} + \frac{\overline{c-d}\cdot\overline{o-p}\cdot\overline{s-u}}{2ams}$, and so on. These fractions being expanded will form twenty-two series, the sum of which will at last be found

$$= \frac{1-\overline{r-1}.\ AB.}{2r} - \frac{b.}{2ar} \times \overline{1 + HB.} + \frac{n}{2mr} \times \overline{1 + AP.} +$$
$$\frac{1-\overline{r-1}.\ AC.}{2r} - \frac{b}{2ar} \times \overline{1 + HC.} + \frac{t}{2sr} \times \overline{1 + AT.} -$$
$$\frac{2.\times\overline{1-\overline{r-1}.\ ABC.}}{3r} - \frac{n.\ \overline{1 + APC.}}{3mr} - \frac{t.\ \overline{1 + ABT.}}{3sr} + \frac{2b.\ \overline{1 + HBC.}}{3ar}$$
$$+ \frac{bt.\ \overline{1 + HBT.}}{3asr} - \frac{2nt.\ \overline{1 + APT.}}{3msr} + \frac{nb.\ \overline{1 + HPC.}}{3amr}.$$

The first three fractions express the value of S. on the contingency of B. surviving A. (= D.*) The next three fractions express the same value on the contingency of C. surviving A. (= E.*) The remaining fractions are equal to twice the value of S. by the preceding Problem with a negative sign. Let such value be denoted by M, and the required value will be equal to D. + E. − 2 M. When the three lives are of equal age, the value will be S. $\frac{\overline{1-\overline{r-1}.\ 3CC. - 2CCC}}{3r}$.

## NOTE XXII. (Prob. xxx.)

The value of the given sum in the first year depends only on the extinction of the three lives, A. dying last; and this contingency is expressed by the single fraction $\frac{\overline{a-b}.\ \overline{m-n}.\ \overline{s-t}.}{8ams}$. In the second and following years the value depends on either of four events.—1st. On the contingency of the three lives failing in the year, A. dying last;—2dly. On the contingency of A.'s dying *after* B. in the year, C. having died in either of the preceding years;—3dly. On the contingency of A.'s dying after C. in the year, B. having died in either of the preceding years;—4thly. On the contingency of A.'s dying in the year, B. and

* See Note XVII.

C. having *both* died in either of the preceding years. In the second year therefore the fractions expressing those contingencies will be $\frac{\overline{b-c}\cdot\overline{n-o}\cdot\overline{t-u}}{3ams}+\frac{\overline{b-c}\cdot\overline{n-o}\cdot\overline{s-t}.}{2ams}+\frac{\overline{b-c}\cdot\overline{t-u}\cdot\overline{m-n},}{2ans}+\frac{\overline{b-c}\cdot\overline{m-n}.\overline{s-t}.}{ams}$. In the third year they will be $\frac{\overline{c-d}\cdot\overline{o-p}\cdot\overline{u-w}}{3ams}+\frac{\overline{c-d}\cdot\overline{o-p}\cdot\overline{s-u}}{2ams}+\frac{\overline{c-d}\cdot\overline{u-w}\cdot\overline{m-o}.}{2ams}+\frac{\overline{c-d}\cdot\overline{m-o}.\overline{s-u}}{ams}$, and so on. Multiplying these fractions respectively into $\frac{1}{r}$, $\frac{1}{r^2}$, $\frac{1}{r^3}$, &c., and expanding them into twenty-two different series, the sum of the whole will at last be found equal to $\frac{1-\overline{r-1}.\ A.}{r}-\frac{1-\overline{r-1}.\ AB.}{2r}+\frac{b.\ \overline{1+HB}.}{2ar}-\frac{n.\ \overline{1+AP}.}{2mr}-\frac{1-\overline{r-1}.\ AC.}{2r}-\frac{t.\ \overline{1+AT}.}{2sr}+\frac{b}{2ar}\times\overline{1+HC}+\frac{1-\overline{r-1}.\ ABC.}{3r}-\frac{b.\ \overline{1+HBC}.}{3ar}+\frac{n.\ \overline{1+APC}.}{6mr}-\frac{nb.\ \overline{1+HPC}.}{6amr}+\frac{t.\ \overline{1+ABT}.}{6sr}-\frac{bt.\ \overline{1+HBT}.}{6asr}+\frac{nt.\ \overline{1+APT}.}{3msr}$. The first fraction expresses the value of S. after A.'s decease; the next six fractions are equal $-\overline{D.+E.}$; and the remaining ones are equal to M.* Let the absolute reversion after A. be denoted by F., and the value required will be $=\overline{F.+M.}-\overline{D.+E.}$ When the three lives are of equal age, the value will be $\frac{S.\ \overline{1+r-1}\cdot\overline{3C-3CC.+CCC.}}{3r}$, or one-third of the value of the reversion after the longest of the three lives.

This Problem may be solved without the assistance of an algebraical process; the value in this case being evidently the *difference* between the value of the *absolute* reversion after death of A., and the values of the two

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* See Note XX.

*contingent* reversions depending on his life having been the second or the first that failed;—that is, it will be = F. − $\overline{\text{D.}+\text{E.}}$ + 2 M. − M. = F. + M. − $\overline{\text{D.}+\text{E.}}$ (M. D. E. denoting the same quantities as in the foregoing Note.)

## NOTE XXIII. (Prob. xxxi.)

The payment of the given sum in the first year depends upon either of two events:—1st. On the extinction of the three lives, C having died last; 2dly. On the extinction only of the two lives A and B, C having survived the year. In the second and following years the payment will depend on either of six events:—1st. On the extinction of the three lives in the year, C having died last; 2dly. On the extinction only of the two lives A and B, C having survived that year; 3dly. On the decease of B in the year, A having died before the beginning, and C living to the end of it; 4thly. On the decease of A in the year, B having died before the beginning, and C living to the end of it; 5thly. On the extinction of the life of C *after* A in the year, B having died before the beginning of it; 6thly. On the extinction of the life of C *after* B in the year, A having died before the beginning of it. By reasoning as in Note XIX, the different fractions expressing the value of the reversion in the respective years will be, for the first year, $\frac{\overline{a-b}\cdot\overline{s-t}\cdot\overline{m-n}}{3ams} + \frac{\overline{a-b}\cdot\overline{m-n}\cdot t}{2ams}$; for the second year $\frac{\overline{b-c}\cdot\overline{n-o}\cdot\overline{t-u}}{3ams} + \frac{\overline{b-c}\cdot\overline{n-o}\cdot u}{2ams} + \frac{\overline{n-o}\cdot\overline{a-b}\cdot u}{ams} + \frac{\overline{b-c}\cdot\overline{m-n}\cdot u}{ams} + \frac{\overline{b-c}\cdot\overline{t-u}\cdot\overline{m-n}}{2ams} + \frac{\overline{t-u}\cdot\overline{n-o}\cdot\overline{a-b}}{2ams}$; for the third year $\frac{\overline{c-d}\cdot\overline{o-p}\cdot\overline{u-w}}{3ams} + \frac{\overline{c-d}\cdot\overline{o-p}\cdot w}{2ams} + \frac{\overline{o-p}\cdot\overline{a-c}\cdot w}{ams} + \frac{\overline{c-d}\cdot\overline{m-o}\cdot w}{ams}$

$+ \frac{\overline{c-d}.\,\overline{u-w}.\,\overline{m-o}.}{2ams} + \frac{\overline{o-p}.\,\overline{u-w}.\,\overline{a-c}}{2ams}$, and so on for the othe

years, $= \frac{\overline{a-b}.\,t}{2asr} + \frac{\overline{b-c}.\,u}{2asr^2} + \frac{\overline{c-d}.\,w}{2asr^3}$, &c. $+ \frac{\overline{a-b}.u}{2asr} + \frac{\overline{b-c}.}{2asr^2}$

$+ \frac{\overline{c-d}.\,x}{2asr^3}$, &c. $- \frac{\overline{a-b}.\,ms}{6amsr} - \frac{\overline{b-c}.\,nt}{6amsr^2} - \frac{\overline{c-d}.\,ou}{6amsr^3}$, &c. $+ \frac{\overline{a-b}.}{2amsr}$

$+ \frac{\overline{a-c}.\,ou}{2asmr^3} + \frac{\overline{a-d}.\,pw}{2amsr^4} +$, &c. $- \frac{\overline{a-b}.\,ns}{8amsr} - \frac{\overline{b-c}.\,ot}{3amsr^2} - \frac{\overline{c-d}.\,pu}{3amsr^3} -$

&c. $- \frac{\overline{a-b}.\,ot}{2amsr^2} - \frac{\overline{a-c}.\,pu}{2amsr^3} - \frac{\overline{a-d}.\,qw}{2ams^4} -$, &c. $+ \frac{\overline{a-b}.\,mt}{6amsr} +$

$\frac{\overline{b-c}.\,nu}{6amsr^2} + \frac{\overline{c-d}.\,ow}{6amsr^3} +$, &c. $+ \frac{\overline{a-b}.\,nu}{2amsr^2} + \frac{\overline{a-c}.\,ow}{2amsr^3} + \frac{\overline{a-d}.\,px}{2amsr^4} +$, &c

$- \frac{2.\,\overline{a-b}.\,nt.}{3amsr} - \frac{2.\,\overline{b-c}.\,ou}{3amsr^2} - \frac{2.\,\overline{c-d}.\,pw}{3amsr^3} -$, &c. $- \frac{\overline{a-b}.\,ou}{2amsr^2} -$

$\frac{\overline{a-c}.\,pw}{2amsr^3} - \frac{\overline{a-d}.\,qx}{2amsr^4} -$, &c. From the demonstration in Not XVII. it appears that the first two series are equal to D or the value of the given sum on the contingency of C surviving A. Pursuing the same steps as in Note XIX. the other series may be found equal to R $- \frac{2}{3r} \times \overline{1-\overline{r-1}.\,\text{ABC}.} + \frac{n}{6mr} \times \overline{1+\text{ACP}.} + \frac{b}{6ar}\,\overline{1+\text{HBC}.} + \frac{nb.\,\overline{1+\text{HPC}}.}{2amr} - \frac{nt.\,\overline{1+\text{APT}}.}{6msr} - \frac{bt.\,\overline{1+\text{HBT}}.}{6asr} - \frac{t.\,\overline{1+\text{ABT}}.}{3sr}$. The whole value therefore will be $\text{D}+\text{R}-\text{S}. \times \left(\frac{2.\,\overline{1-\overline{r-1}.\,\text{ABC}}.}{3r} + \frac{n.\,\overline{1+\text{ACP}}.}{6mr}, \text{\&c. \&c.}\right)$

The solution of this Problem may be derived from that of the twenty-seventh Problem; the value in this case being the value of the given sum payable if C lives to the decease of B. in case A is then extinct; and payable also if C lives to the decease of A, in case B. is then extinct.— On the first contingency the value is R $-$ S $\left(\frac{\overline{1-\overline{r-1}\ \text{ABC}}.}{3r} + \frac{n}{3mr} \times \overline{1+\text{APC}.} + \frac{nb.\,\overline{1+\text{HPC}}.}{6amr} + \frac{nt.\,\overline{1+\text{APT}}.}{6msr} - \frac{b.}{6ar} \times\right.$

$\overline{1 + HBC}. - \frac{bt.\,\overline{1 + HBT}.}{3asr} - \frac{t.\,\overline{1 + ABT}.}{6sr}\Big)$. By substituting $a$, $b$, A, H, respectively for $m$, $n$, B, P, in the above formula, the value on the second contingency will be

$$D - S. \times \Big(\frac{\overline{1-r-1}.\,ABC.}{3r} + \frac{b}{3ar} \times \overline{1 + HBC} + \frac{nb.\,\overline{1 + HPC}.}{6amr} + \frac{bt.\,\overline{1 + HBT}.}{6asr} - \frac{n}{6mr} \times \overline{1 + APC} - \frac{nt.\,\overline{1 + APT}.}{3msr} - \frac{t.\,\overline{1 + ATB}.}{6sr}\Big).$$

Adding these two values we have $D + R - S. \times$

$$\Big(\frac{2.\,\overline{1-r-1}\,ABC.}{3r} + \frac{n}{6mr} \times \overline{1 + APC} + \frac{b}{6ar} \times \overline{1 + HBC} + \frac{nb.\,\overline{1 + HPC}.}{3amr.} - \frac{nt.\,\overline{1 + APT}.}{6msr} - \frac{bt.\,\overline{1 + HBT}.}{6asr} - \frac{t.\,\overline{1 + ABT}.}{3sr}\Big)$$

for the value required. When the three lives are of equal age the value becomes $= \frac{S}{3r} \times \overline{1-r-1.\ 3\,BB - 2\,BBB}$.

## NOTE XXIV. (Prob. xxxiv.)

In the first year the payment of the given sum depends upon either of four events:—1st. That the three lives become extinct, A having been the first or second that failed; —2dly. That A and B both die, and that C lives to the end of the year;—3dly. That A and C both die, and that B lives to the end of the year;—4thly. That only A dies, and that B and C both live to the end of the year; which contingencies are $\frac{2.\,\overline{a-b}.\,\overline{m-n}.\,\overline{s-t}.}{3ams} + \frac{\overline{a-b}.\,\overline{m-n}.\,t}{ams} + \frac{\overline{a-b}.\,\overline{s-t},\,n}{abc} + \frac{\overline{a-b}.\,nt}{ams}$. In the second and following years the payment will depend upon either of eight events:—1st. That all the three lives fail in the year, A having been the first or second that failed; 2dly. That C survives, and that A and B both die in the year; 3dly. That B survives, and

that A and C both die in the year; 4thly. That both B and C survive, and A only dies in the year; 5thly. That A dies in the year, B having died before the beginning, and C lived to the end of it; 6thly. That A dies in the year, C having died before the beginning, and B lived to the end of it; 7thly. That C dies *after* A in the year, B having died in either of the preceding years; 8thly. That B dies after A in the year, C having died in either of the preceding years. The fractions denoting these contingencies for the second year are $\frac{2.\ \overline{b-c}.\ \overline{n-o}.\ \overline{t-u}}{3ams} + \frac{\overline{b-c}.\ \overline{n-o}.\ u}{ams} + \frac{\overline{b-c}.\ \overline{t-u}.\ o}{ams} + \frac{\overline{b-c}.\ ou}{ams}$ $+ \frac{\overline{b-c}.\ \overline{m-n}.\ u}{ams} + \frac{\overline{b-c}.\ \overline{s-t}.\ o}{ams} + \frac{\overline{b-c}.\ \overline{t-u}.\ \overline{m-n}}{2ams} + \frac{\overline{b-c}.\ \overline{n-o}.\ \overline{s-t}}{2ams}$ and so on for the other years. These fractions being expanded into twenty different series, their sum at last may be found $= \frac{S}{2r} \times \left(\overline{1-\overline{r-1}.\ AB.} + \frac{n.\ \overline{1+AP.}}{m} - \frac{b.\ \overline{1+HB.}}{a}\right) + \frac{S}{2r} \times \left(\overline{1-\overline{r-1}.\ AC.} + \frac{t.\ \overline{1+AT.}}{s} - \frac{b.\ \overline{1+HC.}}{m}\right)$ $- \frac{S}{r} \times \left(\frac{\overline{1-\overline{r-1}.\ ABC.}}{3} + \frac{n.\ \overline{1+APC.}}{6m} + \frac{t.\ \overline{1+ABT.}}{6s} + \frac{nt.\ \overline{1+APT.}}{3ms}\right.$ $\left.- \frac{b.\ \overline{1+HBC.}}{3a} - \frac{bt.\ \overline{1+HBT.}}{6as} - \frac{nb.\ \overline{1+HPC.}}{6ma}\right)$. Retaining the same symbols as in Note XXI. the above expressions become $D+E-M.$, which might also have been obtained from the solutions of the twenty-seventh and twenty-eighth Problems, by adding the sum of the values of the reversion depending on the contingency of A. being the first, and also on the contingency of A. being the second that fails; that is by adding M to $D + E - 2M = D + E - M$. When the lives are all equal the value is $= \frac{S}{2r} \times \overline{2-\overline{r-1}.\ 3CC-CCC.}$

## NOTE XXV. (PROB. XXXV.)

ıe payment of the given sum in the first year will de- on either of three events:—1st. That all the three become extinct, A having been the second or third ıas failed; 2dly. That A dies *after* B and C lives to ıd of the year;—3dly. That A dies after C and B to the end of the year. These three contingencies $= \frac{2.\overline{a-b}.\overline{m-n}.\overline{s-t}}{3ams} + \frac{\overline{a-b}.\overline{m-n}.t}{2ams} + \frac{\overline{a-b}.\overline{s-t}.n}{2ams}$. In econd and following years the payment will depend ther of eight events:—1st. That all the three lives ı the year, A having been the second or third that ıiled; 2dly. That A dies *after* B in the year and C to the end of it; 3dly. That A dies *after* C in the ınd B lives to the end of it; 4thly. That A and C die in the year, B having died before the beginning ; 5thly. That A and B both die in the year, C g died before the beginning of it; 6thly. That A dies in the year, B and C having both died e the beginning of it; 7thly. That A dies in the B having died before the beginning and C lived e end of it; 8thly. That A dies in the year, C g died before the beginning and B lived to the f it. The fractions denoting these contingencies for ıecond year are $\frac{2.\overline{b-c}.\overline{n-o}.\overline{t-u}}{3ams} + \frac{\overline{b-c}.\overline{n-o}.u}{2ams} +$ $\frac{\overline{-u}.o}{\text{ıs}} + \frac{\overline{b-c}.\overline{t-u}.\overline{m-n}}{ams} + \frac{\overline{b-c}.\overline{n-o}.\overline{s-t}}{ams} + \frac{\overline{b-c}.\overline{m-n}.\overline{s-t}}{ams}$ $\frac{\overline{-c}.\overline{m-n}.u}{ams} + \frac{\overline{b-c}.\overline{s-t}.o}{ams}$; for the third year $=$ $\frac{\overline{l}.\overline{o-p}.\overline{u-w}}{3ams} + \frac{\overline{c-d}.\overline{o-p}.w}{2ams} + \frac{\overline{c-d}.\overline{u-w}.p}{2ams} + \frac{\overline{c-d}.\overline{u-w}.\overline{m-o}}{ams}$ $\frac{\overline{-d}.\overline{o-p}.\overline{s-u}}{ams} + \frac{\overline{c-d}.\overline{m-o}.\overline{s-u}}{ams} + \frac{\overline{c-d}.\overline{m-o}.w}{ams} + \frac{\overline{c-d}.\overline{s-u}.p}{ams}$.

These fractions being expanded will at last be found $=$

$$\frac{s}{r} \times \overline{1 - \overline{r - 1}.\ A} - \frac{s}{r} \times \left( \frac{\overline{1-\overline{r-1}.\ ABC.}}{3} + \frac{n.\ \overline{1+APC.}}{6m} + \frac{t.\ \overline{1+ABT.}}{6s} + \frac{nt.\ \overline{1+APT.}}{3ms} - \frac{b.\ \overline{1+HBC.}}{3a} - \frac{bt.\ \overline{1+HBT.}}{6as} - \frac{nb.\ \overline{1+HPC.}}{6ma} \right).$$

The first of these expressions is equal to the reversion of S after A, and the second is equal to $-$ M. (see the preceding Note.) Hence the truth of the general rule is manifest.

The solution of this Problem may also be obtained from the twenty-third and twenty-eighth Problems, and is indeed almost self-evident; but for greater accuracy I have chosen to give the solutions of this and of all the other Problems independent of each other. When the three lives are of equal age the value will be $= \frac{S}{3r} \times \overline{2-\overline{r-1}.\ \overline{3\,C-CCC}}.$

## NOTE XXVI. (Prob. xxxvi.)

In the first year the payment of the given sum depends upon either of four events:—1st. That the three lives fail, A being the first or last that dies; 2dly. That B dies after A, and C lives; 3dly. That C dies after A, and B lives; 4thly. That A only dies, and B and C both live. In the second and following years, the payment will depend on either of seven events:—1st. That the three lives fail in the year, A having been the first or last that died; 2dly. That B dies after A in the year, and C lives to the end of it; 3dly. That C dies after A in the year, and B lives to the end of it; 4thly. That A only dies, B and C having both lived to the end of the year; 5thly. That A's life fails after B in the year, C having died before the beginning of it; 6thly. That A's life fails after C in the year, B having died before the beginning of it; 7thly. That A

only dies in the year, B and C having both died in either of the preceding years. The fractions expressing the probability of these events in the first year are $\frac{2.\overline{a-b}.\overline{m-n}.\overline{s-t}}{3ams} + \frac{\overline{m-n}.\overline{a-b}.t}{2ams} + \frac{\overline{s-t}.\overline{a-b}.n}{2ams} + \frac{\overline{a-b}.tn}{ams}$. In the second year they are $\frac{2.\overline{b-c}.\overline{n-o}.\overline{t-u}}{3ams} + \frac{\overline{n-o}.\overline{b-c}.u}{2ams} + \frac{\overline{t-u}.\overline{b-c}.o}{2ams} + \frac{\overline{b-c}.ou}{ams} + \frac{\overline{b-c}.\overline{n-o}.\overline{s-t}.}{2ams} + \frac{\overline{b-c}.\overline{t-u}.\overline{m-n}.}{2ams} + \frac{\overline{b-c}.\overline{m-n}.\overline{s-t}}{ams}$. In the third year they are $\frac{2.\overline{c-d}.\overline{o-p}.\overline{u-w}}{3ams} + \frac{\overline{o-p}.\overline{c-d}.w}{2ams.} + \frac{\overline{u-w}.\overline{c-d}.p}{2ams} + \frac{\overline{c-d}.p.w}{ams} + \frac{\overline{c-d}.\overline{o-p}.\overline{s-u}}{2ams} + \frac{\overline{c-d}.\overline{u-w}.\overline{m-o}}{2ams} + \frac{\overline{c-d}.\overline{s-u}.\overline{m-o}}{ams}$, and so on for the other years. These fractions being expanded will form twenty-two different series, which may be found $= \frac{S}{r} \times \overline{1-\overline{r-1}.\,A} - \frac{S}{2r} \times \left(\overline{1-\overline{r-1}.\,AB}. + \frac{n.\overline{1+AP}.}{m} - \frac{b}{a} \times \overline{1+HB}.\right) - \frac{S}{2r} \times \left(\overline{1-\overline{r-1}.\,AC}. + \frac{t}{s} \times \overline{1+AT}. - \frac{b}{a} \times \overline{1+HC}.\right) + \frac{S}{r} \left(\frac{2.\overline{1-\overline{r-1}\,ABC}.}{3} + \frac{t}{3s} \times \overline{1+ABT}. + \frac{n}{3m} \times \overline{1+APC}. - \frac{2b}{3a} \times \overline{1+HBC}. - \frac{bt}{3as} \times \overline{1+HBT}. + \frac{2nt}{3ms} \times \overline{1+APT}. - \frac{nb}{3ma} \times \overline{1+HPC}.\right)$. The first of these expressions denotes the value of S. after the decease of A (= F); the second denotes the value on the contingency of B surviving A (= D*); the third denotes the value of S on the contingency of C surviving A (= E*); the fourth denotes *twice* the value of S on the contingency of A's being the *last* that fails of the three lives, by Note XX. (= 2 M).

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* See Note XXI.

Hence the whole value of the reversion will be $F + 2M - \overline{D + E}$.

The solution of this, like some of the preceding Problems, may be obtained more easily from other Problems, supposing those Problems to have been previously solved; the value in the present case being evidently the *difference* between the absolute value after the decease of A, found by Prob. XXIII, and the value on the contingency of A's life having been the *second* that failed, found by Prob. XXIX; that is, it will be $= F + 2M - \overline{D + E}$. *

If the three lives are of equal age the value will be $= \frac{s}{3r} \times \overline{2-r-1} \cdot \overline{3C - 3CC. + 2CCC.}$

## NOTE XXVII. (Prob. XXXVII.)

In each year the reversion will depend upon either of six events:—1st. If the three lives should fail in the year, A or B having died first; 2dly. If A and B should die in the year, and C live; 3dly. If C should die after A in the year, and B live; 4thly. If C should die after B in the year, and A live; 5thly. If A only should die in the year, and B and C both live; 6thly. If B only should die in the year, and A and C should both live. The fractions denoting these several contingencies, in the first year are $\frac{2.\overline{a-b}.\overline{m-n}.\overline{s-t}}{3ams} + \frac{\overline{a-b}.\overline{m-n}.t}{ams} + \frac{\overline{a-b}.\overline{s-t}.n}{2ams} + \frac{\overline{m-n}.\overline{s-t}.b}{2ams} + \frac{\overline{a-b}.nt}{ams} + \frac{\overline{m-n}.bt}{ams}$; in the 2d year they are $\frac{2.\overline{b-c}.\overline{n-o}.\overline{t-u}}{3ams} + \frac{\overline{b-c}.\overline{n-o}.u}{ams} + \frac{\overline{b-c}.\overline{t-u}.o}{2ams} + \frac{\overline{n-o}.\overline{t-u}.c}{2ams} + \frac{\overline{b-c}.ou}{ams} + \frac{\overline{n-o}.cu}{ams}$, and so on in the other years. These fractions may be

* See Note XXI.

expanded into the eight following series:—(first) $\frac{S}{3ams} \times \overline{\frac{amt}{r} + \frac{bnu}{r^2} + \frac{cow}{r^3}, \&c.} = \frac{S}{3r} \times \frac{t.\overline{1+ABT}}{s}$; (second) $\frac{S}{6ams} \times \overline{\frac{bmt}{r} + \frac{cnu}{r^2} + \frac{dow}{r^3}, \&c.} = \frac{S}{6r} \times \frac{bt.\overline{1+HBT}}{as}$; (third) $\frac{S}{6ams} \times \overline{\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3}, \&c.} = \frac{S}{6r} \times \frac{nt.\overline{1+APT}}{as}$; (fourth) $-\frac{2S}{3ams} \times \overline{\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3}, \&c.} = -\frac{2S}{3} \times ABC.$; (fifth) $\frac{2S}{3amsr} \times \overline{\frac{ans}{r} + \frac{bnt}{r^2} + \frac{cou}{r^3}, \&c.} = \frac{2S}{3r} \times \overline{1+ABC}.$; (sixth) $-\frac{S}{6ams} \times \overline{\frac{bms}{r} + \frac{cnt}{r^2} + \frac{dou}{r^3}, \&c.} = -\frac{S}{6r} \times \frac{b.\overline{1+HBC}}{a}$; (seventh) $-\frac{S}{6ams} \times \overline{\frac{ans}{r} + \frac{bot}{r^2} + \frac{cpu}{r^3}, \&c.} = -\frac{S}{6r} \times \frac{n.\overline{1+APC}}{m}$; (eighth) $-\frac{S}{3ams} \times \overline{\frac{bns}{r} + \frac{cot}{r^2} + \frac{dpu}{r^3}, \&c.} = -\frac{S}{3r} \times \frac{nb.\overline{1+HPC}}{am}$. Consequently the whole value will be $\frac{S}{r} \times \left(\frac{2.\overline{1-\overline{r-1}.ABC}}{3} + \frac{t.\overline{1+ABT}}{3s} - \frac{b.\overline{1+HBC}}{6a} - \frac{n.\overline{1+APC}}{6m} + \frac{bt.\overline{1+HBT}}{6as} + \frac{nt.\overline{1+APT}}{6ms} - \frac{nb.\overline{1+HPC}}{3am}\right)$. And when the lives are equal, $\frac{S}{r} \times \frac{2 \times \overline{1-\overline{r-1}\,CCC}}{3}$.

## NOTE XXVIII. (Prob. xxxviii.)

In the first year the payment of the given sum will depend on either of four events:—1st. That the three lives fail, A or B having been the *second* that failed; 2dly. that A dies after C, and B lives; 3dly. That B dies after C, and A lives; 4thly. That A and B both die, and C lives. In the second and following years the given sum will become payable, provided either of eleven events should take place. 1st, if the three lives should fail in

the year, A or B having been the second that failed; 2dly. If A should die after C in the year and B live; 3dly. If B should die after C in the year and A live; 4thly. If A and B both die in the year, and C live; 5thly. If B only should die in the year, A having died before the beginning, and C lived to the end of it; 6thly. If A only should die in the year, B having died before the beginning and C lived to the end of it; 7thly. If C should die after A in the year, B having died in either of the foregoing years; 8thly. If C should die after B in the year, A having died in either of the foregoing years; 9thly. If A and B should both die in the year, C having died in either of the preceding years; 10thly. If B only should die in the year, C having died before the beginning and A lived to the end of it; and lastly, If A only should die in the year, C having died before the beginning and B lived to the end of it. The several fractions expressing those contingencies in the first year will be $\frac{2.\overline{a-b}.\overline{m-n}.\overline{s-t}}{3ams} + \frac{\overline{a-b}.\overline{s-t}.n}{2ams} + \frac{\overline{m-n}.\overline{s-t}.b}{2ams} + \frac{\overline{a-b}.\overline{m-n}.t}{ams}$. In the second year $= \frac{2.\overline{b-c}.\overline{n-o}.\overline{t-u}}{3ams} + \frac{\overline{b-c}.\overline{t-u}.o}{2ams} + \frac{\overline{n-o}.\overline{t-u}.c}{2ams} + \frac{\overline{b-c}.\overline{n-o}.u}{ams} + \frac{\overline{n-o}\ \overline{a-b}.u}{ams} + \frac{\overline{b-c}.\overline{m-n}.u}{ams} + \frac{\overline{b-c}.\overline{t-u}.\overline{m-n}}{2ams} + \frac{\overline{n-o}.\overline{t-u}.\overline{a-b}}{2ams} + \frac{\overline{b-c}.\overline{n-o}.\overline{s-t}}{ams} + \frac{\overline{n-o}.\overline{s-t}.c}{ams} + \frac{\overline{b-c}.\overline{s-t}.o}{ams}$. In the third year $= \frac{2,\overline{c-d}.\overline{o-p}.\overline{u-w}}{3ams} + \frac{\overline{c-d}.\overline{u-w}.p}{2ams} + \frac{\overline{o-p}.\overline{u-w}.d}{2ams} + \frac{\overline{c-d}.\overline{o-p}.w}{ams} + \frac{\overline{o-p}.\overline{a-c}.w}{ams} + \frac{\overline{c-d}.\overline{m-o}.w}{ams} + \frac{\overline{c-d}.\overline{u-w}.\overline{m-o}}{2ams} + \frac{\overline{o-p}.\overline{u-w}.\overline{a-c}}{2ams} + \frac{\overline{c-d}.\overline{o-p}.\overline{s-u}}{ams} + \frac{\overline{o-p}.\overline{s-u}.d}{ams} + \frac{\overline{c-d}.\overline{s-u}.p}{ams}$, and so on. These fractions being expanded will form twenty-five different series, the first, second, third, &c. terms of which being respectively multiplied into $\frac{1}{r}, \frac{1}{r^2}, \frac{1}{r^3},$

&c., their sum will at last be found $= \frac{1-\overline{r-1}.AB}{r} + \frac{1-\overline{r-1}.BC}{2r} + \frac{t.\overline{1+BT}}{2sr} - \frac{n.\overline{1+PC}}{2mr} + \frac{1-\overline{r-1}.AC}{2r} + \frac{t.\overline{1+AT}}{2sr} - \frac{b.\overline{1+HC}}{2ar} - \frac{4.\overline{1-\overline{r-1}.ABC}}{3r} - \frac{2t.\overline{1+ABT}}{3sr} + \frac{b.\overline{1+HBC}}{3ar} + \frac{n.\overline{1+APC}}{3mr} - \frac{bt.\overline{1+HBT}}{3asr} - \frac{nt.\overline{1+APT}}{3msr} + \frac{2nb.\overline{1+HPC}}{3amr}$. The first of these expressions is the value of the given sum after the extinction of the two *joint* lives A and B. The three following fractions denote the value on the contingency of C surviving B. The next three fractions express the value of the given sum on the contingency of C surviving A. The seven remaining fractions are equal to twice the value of the reversion on the contingency of A or B being the *first* that fail with a *negative sign**. Let these several values be respectively represented by K, R, E, and N; then will the required value be $K+R+E-2N$ - - - which is the general rule.

When the three lives are of equal age the above fractions may be reduced to $\frac{2.S.}{3r} \times \overline{1-\overline{r-1}.\overline{3CC-2CCC}}$.

## NOTE XXIX. (Prob. xxxix.)

The given sum can be received in the first year only on the extinction of the three lives, restrained to the contingency of C's life having been the first or second that failed. In the second and following years it may be received, provided either of six events shall have happened:—1st. If the three lives shall have failed, C having been the first or second that died; 2dly. If A and B shall have both died in the year, C having died before the beginning of it; 3dly. If A only should die in the year, B and C having died in either of the preceding years; 4thly. If B

* See the preceding Note.

only should die in the year, A and C having died in either of the preceding years; 5thly. If A should die after C in the year, B having died before the beginning of it; 6thly. If B should die after C in the year, A having died before the beginning of it. The fractions expressing these contingencies in the first year will be $= \frac{2.\,\overline{a-b}.\,\overline{m-n}.\,\overline{s-t}}{3ams}$. In the second year $= \frac{2.\,\overline{b-c}.\,\overline{n-o}.\,\overline{t-u}}{3ams} + \frac{\overline{b-c}.\,\overline{n-o}.\,\overline{s-t}}{ams} + \frac{\overline{b-c}.\,\overline{m-n}.\,\overline{s-t}}{ams} + \frac{\overline{n-o}.\,\overline{a-b}.\,\overline{s-t}}{ams} + \frac{\overline{n-o}.\,\overline{t-u}.\,\overline{a-b}}{2ams} + \frac{\overline{b-c}.\,\overline{t-u}.\,\overline{m-n}}{2ams}$. In the third year $= \frac{2.\,\overline{c-d}.\,\overline{o-p}.\,\overline{u-w}}{3ams} + \frac{\overline{c-d}.\,\overline{o-p}.\,\overline{s-u}}{ams} + \frac{\overline{c-d}.\,\overline{m-o}.\,\overline{s-u}}{ams} + \frac{\overline{o-p}.\,\overline{a-c}.\,\overline{s-u}}{ams} + \frac{\overline{o-p}.\,\overline{u-w}.\,\overline{a-c}}{2ams} + \frac{\overline{c-d}.\,\overline{u-w}.\,\overline{m-o}}{ams}$, and so on in the other years. These fractions being expanded will form 28 different series, the first, second, third, &c. terms of which being multiplied respectively into $\frac{1}{r}$, $\frac{1}{r^2}$, $\frac{1}{r^3}$, &c. as in the preceding Note, their sum will at last be found $= \frac{\overline{\overline{1-\overline{r-1}.\,\overline{A+B-AB}}}}{r} - \frac{\overline{1-\overline{r-1}.\,BC}}{2r} - \frac{t.\,\overline{1+BT}}{2sr} + \frac{n.\,\overline{1+PC}}{2mr} - \frac{\overline{1-\overline{r-1}.\,AC}}{2r} - \frac{t.\,\overline{1+AT}}{2sr} + \frac{b.\,\overline{1+HC}}{2r} + \frac{2.\,\overline{1-\overline{r-1}.\,ABC}}{3r} - \frac{b.\,\overline{1+HBC}}{6ar} - \frac{n.\,\overline{1+APC}}{6mr} - \frac{nb.\,\overline{1+HPC}}{3amr} + \frac{t.\,\overline{1+ABT}}{3sr} + \frac{bt.\,\overline{1+HBT}}{6asr} + \frac{nt.\,\overline{1+APT}}{6msr}$. The first of these fractions is the value of the reversion after the longest of the two lives A and B. The second, third, and fourth express the value depending on the contingency of C surviving A, with negative signs. The fifth, sixth, and seventh express the value depending on the contingency of C surviving B, with negative signs. The remaining fractions are the same with those in Note XXVI., expressing the value of the reversion depending on the contingency of A or B being the first that fail. Assuming V, E, R, and N to denote these values respectively, the general rule will be expressed by V + N −

$\overline{E+R}$; which becomes $\frac{2S.\ \overline{1-r+1}.\ \overline{3C-3CC+CCC}}{3r}$ when the lives are of equal age, or "two-thirds the value of the absolute reversion after the extinction of the three lives."

The solution of this problem may also be derived from that of the 30th Problem; the value of the given sum being equal to "the difference between the value of the "absolute reversion after the extinction of the three lives, "and the value of the same depending on the contingency "of C being the *last* that dies." Changing the symbols A, H, *a*, *b*, *c*, *d*, &c. in Note XXII. for C, T, *s*, *t*, *u*, &c. and *vice versâ*, the general rule given above may be obtained, by subtracting the whole from $\frac{S.}{r}\overline{1-r-1}.\ \overline{A+B+C-AB-BC-AC+ABC}$; that is calling the longest of the three lives L, and the value by Prob. xxx. O, it will be $\frac{S.\ \overline{1-r-1}.L}{r}-O$; but the operation is not shortened by this rule.

## NOTE XXX.

LEMMA I. Supposing the ages of A and B to be given, to determine, from any table of observations, the probabilities of survivorship between them.

SOLUTION. Retaining the same symbols as in the preceding notes, let it be required to determine the probability of B surviving A. In the first year this event may take place either by the extinction of both lives, A having died first; or by the death of A only, B having survived to the end of the year. These probabilities are expressed by the fractions $\frac{\overline{a-b}.\ \overline{m-n}}{2am}+\frac{\overline{a-b}.\ n}{ab}=\frac{\overline{a-b}.\ \overline{m+n}}{2am}$. In the second and following years the event depends on the same con-

tingencies; that is, on both lives failing in the year, A having died first, or on A's dying in the year, B having lived to the end of it. The fractions therefore denoting these probabilities in the second year will be $\frac{\overline{b-c}.\overline{n-o}}{2am} + \frac{\overline{b-c}.o}{am} = \frac{\overline{b-c}.\overline{n+o}}{2am}$, in the third year $\frac{\overline{c-d}.\overline{o-p}}{2am} + \frac{\overline{c-d}.p}{am} = \frac{\overline{c-d}.\overline{o+p}}{2am}$, and so on. Hence the whole probability of B's surviving A will be $\frac{1}{2am} \times (\overline{m+n}.\overline{a-b} + \overline{n+o}.\overline{b-c} + \overline{o+p}.\overline{c-d}$ + &c.). Having found by this series the probability of B the elder surviving A the younger, the other expression denoting the probability of A's surviving B is well known to be the difference between the foregoing series and *unity*. Were we possessed of a table of the *expectations* of two joint lives the chance of survivorship might easily be obtained from the above series *; nor will it be found a laborious undertaking to compute from it a table of the probabilities of survivorship between two persons of all ages without having recourse to such expectations. For the probability of survivorship between any two persons being found, the probability between two persons one year younger is obtained with little difficulty, and a table for all the older lives, whose difference of age is the same, may be formed in rather less time than would be necessary for computing the expectations of the same number of joint lives. This will be best exemplified by the following operations, founded on the *Northampton* Table of Observations.

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* Supposing the expectation of the joint lives AP to be A′P′; and the expectation of the joint lives HB to be H′B′, the sum of the series in this Lemma will be $\frac{n.\overline{1+A'P'}}{2m} - \frac{a.H'B'}{2b}$.

| Ages of B | Ages of A | Series expressing the probability of B surviving A. | | Probability of A surviving B. |
|---|---|---|---|---|
| 96 | 86 | $\frac{1}{1 \times 145} \times \overline{\frac{0+1}{2}} \times 34$ - - | = .1173 | $1 - .1173 = .8827$ |
| 95 | 85 | $\frac{1}{4 \times 186} \times \left(\overline{\frac{4+1}{2} \times 41} + 17\right)$ - | = .1606 | $1 - .1606 = .8394$ |
| 94 | 84 | $\frac{1}{9 \times 234} \times \left(\overline{\frac{9+4}{2} \times 48} + 119.5\right)$ - | = .2049 | $1 - .2049 = .7951$ |
| 93 | 83 | $\frac{1}{16 \times 289} \times \left(\overline{\frac{16+9}{2} \times 55} + 431.5\right)$ - | = .2420 | $1 - .2420 = .7580$ |
| 92 | 82 | $\frac{1}{24 \times 346} \times \left(\overline{\frac{24+16}{2} \times 57} + 1119\right)$ - | = .2720 | $1 - .2720 = .7280$ |
| 91 | 81 | $\frac{1}{34 \times 406} \times \left(\overline{\frac{34+24}{2} \times 60} + 2259\right)$ - | = .2897 | $1 - .2897 = .7103$ |
| 90 | 80 | $\frac{1}{46 \times 469} \times \left(\overline{\frac{46+34}{2} \times 63} + 3999\right)$ - | = .3022 | $1 - .3022 = 6978$ |

It will readily be seen from these specimens in what manner the probabilities of survivorship between two younger lives are deduced from the probabilities between two older lives whose difference of age is the same; for the numbers 17, - - 119.5 - - - - 431.5, &c. in the second, third, fourth, &c. lines, are the sums of the series next preceding; that is, $17 = \frac{1}{2} \times 34$ - - - $119.5 = \frac{5}{2} \times 41 + 17$: - - - - $431.5 = \frac{13}{2} \times 48 + 119.5$, &c. If the lives be of equal age the above series becomes $= \frac{1}{2}$, which is known to express the probability in that case, and therefore proves the truth of this solution.

Mr. *Simpson* in his Treatise on Annuities and Reversions * has given a curve, whose area determines the probability of survivorship between two persons according to any table of observations. If one of the lives be not very young, so that the equidistant ordinates may not be too few, this area is sufficiently correct. But if the eldest of the two lives is under twenty years of age, it becomes necessary to assume so many equidistant ordinates, to render the solution accurate when the decrements of life are unequal, that the operation is rendered too laborious for practice; nor do I know that it can be necessary to have recourse to this area in any case, especially as the two probabilities of survivorship are so easily computed from the preceding series. As no table of these probabilities according to the real decrements of life has ever been attempted, I have been induced to compute the 9th Table, in the manner described above, which gives the real probabilities of survivorship between two persons of all ages, whose common difference of age is not less than ten years.

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* Lemma 2. page 100.

Lemma II. To determine from any table of observations the probability that B the *elder* of two lives dies *after* A the *younger*, either in any number of years, or during the continuance of the life of B.

Solution. This event can take place in the first year only by the extinction of both lives, A having died first; the probability of which event is expressed by the fraction $\frac{\overline{a-b}.\overline{m-n}}{2am} = \frac{\overline{m+n}.\overline{a-b}}{2am} - n.\overline{a-b}$. In the second year the probability will be increased; for the event may have taken place as above mentioned in the preceding year, or the lives may have failed in the second year, A having died first, or B may have died in this year and A in the first year. The fractions therefore expressing the probability of the event's happening in two years will be $\frac{\overline{a-b}.\overline{m-n}}{2am} + \frac{\overline{b-c}.\overline{n-o}}{2am} + \frac{\overline{n-o}.\overline{a-b}}{am} = \frac{1}{am} \times \left(\frac{\overline{m+n}.\overline{a-b}}{2} + \frac{\overline{n+o}.\overline{b-c}}{2} - o.\overline{a-c}\right)$. In the third year the probability will be still further increased; for in addition to the foregoing contingencies, the event may have taken place by the extinction of the two lives in the third year, A having died first, or by the extinction of the life of A in the first or second year, and of the life of B in the third year. Hence the probability of the events happening in three years will be $\frac{\overline{a-b}.\overline{m-n}}{2am} + \frac{\overline{b-c}.\overline{n-o}}{2am} + \frac{\overline{n-o}.\overline{a-b}}{am} + \frac{\overline{o-p}.\overline{c-d}}{2am} + \frac{\overline{o-p}.\overline{a-c}}{am} = \frac{1}{am} \times \left(\frac{\overline{m+n}.\overline{a-b}}{2} + \frac{\overline{n+o}.\overline{b-c}}{2} + \frac{\overline{o+p}.\overline{c-d}}{2} - p.\overline{a-d}\right)$. By proceeding in the same manner for the fourth year the probability will be found $= \frac{1}{am} \times \left(\frac{\overline{m+n}.\overline{a-b}}{2} + \frac{\overline{b-c}.\overline{n+o}}{2} + \frac{\overline{o+p}.\overline{c-d}}{2} + \frac{\overline{p+q}.\overline{d-e}}{2} - q.\overline{a-e}\right)$, &c. Supposing $\beta$ to be the number of persons living at the age of A when the

life of B becomes extinct by the table, and $\alpha$ the number living in the preceding year; supposing also $\mu$ and $\nu$ to be the number of persons living at the last two ages in the table, then will the whole probability of B.'s dying *after* A be $= \frac{1}{am} \times \left( \frac{\overline{m+n}.\,\overline{a-b}}{2} + \frac{\overline{n+o}.\,\overline{b-c}}{2} + \frac{\overline{o+p}.\,\overline{c-d}}{2} + \frac{\overline{p+q}.\,\overline{d-e}}{2} \right.$

$\left. \text{- - - - - - - -} + \frac{\overline{\mu+\nu}.\,\overline{\alpha-\beta}}{2} - \nu.\,\overline{a-\beta} \right)$. Now, since it is well known that the probability of both lives failing, without any regard to the order of their extinction, during the continuance of the life of B, is $\frac{\overline{a-\beta}.\,\overline{m-\nu}}{am}$, it follows, that the probability of A.'s dying *after* B will be the difference between this fraction and the foregoing series.

As this contingency is of considerable importance, and the solutions of a great number of Problems require that it should be previously ascertained, I have, as in the case of the former Lemma, computed a Table for all ages whose common difference is not less than ten years *, which however has been accomplished with much less difficulty in consequence of having already computed the Table in that Lemma.

This will readily appear by comparing the two series expressing the different contingencies in these Lemmas; the terms in the first, as far as they are continued, being uniformly the same with those in the second Lemma; so that if the fraction $\frac{\nu.\,\overline{a-\beta}}{am}$ in the latter be subtracted from the sum of that series, the remainder will always express the probability of survivorship in the present case. Thus, the probability of a life of 92 surviving another life of 82 by the 1st Lemma is .2720; the probability therefore that a life of 92 dies *after* another life of 82 will be .2720 −

* Tab. 10.

$\frac{201}{24 \times 346} = .2478$. If the ages were 80 and 60 the probability of the former life becoming extinct *after* the latter would be $.1890 - \frac{1286}{2038 \times 469} = .1876$, and if the ages were 85 and 55 the probability would be $.1130 - \frac{896}{186 \times 2448} = .1110$. The probability that two lives aged 92 and 82 fail in four years is $\frac{23 \times 201}{24 \times 346} = .5569$, that two lives aged 80 and 60 fail in sixteen years is $\frac{468 \times 1286}{469 \times 2038} = .6296$, and that two lives aged 85 and 55 fail in eleven years is $\frac{185 \times 896}{186 \times 2448} = .3640$. The probability therefore that a life of 82 fails *after* a life of 92 is $.5569 - .2480 = .3089$, that a life of 60 fails *after* a life of 80 is $.6296 - .1876 = .4420$, and the probability that a life of 55 fails after another life of 85 is $.3640 - .1110 = .2530$.

In this manner the 10th Table has been computed. But if the probability of survivorship, either in this or the preceding Lemma, be confined to any given number of years, I know of no other method of ascertaining it than by a direct summation of the series; which will be composed of an equal number of terms with the years in which the survivorship is to happen. This would be a work of so much labour as to render the solution of any Problem, involving such a contingency, impracticable. The usual method in this case has hitherto been, to take half the probability of the two lives becoming extinct in the given time, without having any regard to the order of survivorship. But when the ages of the two lives are very different, this method must be incorrect; and in order to ascertain the extent of the inaccuracy, I have computed the following Table:

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* See Tab. 9.

# TABLE,

**Shewing the value, at 4 per cent. of £1, payable at the end of a given time, provided the life of B shall have failed after the life of A.**

| Ages. | | 5 years. | | 10 years. | | 15 years. | | 20 years. | | 25 years. | | 30 years. | | 40 years. | | Life. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B* | A | True val. | Approx. | True val. | Approx. | True val. | Approx. | True val. | Approx. | True val. | Approx. | True val. | Approx. | True val. | Approx. | True val. | Approx. |
| 20 | 10 | .00154 | .00153 | .00448 | .00472 | .00885 | .00979 | .01382 | .01513 | .01879 | .02030 | .02504 | .02457 | .02978 | .03118 | .02099 | .02473 |
| 35 | 15 | .00194 | .00206 | .00732 | .00785 | .01452 | .01528 | .02245 | .02316 | .02977 | .03041 | .03588 | .03667 | .04392 | .04523 | .03153 | .03936 |
| 45 | 15 | .00250 | .00266 | .00883 | .01017 | .01882 | .01980 | .02835 | .02958 | .03710 | .03858 | .04456 | .04599 | | | .03663 | .04829 |
| 50 | 30 | .00504 | .00503 | .01654 | .01656 | .03057 | .03087 | .04454 | .04523 | .05723 | .05872 | .06639 | .06897 | | | .05197 | .06819 |
| 55 | 15 | .00347 | .00369 | .01173 | .01377 | .02464 | .02640 | .03710 | .03925 | .04683 | .05000 | | | | | .04022 | .05644 |
| 65 | 20 | .00725 | .00730 | .02402 | .02410 | .04203 | .04326 | .05460 | .05898 | .05525 | .06692 | | | | | .04696 | .06800 |
| 70 | 40 | .01413 | .01421 | .04303 | .04477 | .06948 | .07697 | | | | | | | | | .06688 | .10334 |
| 75 | 60 | .03357 | .03572 | .09545 | .10372 | .12293 | .15521 | | | | | | | | | .11524 | .17570 |
| 80 | 70 | .07653 | .08051 | .15576 | .18868 | | | | | | | | | | | .15915 | .23303 |
| 85 | 10 | .01136 | .01374 | | | | | | | | | | | | | .01987 | .03300 |

* In these specimens (which are sufficient to give an idea of the difference between the true values and the approximation in all cases) I have constantly supposed the life of B to be the eldest. But the differences would have been the same if his life had been the youngest; only that in this case the true values would have varied as much in *excess* as they here do in *defect*. This is obvious from the nature of the approximation.

From this table it appears that the approximated and exact values do not materially differ from each other, till the last years of B's life; and that the principal inaccuracy in adopting the approximation will arise after the extinction of the life of B, when it becomes necessary to multiply the fraction expressing the probability of B's dying after A into the remaining series of the solution. But this is obviated by having recourse to the 9th Table, which gives the correct probability of survivorship during the whole continuance of B's life, and which may be as easily applied to the solution of the Problem after his decease as the approximation.

## NOTE XXXI. (PROB. XL.)

As the approximation appears from the preceding table to be always sufficiently correct till the latter part of B's life, it is evident that no great inaccuracy will arise from having recourse to it, unless the ages of A and B are materially different,—more especially when C is the oldest of the three lives. But when B or A is the oldest, and their ages are very unequal, the fractions expressing the probability of the one surviving the other, or of the one dying after the other, are combined with a series which is often of considerable importance, and consequently the common method of solution fails in these instances. Being however possessed of the tables deduced from the foregoing Lemmas, even this circumstance is attended with little or no difficulty, and a general rule is obtained as short and accurate as in any other case. But this will be more satisfactorily proved by the following operations:—

1st. *Let C be the oldest of the three lives.* The payment of the annuity in the first year depends on one or other of

two events; either on the event of B's having died *after* A and of C's living to the end of the year, or on the event of A's dying, and of B and C both living to the end of the year. The value therefore of the annuity for the first year will be $\frac{\overline{m-n}.\overline{a-b}.t}{2amsr} + \frac{\overline{a-b}.nt}{amsr}$. In the second year the payment of the annuity depends nearly on the same events:—1st. On B's having died *after* A in the first or second year, and C's living to the end of that term; 2dly. On A's *dying* before the end of the second year, and B and C's both *living* to the end of it. Hence the value of the annuity for the second year will be $\frac{\overline{a-c}.\overline{m-o}.u}{2amsr^2}$ $+ \frac{\overline{a-c}.ou}{amsr^2}$. In the third year, by following the same steps, the value of the annuity will be found $= \frac{\overline{a-d}.\overline{m-p}.w}{2amsr^3} +$ $\frac{\overline{a-d}.pw}{amsr^3}$; and in like manner may be found the value of the annuity in each of the remaining years of C's life. The whole value of the annuity therefore will be expressed by the four following series: $\frac{t}{2sr} + \frac{u}{2sr^2} + \frac{w}{2sr^3} +$, &c. $-$ $\frac{bt}{2asr} - \frac{cu}{2asr^2} - \frac{dw}{2asr^3} -$, &c. $+ \frac{nt}{2msr} + \frac{ou}{2msr^2} + \frac{pw}{2msr^3} +$, &c. $- \frac{bnt}{2amsr} - \frac{cou}{2amsr^2} - \frac{dpw}{2amsr^3} -$, &c. $= \frac{C-AC.}{2} + \frac{BC-ABC.}{2}$.

If *A be the oldest of the three lives*, let $z$ denote the number of years between the age of A and that of the last person in the table, $C'$ the value of an annuity on the life of C, and $B'C'$ the value of an annuity on the joint lives of B and C for $z$ years; then will the value of the annuity for the first $z$ years be $\frac{C'-AC.}{z} + \frac{B'C'-ABC.}{z}$. At the expiration of this term the life of A becomes extinct, and the value of the a nuity for the remaining years of C's life

(supposing $\sigma$, $\tau$, $\upsilon$, &c. to denote the number of persons living in the table at the age of a person $\overline{z+1}$, $\overline{z+2}$, $\overline{z+3}$, &c. years older than C, and $\varphi$ to denote the probability by Table 9 of B's surviving A), will be $= \varphi \times \overline{\frac{\sigma}{sr^{z}+1} + \frac{\tau}{sr^{z}+2} + \frac{\upsilon}{sr^{z}+3} +, \&c.} = \varphi . \overline{C - C'}$. The whole value therefore of the annuity in this case will be $\frac{C' + B'C'.}{2} + \varphi . \overline{C - C'} - \frac{AC + ABC.}{2}$.

If *B be the oldest of the three lives*, let $x$ denote the number of years between the ages of B and of the last person in the table, C′ the value of an annuity on the life of C, and A′C′ the value of an annuity on the joint lives of A and C for $x$ years, and $\pi$ the probability (found by Table 10) that B dies after A. Then, by proceeding as above, the value of the annuity in this case will be $\frac{C' - C'A'.}{2} + \pi . \overline{C - C'} + \frac{BC - ABC.}{2}$. When the three lives are of equal age the value will be $\frac{C - CCC.}{2}$.

As this method of solution is applicable to a great number of problems, I have thought it necessary to make the following computations with the view of determining how far it may be depended upon. It is to be observed, that the fractions $\frac{\overline{a-b} \cdot \overline{m-n} . t}{2amsr} + \frac{\overline{a-c} \cdot \overline{m-o} . u}{2amsr^2} + \frac{\overline{a-d} \cdot \overline{m-p} . w}{2amsr^3} +$, &c., should have been, according to Lemma II., in order to express the exact value, $\frac{\overline{a-b} \cdot \overline{m-n} . t}{2amsr} + \frac{\overline{a-b} . \overline{m-o} . + \overline{a-c} . \overline{n-o}}{2amsr^2} \times u +$, &c.; and that it is impossible to find such a general expression for this series as shall render it fit for practice. It has therefore become necessary to have recourse to the preceding method of approximation, and by

comparing the values in the following examples, a correct idea may be formed of its accuracy.

| Ages. | | | * True Value in Years purchase. | Approximated Value in Years purchase. |
|---|---|---|---|---|
| C. | A. | B. | | |
| 78. | 15. | 75 | - - - - .138 | - - - - . 146 |
| 70. | 20. | 65 | - - - - .406 | - - - - . 408 |
| 81. | 70. | 80 | - - - - .530 | - - - - . 550 |
| 15. | 85. | 10 | - - - 13.833 | - - - 13.782 |
| 15. | 75. | 15 | - - - 11.385 | - - - 11.230 |
| 35. | 75. | 15 | - - - 8.968 | - - - 8.834 |
| 15. | 65. | 20 | - - - 8.485 | - - - 8.379 |
| 15. | 80. | 70 | - - - 9.893 | - - - 9.529 |
| 15. | 10. | 85 | - - - .647 | - - - .698 |
| 15. | 15. | 75 | - - - 1.038 | - - - 1.193 |
| 35. | 15. | 75 | - - - .786 | - - - .920 |
| 15. | 20. | 65 | - - - 1.769 | - - - 1.876 |
| 15. | 70. | 80 | - - - 4.307 | - - - 4.671 |

From these examples it appears that when C or A is the oldest of the three lives, the approximated and the true values agree sufficiently near for any useful purpose; and even when B is the oldest, that the difference is by no means considerable. It should likewise be observed, that these examples are cases in which the difference is likely to be greatest, and therefore a nearer approxima-

* In these values each term of the series has been separately computed at 4 per cent. from the Northampton Table of Observations.

tion need not be required. Both Mr. *Simpson*, in his Select Exercises, and myself (in a former edition of this work), have given solutions of this Problem; but these being founded on a wrong hypothesis are not so correct as the present, except when C is the oldest of the three lives, nor are they even more simple; so that it can seldom be necessary to have recourse to them. Without the assistance of the preceding Lemmas, and the computations which have been just made, it would not have been possible to have ascertained the degree of accuracy of any approximation; and therefore were no other end answered by them, this of itself would have been of sufficient consequence to deserve the time and labour which I have bestowed upon them. But it will appear in the succeeding Notes that the use and application of these Lemmas, and especially of the tables deduced from them, are much more extensive and important.

## NOTE XXXII. (Prob. xli.)

The payment of this annuity depends only on one contingency; and that is, the extinction of the two lives of A and B before the end of each year (B having died first), and the continuance of the life of C to the end of those respective years. The value therefore for the first year will be $\frac{\overline{m-n}.\overline{a-b}.t}{2amsr}$; for the second year $\frac{\overline{m-o}.\overline{a-c}.u}{2amsr^2}$; for the third year $\frac{\overline{m-p}.\overline{a-d}.w}{2amsr^3}$, and so on for the remaining years, Consequently, when *C is the oldest of the three lives*, the whole value of the annuity will be $\frac{t}{2sr} + \frac{u}{2sr^2} + \frac{w}{2sr^3} +$, &c. $- \frac{bt}{2asr} - \frac{cu}{2asr^2} - \frac{dw}{2asr^3} -$, &c. $+ \frac{bnt}{2amsr} + \frac{cou}{2amsr^2}$

$+ \frac{dpw}{2amsr^3} +$, &c. $- \frac{nt}{2msr} - \frac{ou}{2msr^2} - \frac{pw}{2msr^3} -$, &c. $= \frac{C-AC.}{2} - \frac{BC-ABC.}{2}$.

If *A be the oldest of the three lives*, let $z$, $C'$, and $B'C'$, denote the same quantities as in the second part of the preceding Note; and let $\varphi$ denote, by Table 8, the probability of A's surviving B; then will the value in this case be $\frac{C'-B'C'.}{2} - \frac{AC-ABC.}{2} + \varphi. \overline{C-C'}$.

If *B be the oldest of the three lives*, let $x$, $\pi$, $C'$, $A'C'$, $\sigma$, $\tau$, $\upsilon$, &c. denote the same quantities as in the third part of the preceding Note; and let $\alpha$, $\beta$, $\gamma$, &c. be the number of persons living in the table at the end of $\overline{x+1}$, $\overline{x+2}$, $\overline{x+3}$, &c. years at the age of A. The value of the annuity for the first $x$ years will be $= \frac{C'-A'C'.}{2} - \frac{BC-ABC.}{2}$. In the $\overline{x+1}$.[st] year, the payment of it will depend on the contingency of A's having died after B in $\overline{x+1}$. years, and C's having lived to the end of this term. As the life necessarily becomes extinct in $x$ years, it is plain that the probability of A's dying after him in $\overline{x+1}$ years must be $= \frac{a-\alpha}{a} - \pi$; and therefore that the value of the annuity in this year will be $\frac{\overline{a-\alpha}.\,\sigma}{asr^{x+1}} - \frac{\pi.\,\sigma}{sr^{x+1}}$. In the same manner the value of the annuity in the $\overline{x+2}$[d], $\overline{x+3}$[d], &c. years, will be $\frac{\overline{a-\beta}.\,\tau}{asr^{x+2}} - \frac{\pi\tau}{sr^{x+2}}$ - - - - $\frac{\overline{a-\gamma}.\,\upsilon}{asr^{x+3}} - \frac{\pi.\,\upsilon}{sr^{x+3}}$ - - - $+$, &c. But the series $\frac{\overline{a-\alpha}.\,\sigma}{asr^{x+1}} + \frac{\overline{a-\beta}.\,\tau}{asr^{x+2}} + \frac{\overline{a-\gamma}.\,\upsilon}{asr^{x+3}} +$, &c. $= \overline{C-C'} + \overline{AC-A'C'}$, and the series $- \frac{\pi\sigma}{sr^{x+1}} - \frac{\pi\tau}{sr^{x+2}} - \frac{\pi\upsilon}{sr^{x+3}} -$, &c. $= - \pi. \overline{C-C'}$; the whole value of the annuity therefore, when B is the oldest, will be $= C-AC. - \frac{C'-A'C'.}{2} - \frac{BC-ABC.}{2} - \pi. \overline{C-C'}$.

If the solution either of this, or of the preceding Problem, be given $=$ Q. the solution of the other Problem will be immediately obtained, being always $= \overline{\mathrm{C}-\mathrm{AC}}-\mathrm{Q}$.

When the three lives are of equal age the value will be $= \frac{\mathrm{C}+\mathrm{CCC}}{2} - \mathrm{CC}$.

## NOTE XXXIII. (Prob. xlii.)

The payment of the annuity in the first year will depend on either of three events:—1st. On the contingency of B and C both surviving A; 2dly. On the contingency of B.'s dying after A and C living; 3dly. On the contingency of C.'s dying after A and B living. Hence the value of the annuity in the first year will be $\frac{\overline{a-b}.\,nt}{samr} + \frac{\overline{a-b}.\,\overline{m-n}.\,t}{2amsr} + \frac{\overline{a-b}.\,\overline{s-t}.\,n}{2amsr}$. In the second, third, fourth, &c. years the payment of the annuity will depend on the same number of events:—1st. On A.'s having died, and B and C.'s having both lived to the end of the year; 2dly. On B.'s having died after A, and C.'s having lived; 3dly. On C.'s having died after A and B.'s having lived to the end of the respective years. The fractions therefore expressing the value of the annuity in the second year will be $\frac{\overline{a-c}.\,ou}{amsr^2} + \frac{\overline{a-c}.\,\overline{m-o}.\,u}{2amsr^2} + \frac{\overline{a-c}.\,\overline{s-u}.\,o}{2amsr^2}$; in the third year $\frac{\overline{a-d}.\,pw}{amsr^3} + \frac{\overline{a-d}.\,\overline{m-p}.\,w}{2amsr^3} + \frac{\overline{a-d}.\,\overline{s-w}.\,p}{2amsr^3}$, and so on. These fractions may be reduced to the four following series $\frac{t}{2sr} + \frac{u}{2sr^2} + \frac{w}{2sr^3} +$, &c. $+ \frac{n}{2mr} + \frac{o}{2mr^2} + \frac{p}{2mr^3} +$, &c. $- \frac{bt}{2asr} - \frac{cu}{2asr^2} - \frac{dw}{2asr^3} -$, &c. $- \frac{bn}{2amr} - \frac{co}{2amr^2} - \frac{dp}{2amr^3} -$, &c. $= \frac{\mathrm{C}+\mathrm{B}}{2} - \frac{\mathrm{AB}+\mathrm{AC}}{2}$, which is the true value when *A is the youngest of the three lives, and B and C are nearly of the*

*same age.* If C is much younger the values of C and AC must be continued only for as many years as are equal to the difference between the age of B and the oldest life in the Table.

*If B and C be* BOTH *younger than A,* let $z$ denote the same quantity as in Note XXXI, and the value in that case will be $\frac{C+B}{2} - \frac{AB+AC}{2} + \frac{C^z+B^z}{2} - BC^z$; $C^z$, $B^z$, and $BC^z$, denoting the values of the lives *after* $z$ years.

*If one of the lives (B) be older than A and the other younger,* let $x$ denote the number of years between the ages of B and of the last person in the Table, $C^x$ the value of an annuity on the life of C after $x$ years, $C'$ and $A'C'$ the value of the single and two joint lives for $x$ years, $\pi$ the probability found by Tab. x. that B dies *after* A, and the value will then be $\frac{C'+B}{2} - \frac{AB+A'C'}{2} + \pi . C^x$; $C^x$ denoting the value of the life *after* $x$ years. When the three lives are of equal age the value will be C — CC.

## NOTE XXXIV. (PROB. XLIII.)

In the first year the payment of the given sum will depend on either of two events:—1st. That all the three lives shall become extinct (B having survived A) which is expressed by $\frac{\overline{a-b}.\overline{m-n}.\overline{s-t}}{2mas}$; 2dly. That B shall live, and only A and C die, which is expressed by $\frac{n.\overline{a-b}.\overline{s-t}}{mas}$. The value therefore of the given sum in the first year will be $\frac{S}{amsr} \times \overline{\frac{ams}{2} - \frac{bms}{2} + \frac{ans}{2} - \frac{bns}{2} - \frac{amt}{2} + \frac{bmt}{2} - \frac{ant}{2} + \frac{bnt}{2}}$. In the second and following years the payment of S will depend upon either of seven events:—1st. That all the three lives become extinct in the year, B having survived A; 2dly. That B lives, and A and C die in the year; 3dly.

hat A dies in the year, that C dies in any of the preeding years, and B lives; 4thly. That B dies after A in e year, C having died in any of the preceding years; thly. That C dies in the year, B having died after A in ny of the preceding years; 6thly. That B and C both die the year, A having died in any of the preceding years; nd 7thly, That C dies in the year, A having died in any f the preceding years, and that B lives.

These contingencies in the second year are expressed by he fractions $\frac{\overline{b-c}.\overline{n-o}.\overline{t-u}}{2ams} + \frac{o.\overline{b-c}.\overline{t-u}}{ams} + \frac{\overline{b-c}.\overline{s-t}.o}{ams} + \frac{\overline{n-o}.\overline{b-c}.\overline{s-t}}{2ams} + \frac{\overline{t-u}.\overline{m-n}.\overline{a-b}}{2ams} + \frac{\overline{t-u}.\overline{n-o}.\overline{a-b}}{ams} + \frac{\overline{t-u}.\overline{a-b}.o}{ams}$, n the third year by the fractions $\frac{\overline{c-d}.\overline{o-p}.\overline{u-w}}{2ams} + \frac{\overline{c-d}.\overline{u-w}.p}{ams} + \frac{\overline{c-d}.\overline{s-u}.p}{ams} + \frac{\overline{o-p}.\overline{c-d}.\overline{s-u}}{2ams} + \frac{\overline{u-w}.\overline{m-o}.\overline{a-c}}{2ams} + \frac{\overline{u-w}.\overline{o-p}.\overline{a-c}}{ams} + \frac{\overline{u-w}.\overline{a-c}.p}{ams}$, and so on in the succeeding years; which fractions may be reduced into the following series: (first) $\frac{S}{2ams} \times \left(\frac{ams}{r} + \frac{amt}{r^2} + \frac{amu}{r^3}, \text{\&c.}\right)$; (second) $- \frac{S}{2ams} \times \left(\frac{bms}{r} + \frac{cns}{r^2} + \frac{dos}{r^3}, \text{\&c.}\right)$; (third) $+ \frac{S}{2ams} \times \left(\frac{ans}{r} + \frac{bos}{r^2} + \frac{cps}{r^3}, \text{\&c.}\right)$; (fourth) $- \frac{S}{2ams} \times \left(\frac{bns}{r} + \frac{cos}{r^2} + \frac{dps}{r^3}, \text{\&c.}\right)$; (fifth) $- \frac{S}{2amsr} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3}, \text{\&c.}\right)$; (sixth) $- \frac{S}{2amsr} \times \left(\frac{amu}{r} + \frac{amw}{r^2} + \frac{amx}{r^3}, \text{\&c.}\right)$; (seventh) $+ \frac{S}{2amsr} \times \left(\frac{ant}{r} + \frac{aou}{r^2} + \frac{apw}{r^3}, \text{\&c.}\right)$; (eighth) $- \frac{S}{2amsr} \times \left(\frac{anu}{r} - \frac{aow}{r^2} - \frac{apx}{r^3}, \text{\&c.}\right)$; (ninth) $- \frac{S}{2amsr} \times \left(\frac{bmt}{r} + \frac{cmu}{r^2} + \frac{dmw}{r^3}, \text{\&c.}\right)$; (tenth) $+ \frac{S}{2amsr} \times \left(\frac{bmu}{r} + \frac{cmw}{r^2} + \frac{dmx}{r^3} +, \text{\&c.}\right)$; (eleventh) $+ \frac{S}{2amsr} \times \left(\frac{bnu}{r} + \frac{cow}{r^2} + \frac{dpx}{r^3} +, \text{\&c.}\right)$; (twelfth) $+ \frac{S}{2amsr} \times \left(\frac{bns}{r} + \frac{cos}{r^2} + \frac{dps}{r^3}, \text{\&c.}\right)$; (thirteenth) $- \frac{S}{2ams} \times$

$\left(\frac{amt}{r} + \frac{bnx}{r^2} + \frac{cow}{r^3} +, \text{\&c.}\right)$; (fourteenth) $+ \frac{S}{2ams} \times \left(\frac{bmt}{r} + \frac{cnu}{r^2} + \frac{dow}{r^3} +, \text{\&c.}\right)$; (fifteenth) $- \frac{S}{2ams} \times \left(\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3} + \text{\&c.}\right)$; (sixteenth) $- \frac{S}{2ams} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} +, \text{\&c.}\right)$ The first four and the twelfth series are $= \frac{S}{2r} \times \left(\overline{1 - \overline{r-1}. AB} + \frac{n.\overline{1+AP}}{m} - \frac{b.\overline{1+HB}}{a} - C.\right)$ The fifth and sixteenth series are $= \frac{S.\overline{r-1}.ABC}{2r}$. The sixth, eleventh, and thirteenth, are $= - \frac{S \times C}{2}$. The seventh and ninth are $= \frac{S.\overline{BC - AC}}{2r}$. The eighth and tenth $= \frac{S.t}{2sr} \times \overline{AT - BT}$. The fourteenth is $= \frac{S}{2r} \times \frac{bt.\overline{1+HBT}}{as}$; and the fifteenth $= - \frac{S.}{2r} \times \frac{nt.\overline{1+APT}}{ms}$. Let D denote the value of S on the contingency of B surviving A, and the whole value of the reversion, when C is the oldest of the three lives, will be D $+ \frac{S}{2r}$ into $\left(\overline{r-1}.\overline{ABC - C} + \overline{BC - AC} + \frac{t}{s} \times \overline{AT - BT} + \frac{b.\overline{1+HBT}}{a} - \frac{n.\overline{1+APT}}{m}\right)$.

*If A be the oldest of the three lives,* let $z$, C′, B′C′, and $\varphi$, denote the same quantities as in the second part of Note XXXI, let $\sigma$ be the number of persons living in the Table at the ages of a person $z$ years older than C, and let B′T′ be the value of the joint lives of B and T for $z$ years, then will the value of the reversion in this case be $=$ D $+ \frac{S}{2r} \times \left(\overline{r-1}.\overline{ABC - C'} + \overline{B'C' - AC} + \frac{t}{s} \times \overline{AT - B'T'} + \frac{b.\overline{1+HBT}}{a} - \frac{n.\overline{1+APT}}{m}\right) + \frac{S.\overline{r-1}}{r^z+1} \times \frac{\varphi\sigma}{s} \times \overline{V - C^z}$; $C^z$ being the value of a life $z$ years older than C.

*If B is the oldest of the three lives,* let $x$ and $\pi$ denote the same quantities as in the third part of Note XXXI,

and let $\tau$ be the number of persons living in the table at the age of a person $x$ years older than C, the value of the reversion will then be $D + \frac{S}{2r} \times \left(\overline{r-1}.\ \overline{ABC-C'+BC-A'C'} + \frac{t}{s} \times \overline{'A'T-BT + \frac{b.\overline{1+HBT}}{a} - \frac{n.\overline{1+APT}}{m}}\right) + \frac{S.\overline{r-1}}{r^{x}+1} \times \frac{\pi.\tau}{s} + \overline{V-C^{x}}$; $C^{x}$ being the value of a life $x$ years older than C.

When the three lives are of equal age the value becomes $= \frac{S.\overline{r-1}}{2r} \times \overline{V - CC - C + CCC}$.

The solution of this Problem may be derived from the 25th and 40th Problems; the value of S being equal to the difference between its value on the contingency of B surviving A, and the value of an annuity of £1 during the life of C after A, provided A should die before B, multiplied into the interest of S, and divided by £1 increased by its interest for a year: that is, let D be the former and $\Delta$ the latter value, the reversion will be $= D - \frac{S.\overline{r-1}.\Delta}{r}$.

## NOTE XXXV. (Prob. xliv.)

The payment in the first year depends on the contingency of the three lives having become extinct (A having survived B), which is expressed by $\frac{\overline{m-n}.\overline{a-b}.\overline{s-t}}{2ams}$. In the 2d and following years the given sum will become payable if either of five events should take place:—1st. If the three lives should fail in the year (B having died before A); 2dly. If C should die in any of the preceding years, and A die after B in that particular year; 3dly. If B and C should die in any of the preceding years, and only A die in that year; 4thly. If B should die in any of the pre-

ceding years, and A and C both die in that year; and 5thly. If A should die after B in any of the preceding years, and C die in that year. The different fractions expressing those probabilities in the second year are $\frac{\overline{n-o}\cdot\overline{b-c}\cdot\overline{t-u}}{2ams} + \frac{\overline{s-t}\cdot\overline{n-o}\cdot\overline{b-c}}{2ams} + \frac{\overline{s-t}\cdot\overline{m-n}\cdot\overline{b-c}}{ams} + \frac{\overline{m-n}\cdot\overline{b-c}\cdot\overline{t-u}}{ams} + \frac{\overline{a-b}\cdot\overline{m-n}\cdot\overline{t-u}}{2ams}$, in the third year $\frac{\overline{o-p}\cdot\overline{c-d}\cdot\overline{u-w}}{2ams} + \frac{\overline{s-u}\cdot\overline{o-p}\cdot\overline{c-d}}{2ams} + \frac{\overline{s-u}\cdot\overline{m-o}\cdot\overline{c-d}}{ams} + \frac{\overline{m-o}\cdot\overline{c-d}\cdot\overline{u-w}}{ams} + \frac{\overline{a-c}\cdot\overline{m-o}\cdot\overline{u-w}}{2ams}$, and so on in the other years. Hence the value of the given sum will be $=$ S into

$$\left(\frac{1}{r} + \frac{t}{sr^2} + \frac{u}{sr^3} + \frac{w}{sr^4} + \&c.\right) - \left(\frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3}\ \&c.\right) - \left(\frac{t}{sr^2} + \frac{u}{sr^3} + \frac{w}{sr^4} + \&c.\right) - \left(\frac{bt}{asr^2} + \frac{cu}{asr^3} + \frac{dw}{asr^4} + \&c.\right) + \left(\frac{bt}{asr} + \frac{cu}{asr^2} + \frac{dw}{asr^3} + \&c.\right) + \left(\frac{b}{ar^2} + \frac{c}{ar^3} + \frac{d}{ar^4} + \&c.\right) -$$

$$\frac{S}{2ams} \times \left(\frac{ams}{r} + \frac{amt}{r^2} + \frac{amu}{r^3} + \&c.\right) + \frac{S}{2ams} \times \left(\frac{bms}{r} + \frac{cns}{r^2} + \frac{dos}{r^3} + \&c.\right) - \frac{S}{2ams} \times \left(\frac{ans}{r} + \frac{bos}{r^2} + \frac{cps}{r^3} + \&c.\right) + \frac{S}{2ams} \times \left(\frac{bns}{r} + \frac{cos}{r^2} + \frac{dps}{r^3} + \&c.\right) + \frac{S}{2amsr} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dqw}{r^3}\ \&c.\right)$$

$$+ \frac{S}{2amsr} \times \left(\frac{amu}{r} + \frac{amw}{r^2} + \frac{amx}{r^3}\ \&c.\right) - \frac{S}{2amsr} \times \left(\frac{ant}{r} + \frac{aou}{r^2} + \frac{apw}{r^3}\ \&c.\right) + \frac{S}{2amsr} \times \left(\frac{anu}{r} + \frac{aow}{r^2} + \frac{apx}{r^3}\ \&c.\right) + \frac{S}{2amsr} \times \left(\frac{bmt}{r} + \frac{cmu}{r^2} + \frac{dmw}{r^3}\ \&c.\right) - \frac{S}{2amsr} \times \left(\frac{bmu}{r} + \frac{cmw}{r^2} + \frac{dmx}{r^3}\ \&c.\right)$$

$$- \frac{S}{2amsr} \times \left(\frac{bnu}{r} + \frac{cow}{r^2} + \frac{dpx}{r^3}\ \&c.\right) - \frac{S}{2amsr} \times \left(\frac{bns}{r} + \frac{cos}{r^2} + \frac{dps}{r^3}\ \&c.\right) + \frac{S}{2ams} \times \left(\frac{amt}{r} + \frac{bnu}{r^2} + \frac{cow}{r^3} + \&c.\right) - \frac{S}{2ams} \times \left(\frac{bmt}{r} + \frac{cnu}{r^2} + \frac{dow}{r^3}\ \&c.\right) + \frac{S}{2ams} \times \left(\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3}\ \&c.\right) + \frac{S}{2ams} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} + \&c.\right)$$

The first six series are

$= \frac{S}{r} \times (1 - \overline{r-1}. \overline{A + C - AC})$. The remaining sixteen series denote the value of S by the preceding Problem with contrary signs. Let this value be called Y, and the value of an annuity of £1 on the longest of the two lives of A and C be called E, then will the value required be $= \frac{S. \overline{1 - \overline{r-1}. E}}{r} - Y$. When the three lives are equal, the value will become $= \frac{S. \overline{1 - \overline{r-1}. L}}{2r}$. L denoting the value of an annuity on the longest of the three lives. The solution of this problem may also be deduced from the 26th and 41st problems, "the value of S being equal in this case "to the *difference* between its value after the death of A "and B, provided B should die before A, and the value "of an annuity of £1 during the life of C after A, pro-"vided A should survive B, multiplied into the interest "of S," that is, let F be the former and X the latter value, the reversion will be $= F - \frac{S. \overline{r-1}. X}{r}$.

## NOTE XXXVI. (Prob. xlv.)

In the first year the given sum can be received only provided the three lives shall have failed and the life of A have been the first that became extinct, which contingency is expressed by the fraction $\frac{\overline{a-b}. \overline{m-n}. \overline{s-t}}{3ams}$. In the second, third, and following years it may be received provided either of four events shall have happened:—1st. If all the three lives shall have failed in the year, A dying first; 2dly. If A shall have died in either of the foregoing years, and B and C both died in that year; 3dly. If A and B shall have both died in the foregoing years (B dying

last) and C died in that year; 4thly. If C and A shall have both died in the foregoing years (C dying last) and B died in that year. The fractions expressing these contingencies in the second year are $\frac{\overline{b-c}.\,\overline{n-o}.\,\overline{t-u}}{3ams} + \frac{\overline{a-b}.\,\overline{n-o}.\,\overline{t-u}}{ams} + \frac{\overline{a-b}.\,\overline{m-n}.\,\overline{t-u}}{2ams} + \frac{\overline{a-b}.\,\overline{s-t}.\,\overline{n-o}}{2ams}$; in the 3d year $\frac{\overline{c-d}.\,\overline{o-p}.\,\overline{u-w}}{3ams} + \frac{\overline{a-c}.\,\overline{o-p}.\,\overline{u-w}}{ams} + \frac{\overline{a-c}.\,\overline{m-o}.\,\overline{u-w}}{2ams} + \frac{\overline{a-c}.\,\overline{s-u}.\,\overline{o-p}}{2ams}$, and so on in the following years. These fractions being expanded will be formed into the twenty-two following series: $=$ (first) $\frac{S}{3ams} \times \overline{\frac{ams}{r} + \frac{bnt}{r^2} + \frac{cou}{r^3} + \&c.}$ (second) $- \frac{S}{3ams} \times \overline{\frac{ans}{r} + \frac{bot}{r^2} + \frac{cup}{r^3}\ \&c.}$ (third) $- \frac{S}{3ams} \times \overline{\frac{amt}{r} + \frac{bnu}{r^2} + \frac{cow}{r^3}\ \&c.}$ (fourth) $+ \frac{S}{3ams} \times \overline{\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3}\ \&c.}$ (fifth) $- \frac{S}{3ams} \times \overline{\frac{bms}{r} + \frac{cnt}{r^2} + \frac{dou}{r^3}\ \&c.}$ (sixth) $+ \frac{S}{3ams} + \overline{\frac{bns}{r} + \frac{cot}{r^2} + \frac{dup}{r^3}\ \&c.}$ (seventh) $+ \frac{S}{3ams} \times \overline{\frac{bmt}{r} + \frac{cmu}{r^2} + \frac{dmw}{r^3}\ \&c.}$ (eighth) $- \frac{S}{3ams} \times \overline{\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} + \&c.}$ (ninth) $- \frac{S}{2amsr} \times \overline{\frac{amu}{r} + \frac{amw}{r^2} + \frac{amx}{r^3}\ \&c.}$ (tenth) $+ \frac{S}{amsr} \times \overline{\frac{aou}{r} + \frac{apw}{r^2} + \frac{aqx}{r^3}\ \&c.}$ (eleventh) $- \frac{S}{amsr} \times \overline{\frac{bou}{r} + \frac{cpw}{r^2} + \frac{dqx}{r^3}\ \&c.}$ (twelfth) $- \frac{S}{2amsr} \times \overline{\frac{bmt}{r} + \frac{cnu}{r^2} + \frac{dow}{r^3} + \&c.}$ (thirteenth) $+ \frac{S}{2amsr} \times \overline{\frac{bmu}{r} + \frac{cmw}{r^2} + \frac{dmx}{r^3}\ \&c.}$ (fourteenth) $+ \frac{S}{2amsr} \times \overline{\frac{ans}{r} + \frac{aot}{r^2} + \frac{apu}{r^3} + \&c.}$ (fifteenth) $- \frac{S}{2amsr} \times \overline{\frac{aos}{r} + \frac{aps}{r^2} + \frac{aqs}{r^3}\ \&c.}$ (sixteenth) $- \frac{S}{2amsr} \times \overline{\frac{bns}{r} + \frac{cos}{r^2} + \frac{dps}{r^3} + \&c.}$ (seventeenth) $+ \frac{S}{2amsr} \times \overline{\frac{bos}{r} + \frac{cps}{r^2} + \frac{dqs}{r^3} + \&c.}$ (eighteenth) $- \frac{S}{2amsr} \times \overline{\frac{anu}{r} + \frac{aow}{r^2} + \frac{apx}{r^3}}$

$\overline{+ \text{ \&c.}}$ (nineteenth) $- \frac{S}{2amsr} \times \overline{\frac{aol}{r} + \frac{apu}{r^2} + \frac{aqw}{r^3} \text{ \&c.}}$ (twentieth) $+ \frac{S}{2amsr} \times \overline{\frac{bot}{r} + \frac{cpu}{r^2} + \frac{dqw}{r^3} + \text{\&c.}}$ (twenty-first) $+ \frac{S}{2amsr} \times \overline{\frac{bnu}{r} + \frac{cow}{r^2} + \frac{dpx}{r^3} + \text{ \&c.}}$ (twenty-second) $\frac{S}{2amsr} + \overline{\frac{amt}{r} + \frac{amw}{r^2} + \frac{amx}{r^3}}$ &c. These several series may be found equal to S into $\frac{1 - \overline{r-1}.\, ABC}{3r} - \frac{AB + AC}{2r} - \frac{\overline{r-1}\, \overline{C+B}}{2r} + BC + \frac{n.\, \overline{1 + APC}}{6mr} + \frac{t.\, \overline{1 + ABT}}{6sr} - \frac{b.\, \overline{1 + HBC}}{3ar} - \frac{2nt.\, \overline{1 + APT}}{3msr} + \frac{nb.\, \overline{1 + HPC}}{3amr} + \frac{bt.\, \overline{1 + HBT}}{3asr} + \frac{t.\, \overline{AT - BT}}{2sr} - \frac{n.\, \overline{CP - AP}}{2mr}$ which is the value required when B and C are nearly of the same age, and both older than A. But when C is much younger than B or A (B being older than A) the annuities on the single life of C, and on the joint lives of AC and AT must be continued only for as many years as are equal to the difference between the age of B and that of the oldest life in the Table. Let these values be denoted by C′, A′C′ and A′T′. Let $\pi$ denote the probability that B has died after A by Tab. 10th, $z$ the difference between the age of B and that of the oldest person in the Table, $x$ the number of persons living at the age of C after $z$ years, and $C^z$ the value of C.'s life after $z$ years, then will the value in this case be S into $\frac{1 - \overline{r-1}\, ABC}{3r} - \frac{AB + A'C'}{2r} - \frac{\overline{r-1}.\, \overline{C' + B}}{2r} + BC + \frac{n.\, \overline{1 + APC}}{6mr} + \frac{t.\, \overline{1 + ABT}}{6sr} - \frac{b.\, \overline{1 + HBC}}{3ar} - \frac{2nt.\, \overline{1 + APT}}{3msr} + \frac{nb.\, \overline{1 + HPC}}{3amr} + \frac{bt.\, \overline{1 + HBT}}{3asr} + \frac{t.\, \overline{A'T' - BT}}{2sr} - \frac{n.\, \overline{CP - AP}}{2mr} + \frac{\pi.\, x}{sr^z + 1} \times \overline{r-1}.\, \overline{V - C^z}$.

If both the lives of B and C are younger, let $\pi$ express as above, let $\mu$ and $x$ express the number of persons

living at the age of B and C respectively at the end of $y$ years (or the difference between the age of A and that of the oldest life in the Table), and let $\varphi$ express the probability that C dies after A by Tab. 10, then will the value required in this case be S into $\frac{1-\overline{r-1}.\,ABC}{3r} - \frac{AB+AC}{2r} - \frac{\overline{r-1}.\,\overline{C'+B'}}{2r} + B'C' + \frac{n.\,\overline{1+APC}}{6mr} + \frac{t.\,\overline{1+ABT}}{6sr} - \frac{b.\,\overline{1+HBC}}{3ar} - \frac{2nt.\,\overline{1+APT}}{3msr} + \frac{nb.\,\overline{1+HPC}}{3amr} + \frac{bt.\,\overline{1+HBT}}{3asr} + \frac{t.\,\overline{AT-BT}}{2sr} - \frac{n.\,\overline{C'P'-AP}}{2mr} + \frac{x\mu.\,\overline{r-1}.\,\overline{V-L^y}}{msr^{y+1}} + \frac{x\pi.\,\overline{r-1}.\,\overline{V-C^y}}{sr^{y+1}} + \frac{\varphi\mu.\,\overline{r-1}}{mr^{y+1}} + \overline{V-B^y}$; $L^y$ being the value of an annuity on the longest of two lives and $B^y$ and $C^y$ the value of an annuity on single lives, $y$ years older than B and C.

If the three lives are of equal age the value becomes $= \frac{S.\,\overline{r-1}.\,\overline{V-L}}{3r}$; L representing the value of an annuity on the longest of the three lives.

The solution of this Problem like that of the two preceding ones may be derived from the twenty-eighth and the forty-second Problems. Let G be the value of S on the contingency of A's life being the first that fails, $\Gamma$ the value of an annuity of £1 on the lives of B and C after A, provided *both* of them survive A, then will the value of the reversion be $= G - \frac{S.\,\overline{r-1}.\,\Gamma}{r}$.

## NOTE XXXVII. (Prob. xlvi.)

The payment of the given sum in the first year depends only on the contingency of the three lives becoming extinct in the order specified in this Problem, which is expressed by the fraction $\frac{\overline{a-b}.\,\overline{m-n}.\,\overline{s-t}}{6ams}$. In the second year the payment of the given sum depends on either of three

events :—1st. That the three lives fail in the order required by the Problem; 2dly. That B dies after A in the first year, and C dies in the second year; 3dly. That A only dies in the first year, and C dies after B in the second year. These contingencies are expressed by the fractions $\frac{\overline{b-c}.\overline{n-o}.\overline{t-u}}{6ams} + \frac{\overline{a-b}.\overline{m-n}.\overline{t-u}}{2ams} + \frac{\overline{a-b}.\overline{n-o}.\overline{t-u}}{2ams}$. The payment of the given sum in the third year depends on the same number of events :—1st. The three lives must become extinct in the year, in the order stated above; or, 2dly. B must die after A in the first or second year, and C die in the third year; or, 3dly. A must die in the first or second year, and C die after B in the third year, which are denoted by the fractions $\frac{\overline{c-d}.\overline{o-p}.\overline{u-w}}{6ams} + \frac{\overline{a-c}.\overline{m-o}.\overline{u-w}}{2ams} + \frac{\overline{a-c}.\overline{o-p}.\overline{u-w}}{2ams}$. By pursuing the same steps in the following years and expanding the different fractions, the whole value, when C is the oldest of the three lives, will become equal to S into

$$\frac{1}{6ams} \times \overline{\frac{ams}{r} + \frac{bnt}{r^2} + \frac{cou}{r^3} + \&c.} - \frac{1}{6ams} \times$$

$$\overline{\frac{ans}{r} + \frac{bot}{r^2} + \frac{cup}{r^3} + \&c.} - \frac{1}{6ams} \times \overline{\frac{amt}{r} + \frac{bnu}{r^2} + \frac{cow}{r^3} + \&c.} +$$

$$\frac{1}{6ams} \times \overline{\frac{ant}{r} + \frac{bou}{r^2} + \frac{owp}{r^3}\ \&c.} - \frac{1}{6ams} \times \overline{\frac{bms}{r} + \frac{cnt}{r^2} + \frac{dou}{r^3} + \&c.}$$

$$+ \frac{1}{6amsr} \times \overline{\frac{bns}{r} + \frac{cot}{r^2} + \frac{dup}{r^3} + \&c.} - \frac{1}{6amsr} \times \overline{\frac{bmt}{r} + \frac{cnu}{r^2} +}$$

$$\overline{\frac{dow}{r^3} + \&c.} - \frac{1}{6amsr} \times \overline{\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dwp}{r^3} + \&c.} + \frac{1}{2amsr} \times$$

$$\overline{\frac{amt}{r} + \frac{amu}{r^2} + \frac{amw}{r^3}\ \&c.} - \frac{1}{2amsr} \times \overline{\frac{amu}{r} + \frac{amw}{r^2} + \frac{amx}{r^3}\ \&c.}$$

$$- \frac{1}{2amsr} \times \overline{\frac{bmt}{r} + \frac{cmu}{r^2} + \frac{dmw}{r^3} + \&c.} + \frac{1}{2amsr} \times \overline{\frac{bmu}{r} +}$$

$$\overline{\frac{cmw}{r^2} + \frac{dmx}{r^3} + \&c.} - \frac{1}{2amsr} \times \overline{\frac{aot}{r} + \frac{apu}{r^2} + \frac{aqw}{r^3}\ \&c.} +$$

$$\frac{1}{2amsr} \times \overline{\frac{aou}{r} + \frac{apw}{r^2} + \frac{aqx}{r^3} + \&c.} + \frac{1}{2amsr} \times \overline{\frac{bpt}{r} + \frac{cpu}{r^2} +}$$

$$\overline{\frac{dqw}{r^3} \text{ \&c.}} - \frac{1}{2amsr} \times \overline{\frac{bou}{r} + \frac{cpw}{r^2} + \frac{dqx}{r^3} \text{ \&c.}} = S \text{ into}$$

$$\frac{1-\overline{r-1}.\ \overline{ABC+3C}}{6r} + \frac{BC}{2} - \frac{AC}{2r} + \frac{n.\ \overline{1+APC}}{3mr} - \frac{t.\ \overline{1+ABT}}{6sr} - \frac{b.\ \overline{1+HBC}}{6ar} - \frac{nt.\ \overline{1+APT}}{3msr} + \frac{nb.\ \overline{1+HPC}}{6amr} + \frac{bt.\ \overline{1+HBT}}{6asr} + \frac{t.\ \overline{1+AT}}{2sr} - \frac{n.\ \overline{1+CP}}{2mr}.$$

*When B is the oldest of the three lives*, the series expressing the value of an annuity on the single life C, and on the joint lives AC. and AT., are to be continued only for as many years as are equal to the difference between the age of B and that of the oldest life in the table. Let this difference be denoted by $z$; the annuities for $z$ years by $C'$ and $A'C'$, and the annuity for $\overline{z-1}$. years by $A''T''$; the number of persons living at the end of $z$ years at the age of C by $\sigma$; and let $\pi$ denote the probability that B dies after A; then will the value in this case be S into

$$\frac{1-\overline{r-1}.\ \overline{ABC+3C'}.}{6r} + \frac{BC.}{2} - \frac{A'C'.}{2r} + \frac{n.\ \overline{1+APC}.}{3mr} - \frac{t.\overline{1+ABT}.}{6sr} - \frac{b.\overline{1+HBC}}{6ar} - \frac{nt.\overline{1+APT}.}{3msr} + \frac{nb.\overline{1+HPC}.}{6amr} + \frac{bt.\overline{1+HBT}.}{6asr} + \frac{t.\ \overline{1+A''T''}.}{2sr}$$

$$-\frac{n.\ \overline{1+CP}.}{2mr} + \frac{\pi.\sigma.\overline{r-1}.\ \overline{V-C^z}}{sr^{z+1}},$$ $C^z$ denoting the value of an annuity on a life $z$ years older than C.

*When A is the oldest of the three lives*, let $x$ denote the difference between the age of A and that of the oldest person in the table. Let $\mu$, $\nu$, $\xi$, $o$, &c. represent the number of the living at the age of B at the end of $x$, $\overline{x+1}$, $\overline{x+2}$, &c. years, and $\sigma$, $\tau$, $\upsilon$, $\phi$, &c. the number of the living at the age of C; let $\pi$ represent the probability that B dies after A, and $C^x$ the value of an annuity on a life $x$ years older than C; then will the value of S in the $\overline{x+1}^{st}$ year, depending on the contingency of C's dying after B in that

ear, or of C's dying in that year, B having died after A in either of the foregoing years, be expressed by $\frac{\overline{\sigma-\tau}.\overline{\mu-\nu}}{2msr^{x+1}} + \frac{S.\pi.\overline{\sigma-\tau}}{sr^{x+1}}$. In the $\overline{x+2}^{d}$, $\overline{x+3}^{d}$, &c. years, the payment of S will depend on either of three events:— 1st. Of C's dying after B in that particular year; 2dly. Of C's only dying in that year, B having died in either of the preceding $\overline{x+1}$, $\overline{x+2}$, &c. years; or, 3dly. Of C's only dying in the year, B having died after A in the first x years. Hence the value in the $\overline{x+2}^{d}$ year will be S into $\frac{\overline{\tau-\upsilon}.\overline{\nu-\xi}}{2msr^{x+2}} + \frac{\overline{\tau-\upsilon}.\overline{\mu-\nu}}{msr^{x+2}} + \frac{\pi.\overline{\tau-\upsilon}}{sr^{x+2}}$; in the $\overline{x+3}^{d}$ year, S into $\frac{\overline{\upsilon-\phi}.\overline{\xi-o}}{2msr^{x+3}} + \frac{\overline{\upsilon-\phi}.\overline{\mu-\xi}}{msr^{x+3}} + \frac{\pi.\overline{\upsilon-\phi}}{sr^{x+3}}$, and so on. The fractions $\frac{\pi.\overline{\sigma-\tau}}{sr^{x+1}} + \frac{\pi.\overline{\tau-\upsilon}}{sr^{x+2}} + \frac{\pi.\overline{\upsilon-\phi}}{sr^{x+3}}$ +, &c. are $= \frac{\pi.\sigma.\overline{r-1}.\overline{V-C^{x}}}{sr^{x+1}}$. The other fractions expanded become $\frac{\mu\sigma}{2msr^{x+1}} + \frac{\nu\tau}{2msr^{x+2}} + \frac{\xi\upsilon}{2msr^{x+3}}$ +, &c. $- \frac{1}{2msr^{x}} \times \overline{\frac{\mu\tau}{r} + \frac{\nu\upsilon}{r^{2}} + \frac{\xi\phi}{r^{3}} +, \&c.} - \frac{1}{2msr^{x}} \times \overline{\frac{\sigma\nu}{r} + \frac{\tau\xi}{r^{2}} + \frac{\upsilon o}{r^{3}} +, \&c.} + \frac{1}{2msr^{x}} \times \overline{\frac{\nu\tau}{r} + \frac{\xi\upsilon}{r^{2}} + \frac{o\phi}{r^{3}} +, \&c.} + \frac{1}{msr^{x}} \times \overline{\frac{\mu\tau}{r} + \frac{\mu\upsilon}{r^{2}} + \frac{\mu\phi}{r^{3}} +, \&c.} - \frac{1}{msr^{x}} \times \overline{\frac{\mu\upsilon}{r} + \frac{\mu\phi}{r^{2}} + \frac{\mu\chi}{r^{3}} +, \&c.} - \frac{1}{msr^{x+1}} \times \overline{\frac{\nu\tau}{r} + \frac{\xi\upsilon}{r^{2}} + \frac{o\phi}{r^{3}} +, \&c.} + \frac{1}{msr^{x+1}} \times \overline{\frac{\nu\upsilon}{r} + \frac{\xi\phi}{r^{2}} + \frac{o\chi}{r^{3}} +, \&c.}$

The first of these series is $= \frac{\mu\sigma.\overline{1+BC^{x}}}{2msr^{x+1}}$; the second $= -\frac{\mu\tau.\overline{1+BT^{x}}}{2msr^{x+1}}$; the third is a continuation of the series $- \frac{1}{2msr} \times \overline{\frac{aot}{r} + \frac{apu}{r^{2}} + \frac{aqw}{r^{3}}}$, &c.; the fourth is a continuation of the expression $\frac{B'C'}{2}$; the fifth is $= \frac{\mu\sigma.C^{x}}{msr^{x+1}}$; the sixth $- \frac{\mu\sigma.C^{x}}{msr^{x}} + \frac{\mu\tau}{msr^{x+1}}$; the seventh $= - \frac{\mu\sigma.BC^{x}}{msr^{x+1}}$; and the

eighth $= \frac{\mu\varpi . BT^x}{msr^x+1}$. Hence the value required in this case will be S into $\frac{1-\overline{r-1}.\overline{ABC.+3C'}}{6r} + \frac{BC.}{2} - \frac{AC.}{2r} + \frac{n.\overline{1+APC}.}{3mr} - \frac{t.\overline{1+ABT}.}{6sr} - \frac{b.\overline{1+HBC}.}{6ar} - \frac{nt.\overline{1+APT}.}{3msr} + \frac{nb.\overline{1+HPC}.}{6amr} + \frac{bt.\overline{1+HBT}.}{6asr} + \frac{t.\overline{1+AT}.}{2sr} - \frac{n.\overline{1+CP}.}{2mr} + \frac{\mu\sigma}{msr^x+1} \times \overline{\frac{\overline{1-BC^x}}{2} - \overline{r-1}.C^x} + \frac{\mu\varpi.\overline{1+BT^x}}{2msr^x+1} + \frac{\varpi.\sigma.\overline{r-1}.\overline{V-C^x}}{sr^x+1}$; $BC^x$ and $BT^x$ being the values of an annuity on the joint continuance of two lives $x$ years older than BC. and BT.

When the three lives are all equal the value becomes $= \frac{S.\overline{r-1}.\overline{V-L}.}{6r}$; L, as in the foregoing Notes, representing the value of an annuity on the longest of the three lives, and V the perpetuity.

It is to be observed, that the fractions $\frac{\overline{a-b}.\overline{m-n}.\overline{t-u}}{2ams}$, $\frac{\overline{a-c}.\overline{m-o}.\overline{u-w}}{2ams}$, &c. do not accurately express the second contingency in this Problem; but that, according to the Lemma, they should have been $\overline{\frac{\overline{a-b}.\overline{m-n}.}{2am} + \frac{\overline{b-c}.\overline{n-o}}{2am} + \frac{\overline{n-o}.\overline{a-b}}{am}} \times \overline{t-u}$, $\overline{\frac{\overline{a-b}.\overline{m-n}.}{2am} + \frac{\overline{b-c}.\overline{n-o}}{2am} + \frac{\overline{n-o}.\overline{a-b}.}{am} + \frac{\overline{o-p}.\overline{c-d}.}{2am} + \frac{\overline{o-p}.\overline{a-c}}{am}} \times \overline{u-w}$, &c. In order to determine how near the former approach to the true values, I have, in the following examples, separately computed each of those latter fractions; and the results appear to differ so little from the approximated values, that I think a greater degree of accuracy need not be required.

*Value of £100 payable on the contingency in this Problem, computed from the Northampton Table, at £4 per cent.*

| A. | Ages of B. | C. | | Value by the rule. | | Correct value. | | Difference. |
|---|---|---|---|---|---|---|---|---|
| 10. | 85. | 80 | - | 1.467 | - | 1.438 | - | 0.029 |
| 15. | 75. | 73 | - | 2.233 | - | 2.150 | - | 0.083 |
| 15. | 75. | 35 | - | 2.761 | - | 2.589 | - | 0.172 |
| 15. | 75. | 78 | - | 1.698 | - | 1.513 | - | 0.185 |
| 20. | 65. | 64 | - | 3.031 | - | 2.912 | - | 0.119 |
| 20. | 65. | 70 | - | 2.588 | - | 2.580 | - | 0.008 |
| 70. | 80. | 78 | - | 9.457 | - | 9.068 | - | 0.389 |
| 70. | 80. | 35 | - | 10.109 | - | 9.618 | - | 0.491 |

I have chosen those ages in which the approximation was likely to have been most inaccurate; for if A and B are both younger, or differ less from each other than they do in these examples, the foregoing rules will be still nearer the truth. I have also uniformly supposed the life of B to be older than that of A, so that the approximated value always errs in excess. If the life of A had been the older of the two, it would have been found to have erred nearly to the same amount in defect. But as in this latter case the value of the reversion is greater than when B is the older life, the error must necessarily bear a less proportion to the whole value than it does in the preceding examples.

With regard to the forty-fifth Problem, the error in some cases is greater, in others less, than in the present Problem. If B and C are both older than A it will be nearly twice as great. If one is older and the other younger, it will be altogether inconsiderable; for the fractions which express the probability of the older of B

and C dying after A will be as much above the truth, as the other fractions expressing the probability that the younger of these two lives die after A will be below it; and thus the errors of one by correcting those of the other will render the computation very nearly accurate.

Mr. *Simpson*, in his Select Exercises *, has given the fluent $\frac{Sq^x}{2abcm} \times \overline{xx - \frac{2x}{m} + \frac{2}{m^2}}$ (when properly corrected), as the true value of the reversion in the forty-sixth Problem: $q$ denoting unity divided by the rate of interest; $m$ the hyperbolic logarithm of $q$; $a$, $b$, $c$, the complements of A, B, C; and $x$ the interval of time during which the value is required. But this fluent by no means expressing the whole value, except in the single case where C is the oldest of the three lives, I was induced in the former edition of this work to complete the solution in all cases; and as the theorems may serve to show the difference between the values derived from M. De Moivre's hypothesis and those derived from the real probabilities of life, it may not be improper to insert them in the present Note.

*When C is the oldest of the three lives*, let A be the value of an annuity for the time $c$; $p$ the value of £1 at the end of $c$ years, and P the perpetuity; then will the value of the sum S be $= \frac{S}{2abc} \times \overline{\overline{A - pc} \times 2P^2 - Ppc^2}$.

*When B is the oldest of the three lives*, let the same symbols, $a$, $b$, $c$, and P, be retained. Let B denote the value of an annuity for $b$ years, N the value of an annuity for $\overline{c-b}$. years, and $\pi$ the value of £1 payable at the end of $b$ years, the required value will then be expressed by $\frac{S}{2abc} \times \overline{\overline{B - \pi b} \times 2P^2 - P\pi b^2} + \frac{S.\,b}{2ac} \times \frac{N}{r^{b+1}}$.

* Page 328.

*When A is the oldest and B the youngest of the three lives*, the value will be $\frac{S}{2abc} \times \overline{\overline{C - xa} \times 2P^2 - Pxa^2} + \frac{S}{bc} \times \frac{\overline{A - \alpha} \times \overline{c - a}}{rx + 1} + \frac{Aa}{2r^a + 1}$, C being the value of an annuity for $a$ years, A the value of an annuity for $\overline{c - a}$. years, $\alpha$ the value of an annuity on a life whose complement is $\overline{c - a}$, and $x$ the value of £1 payable at the end of $a$ years; the other symbols denoting the same quantities as above.

*When A is the oldest and C the youngest of the three lives*, the symbols, $a$, $b$, $c$, $r$, P, A, and $x$, remaining the same as in the preceding theorem, let B denote the value of an annuity for $\overline{b - a}$ years, and $\beta$ the value of an annuity on a life whose complement is $\overline{b - a}$, then will the value be $\frac{S}{2abc} \times \overline{\overline{C - xa} \times 2P^2 - Pxa^2} + \frac{S.A}{2cr^a + 1} + \frac{S.\overline{b - a}.\overline{B - 2\beta}}{2bcr^a + 1}$.

It would be tedious to go through the investigation of each of those theorems, and therefore I shall confine myself to that of the last, which, comprehending in it all the reasoning that has been applied to the solution of the other three, renders a separate investigation unnecessary.

The value, according to Mr. Simpson, during the time $x$ being $\frac{Sq^x}{2abcm} \times \overline{xx - \frac{2x}{m} + \frac{2}{m^2}}$, let the fluent be corrected by first supposing $x$ to vanish and then to become equal to $a$, and the value during the time $a$ will become $\frac{aSq^a}{2bcm} - \frac{Sq^a}{bcm^2} + \frac{S.}{abcm^3} \times \overline{q^a - 1}$. Let P be now substituted for its equal $-\frac{1}{m}$, C for $\overline{1 - q^a} \times -\frac{1}{m}$, and $x$ for $q^a$, and the above expression will be reduced to $\frac{S}{2abc} \times \overline{\overline{C - xa} \times 2P^2 - Pxa^2}$. The value of the expectation after the time $a$ depends on the contingency of C's dying *after* the greatest

limit of A's life, having previously survived B. Now as B may have died *within* the limit of A's life, though after the decease of A, this probability must be taken into consideration, which, by reasoning as Mr. Simpson has done in page 316 of his Select Exercises, will be expressed by $\frac{a}{2b}$. The probability of B's dying in $x$ time after this period is $\frac{x}{b}$. The sum of these two fractions, or $\frac{a+2x}{b}$, give the whole probability of B's dying in $\overline{a+x}$ time, according to the order in the Problem. The value therefore of the reversion during the moment $\dot{x}$ will be $\frac{a+2x}{2b} \times \frac{\dot{x}}{c} \times Sq^x = \frac{aS}{2bc} \times q^x\dot{x} + \frac{S}{bc} \times q^x x\dot{x}$. The fluent of the first expression generated while $x$ is increasing from $a$ to $b$ will be $\frac{aS}{2bc} \times B$ *. The second expression may be changed into $\frac{\overline{b-a}.S}{bcr} \times \frac{q^x.x\dot{x}}{b-a}$, whose fluent generated as above is equal to $\frac{\overline{b-a}.S}{bcr} \times \overline{B-\beta}$ †. As this reversion, expressed by both the preceding quantities, is not to take place till after the expiration of $a$ years, its value will be reduced to $\frac{aS \times B}{2bcr^{a+1}} + \frac{\overline{b-a}.S}{bc} \times \frac{\overline{B-\beta}}{r^{a+1}}$. The third and last part of the reversion depends on the event of C's not dying till after the extinction of the lives of A and B, A having died before B. This expectation during the moment $\dot{x}$ will be $\frac{Sq^x}{2c} \times \dot{x}$, whose fluent, generated while $x$ is increasing from $b$ to $c$, will be the value of the reversion, and is equal to $\frac{S}{2c}$ multiplied into M, the value of an annuity certain for $\overline{c-b}$

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* See Simpson's Select Exercises, pag. 324. † Ibid. pag. 327.

ears *. This being discounted for, $b+1$ years will become $= \frac{S.M}{2cr^{b+1}}$. Hence the whole value of all the expectations $= \frac{S}{2abcr} \times \overline{C-xa.\ 2P^2 - Pxa^2} + \frac{S.a}{2bc} \times \frac{B}{r^{a+1}} + \frac{\overline{b-a}.S}{bc} \times \frac{B-\beta}{r^{a+1}} + \frac{S.M}{2c.r^{b+1}}$; or, by substituting $\frac{A}{r^{a+1}}$ for its equal $\frac{B}{r^{a+1}}$, $+ \frac{M}{r^{b+1}} = \frac{S}{2abcr} \times \overline{C-xa.\ 2P^2 - Pxa^2} + \frac{S.A}{2cr^{a+1}} + \frac{\overline{b-a}.S.\overline{B-2\beta}}{2bcr^{a+1}}$ Q.E.D.

When the lives of B and C are very old, the rules derived from these theorems are altogether wrong, and in some cases they entirely fail. They have been given here merely as a proof (if any proof were necessary) of the extreme incorrectness of M. De Moivre's hypothesis, when applied to complicated cases of this kind.

## NOTE XXXVIII. (Prob. xlviii.)

In the first year the payment of the given sum will depend on either of four events:—1st. That the three lives die; 2dly. That A and B die, and C lives; 3dly. That A dies *after* C, and B lives; 4thly. That B dies *after* C, and A lives. The fractions expressing these several contingencies may be reduced to $\frac{1}{ams} \times \left(ams - \frac{bms}{2} - \frac{ans}{2} - \frac{at}{2} + bnt - \frac{bmt}{2}\right)$. In the second and following years the payment will depend on either of thirteen events:—1st. That the three lives die in the year; 2dly. That A and B die in the year, and C lives; 3dly. That A dies *after* C in the year, and B lives; 4thly. That B dies *after* C in the year, and A lives; 5thly. That B and C both die

* Simpson's Select Exercises, pag. 324.

in the year, and A in either of the preceding years; 6thly. That A and C both die in the year, and B in either of the preceding years; 7thly. That A and B both die in the year, and C in either of the preceding years; 8thly. That B dies in the year, A in either of the preceding years, and C lives; 9thly. That A dies in the year, B in either of the preceding years, and C lives; 10thly. That B dies in the year, C in either of the preceding years, and A lives; 11thly. That A dies in the year, C in either of the preceding years, and B lives; 12thly. That B dies in the year, A having died *before* C in either of the preceding years; and, 13thly. That A dies in the year, B having died *before* C in either of the preceding years. From the several fractions expressing these contingencies may be obtained the seventeen following series: $\frac{S}{ams} \times \left(\frac{ams}{r} - \frac{bnt}{r^2} - \frac{cou}{r^3}, \&c.\right) - \frac{S}{2ams} \times \frac{bms+ans}{r} - \frac{S}{2ams} \times \left(\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3}, \&c.\right) + \frac{S}{ams} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} +, \&c.\right) - \frac{S}{2ams} \times \left(\frac{bmt}{r} + \frac{cnu}{r^2} + \frac{dow}{r^3} +, \&c.\right) + \frac{S}{2amsr} \times \left(\frac{ols}{r} + \frac{pcs}{r^2} + \frac{qds}{r^3} +, \&c.\right) + \frac{S}{amsr} \times \left(\frac{cot}{r} + \frac{dup}{r^2} + \frac{ewq}{r^3} +, \&c.\right) - \frac{S}{amsr} \times \left(\frac{cos}{r} + \frac{dps}{r^2} + \frac{eqs}{r^3}, \&c.\right) + \frac{S}{2amsr} \times \left(\frac{ans}{r} + \frac{aos}{r^2} + \frac{aps}{r^3}, \&c.\right) - \frac{S}{2amsr} \times \left(\frac{aos}{r} + \frac{aps}{r^2} + \frac{aqs}{r^3}, \&c.\right) + \frac{S}{2amsr} \times \left(\frac{bms}{r} + \frac{cms}{r^2} + \frac{dms}{r^3}, \&c.\right) - \frac{S}{2amsr} \times \left(\frac{cms}{r} + \frac{dms}{r^2} + \frac{ems}{r^3}, \&c.\right) + \frac{S}{2amsr} \times \left(\frac{cns}{r} + \frac{dos}{r^2} + \frac{eps}{r^3}, \&c.\right) + \frac{S}{2amsr} \times \left(\frac{ant}{r} + \frac{aou}{r^2} + \frac{apw}{r^3}, \&c.\right) - \frac{S}{2amsr} \times \left(\frac{aot}{r} + \frac{apu}{r^2} + \frac{aqw}{r^3}, \&c.\right) + \frac{S}{2amsr} \times \left(\frac{bmt}{r} + \frac{cmu}{r^2} + \frac{dmw}{r^3}, \&c.\right) - \frac{S}{2amsr} \times \left(\frac{cmt}{r} + \frac{dmu}{r^2} + \frac{emw}{r^3}, \&c.\right)$ These series may at last be reduced to S into $\frac{\overline{1+r-1}.ABC.}{r} - \frac{\overline{r-1}.\overline{A+B}.}{2r} + \frac{AC+BC.}{2r} - AB. +$

$\frac{nb.\overline{1+HPC}.}{amr} - \frac{nt.\overline{1+APT}.}{2mrr} - \frac{bt.\overline{1+HBT}.}{2arr} + \frac{n.\overline{AP-CP}.}{2mr} + \frac{b.\overline{HB-CH}.}{2ar}$. But unless A and B are nearly of the same age, and both older than C, this rule will not be sufficiently correct.

If *B be the oldest of the three lives,* the annuities A, AC, and HC, should be continued only for as many years ($z$) as are equal to the difference between the age of B and that of the oldest life in the Table of Observations. Let those annuities respectively be denoted by A′, A′C′, and H′C′; also let $\varphi$ denote the probability that C survives B, $k$ the number of persons living opposite the age of A at the end of $z$ years, then will the value of S after $z$ years be $\frac{S.\,\varphi k}{ar^z+1} \times \overline{r-1}.\,\overline{V-A^z}$, V representing the perpetuity, and $A^z$ the value of an annuity on a life $z$ years older than A, and the whole value of the reversion in this case will be S into $\frac{1+\overline{r-1}.\,ABC.}{r} - \frac{\overline{r-1}.\,\overline{A'+B}.}{2r} + \frac{A'C'+BC.}{2r} - AB. + \frac{nb.\overline{1+HPC}.}{amr} - \frac{nt.\overline{1+APT}.}{2mrr} - \frac{bt.\overline{1+HBT}.}{2arr} + \frac{n.\overline{AP-CP}.}{2mr} + \frac{b.\overline{HB-H'C'}}{2ar} + \frac{\varphi k}{ar^z+1} \times \overline{r-1}.\,\overline{V-A^z}$.

*If C be the oldest of the three lives,* the given sum, after the necessary extinction of his life, may be received upon either of five events:—1st. If A should have died before C, and B died in the* $\overline{z+1^{st}}$ $\overline{z+2^{d}}$ &c. year; 2dly. If B should have died before C and A died in those years respectively; 3dly, If A and B should both die; 4thly. If A should die and B live; 5thly. If B should die and A live in either of those years. Let $\tau$ denote the proba-

* $z$, being the difference between the age of C and that of the oldest person in the table.

bility that C dies after A, $\pi$ the probability that C dies after B, $\mu$ the number of persons living in the table opposite the age of B at the end of $z$ years, and $\beta$ the same number opposite the age of A, then will these contingencies be equal to $\frac{\pi\mu.\overline{r-1}.\overline{V-B^z}\,*}{mr^z+1} + \frac{\beta\pi.\overline{r-1}.\overline{V-A^z}}{ar^z+1} + \frac{\beta\mu}{amr^z} \times \overline{\frac{1+AB^z}{r} - AB^z}$ and the whole value of the reversion will be S into $\frac{1+\overline{r-1}.ABC}{r} - \frac{\overline{r-1}.\overline{A'+B'}}{2r} + \frac{AC+BC}{2r} - AB + \frac{nb.\overline{1+HPC}}{amr.} - \frac{nt.\overline{1+APT}}{2msr} - \frac{bt.\overline{1+HBT}}{2asr} + \frac{\pi.\overline{A'P'-CP}}{2mr} + \frac{b}{2ar} \times \overline{H'B'-HC} + \frac{\beta\mu}{amr^{z+1}} \times \overline{1+AB^z} + \frac{\pi\mu.\overline{r-1}.\overline{V-B^z}}{mr^z+1} + \frac{\pi\beta.\overline{r-1}.\overline{V-A^z}}{ar^z+1}$.

If the three lives are of equal age the value becomes $\frac{S.\overline{r-1}}{r} \times \overline{V-C-CC+CCC}$.

The solution of this problem may be obtained by the assistance of the 6th and 41st problems. Let Z be the value of an annuity by the 6th problem, X the value of an annuity on the life of A after C, provided C should survive B, and Y the value of an annuity on the life of B after C, provided C should survive A, both found by the forty-first problem, then will the value of the given sum be S into $\frac{\overline{r-1}}{r} \times \overline{V-Z-\overline{X+Y}}$, which becomes $\frac{S.\overline{r-1}}{r} \times \overline{V-C-CC+CCC}$ as in the preceding case when the lives are equal.

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* $B^z$, $A^z$, &c. being the values of annuities on lives $z$ years older than A, B, &c.

## NOTE XXXIX. (Prob. xlix.)

In the first year the payment of the given sum depends upon either of six events:—1st. That the three lives die; 2dly. That A. and B die and C lives; 3dly. That A dies *before* C, and B lives; 4thly. That B dies *before* C, and A lives; 5thly. That A dies, and B and C both live; 6thly. That B dies, and A and C both live; in the second and following years, in addition to the above events, it will depend, 7thly. On A and B both dying in the year, C having died in one or other of the preceding years; 8thly. On B's dying in the year, A having died *after* C in either of the preceding years; and 9thly. On A's dying in the year, B having died *after* C in either of the preceding years. The fractions expressing these several contingencies being expanded, will form the twenty following series:

$$\frac{S}{ams} \times \left(\frac{ams}{r} + \frac{bnt}{r^2} + \frac{cou}{r^3} \text{ \&c.}\right) - \frac{S}{2ams} \times \left(\frac{bms}{r} + \frac{cnt}{r^2} + \frac{dou}{r^3} \text{ \&c.}\right)$$
$$- \frac{S}{2ams} \times \left(\frac{ans}{r} + \frac{bot}{r^2} + \frac{cpu}{r^3} \text{ \&c.}\right) - \frac{S}{ams} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpu}{r^3} \text{ \&c.}\right) + \frac{S}{2ams} \times \left(\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3} + \text{\&c.}\right) + \frac{S}{2ams} \times \left(\frac{bmt}{r} + \frac{cnu}{r^2} + \frac{dow}{r^3} + \text{\&c.}\right) - \frac{S}{2amsr} \times \left(\frac{bos}{r} + \frac{cps}{r^2} + \frac{dqs}{r^3} \text{ \&c.}\right) + \frac{S}{2amsr} \times \left(\frac{bot}{r} + \frac{cpu}{r^2} + \frac{dqw}{r^3} + \text{\&c.}\right) - \frac{S}{2amsr} \times \left(\frac{cns}{r} + \frac{dos}{r^2} + \frac{eps}{r^3} \text{ \&c.}\right)$$
$$+ \frac{S}{2amsr} \times \left(\frac{nct}{r} + \frac{odu}{r^2} + \frac{pew}{r^3} \text{ \&c.}\right) + \frac{S}{amsr} \times \left(\frac{cos}{r} + \frac{dps}{r^2} + \frac{eqs}{r^3} \text{ \&c.}\right) - \frac{S}{2amsr} \times \left(\frac{cot}{r} + \frac{dpu}{r^2} + \frac{eqw}{r^3} \text{ \&c.}\right) + \frac{S}{2amsr} \times \left(\frac{ans}{r} + \frac{aos}{r^2} + \frac{aps}{r^3} \text{ \&c.}\right) - \frac{S}{2amsr} \times \left(\frac{ant}{r} + \frac{aou}{r^2} + \frac{apw}{r^3} \text{ \&c.}\right) - \frac{S}{2amsr} \times \left(\frac{aos}{r} + \frac{aps}{r^2} + \frac{aqs}{r^3} \text{ \&c.}\right) + \frac{S}{2amsr} \times \left(\frac{aot}{r} + \frac{apu}{r^2} + \frac{aqw}{r^3} \text{ \&c.}\right)$$
$$+ \frac{S}{2amsr} \times \left(\frac{bms}{r} + \frac{cms}{r^2} + \frac{dms}{r^3} + \text{\&c.}\right) - \frac{S}{2amsr} \times \left(\frac{bmt}{r} + \frac{cmu}{r^2}\right.$$

$+ \frac{dmw}{r^3} + \&c.\Big) - \frac{S}{2amsr} \times \Big(\frac{msc}{r} + \frac{msd}{r^2} + \frac{mse}{r^3} + \&c.\Big) + \frac{S}{2amsr}$ $\times \Big(\frac{cmt}{r} + \frac{dmu}{r^2} + \frac{emw}{r^3} + \&c.\Big)$ The 1st and 4th series are $= \frac{1-\overline{r-1}.\,ABC.}{r}$, the 2d, 10th, and 19th $= -\frac{A}{2}$, the 3d, 8th, and 15th $= -\frac{B}{2}$, the 5th $= \frac{nt.\,\overline{1+APT}}{2msr}$, the 6th $= \frac{bt.\,\overline{1+BHT}}{2asr}$, the 7th and 16th $= \frac{n.\,\overline{PC-AP}}{2mr}$, the 9th and 20th $= \frac{b.\,\overline{CH-BH}}{2ar}$, the 11th and 12th $= AB - \frac{nb.\,\overline{1+HPC}}{dmr}$, the 13th $= \frac{B}{2r}$, the 17th $= \frac{A}{2r}$, the 14th and 18th $= -\frac{BC+AC}{2r}$. Hence the whole value of the reversion *when A and B are both older than C* will be S into

$$\frac{1-\overline{r-1}.\,ABC + \overline{\frac{A+B}{2}}}{r} + AB - \frac{BC+AC}{2r} + \frac{n.\,\overline{PC-AP}}{2mr} + \frac{b.\,\overline{CH-BH}}{2ar} + \frac{nt.\,\overline{1+APT}}{2msr} + \frac{bt.\,\overline{1+BHT}}{2asr} - \frac{nb.\,\overline{1+HPC}}{amr}.$$

*If B be the oldest of the three lives*, let the same symbols be used as in the preceding Note, and the value will be

$$S \text{ into } \frac{1-\overline{r-1}.\,ABC + \overline{\frac{A'+B}{2}}}{r} + AB - \frac{BC+A'C'}{2r} + \frac{n.\,\overline{PC-AP}}{2mr} + \frac{b.\,\overline{C'H'-BH}}{2ar} + \frac{nt.\,\overline{1+APT}}{2msr} + \frac{bt.\,\overline{1+BHT}}{2asr} - \frac{nb.\,\overline{1+HPC}}{amr} + \frac{\phi k.\,\overline{r-1}.\,\overline{V-A^z}}{ar^z+1}.$$

$\phi$ in this case denoting the probability that B has died *after* C.

*If C be the oldest of the three lives*, the given sum may be received after the necessary extinction of his life, provided either of three events shall happen:—1st. If A shall have died *after* C in the first two years, and B dies in the $\overline{z+1}^{st}$ $\overline{z+2}^{d}$, &c. year; 2dly. If B having died *after*

C in the first $z$ years, A dies in the $\overline{z+1}^{st}$ $\overline{z+2}^{d}$, &c. year; 3dly. If both A and B having survived the first $z$ years, the survivor of them dies in any of the following years. Let $\pi$ denote the probability that A dies *after* C, and $\varphi$, as above, the probability that B dies *after* C, then will the value of the reversion be S into

$$\frac{\overline{1-\overline{r-1}.\,ABC+\frac{A'+B'}{2}}}{r}$$

$$+\,AB-\frac{BC+AC}{2r}+\frac{n.\,\overline{PC-A'P'}}{2mr}+\frac{b.\,\overline{HC-B'H'}}{2ar}+\frac{nt.\,\overline{1+APT}}{2msr}$$

$$+\frac{bt.\,\overline{1+BHT}}{2asr}-\frac{nb.\,\overline{1+HBC}}{amr}+\frac{\varphi.\beta.\overline{r-1}}{ar^{z+1}}\times\overline{V-A^{z}}+\frac{\pi.\mu.\overline{r-1}}{mr^{z+1}}$$

$$\times\,\overline{V-B^{z}}+\frac{\beta\mu}{amr^{z+1}}\times\overline{1-\overline{r-1}.\,\overline{A^{z}+B^{z}}-AB^{z}}.$$

$\mu$ and $\beta$ denoting the number of persons living at the end of $\overline{z+1}$. years opposite to the ages of B and A.

This problem may also be solved by the assistance of the 41st Problem. Let W be the value of an annuity on the life of A after B, provided B survives C, and Z the value of an annuity on the life of B after A, provided A survives C, then will the value of the given sum be $\frac{S.\overline{r-1}}{r}\times\overline{V-AB-\overline{W+Z}}$. If the three lives are of equal age the value may be found either from this or the preceding theorems $=\frac{S.\overline{r-1}}{r}\times\overline{V+CC-\overline{C+CCC}}$.

## NOTE XL. (Prob. l.)

The value of the given sum in the 1st year depends on either of four events:—1st. That the three lives fail, A having died first, or B having died first, A next, and C last; 2dly. That A and B die, and C lives; 3dly. That A dies, and B and C both live; 4thly. That A dies after C and B lives. In the second and following years it depends

upon either of six events:—1st. That the three lives fail in the year in the order above mentioned; 2dly. That A and B die in the year, and C lives; 3dly. That A dies in the year, and B and C both live; 4thly. That A dies after C in the year, and B lives; 5thly. That A dies before C in the year, and that B dies in any of the preceding years; 6thly. That A dies in the year, B having died in one or other of the preceding years, and that C lives. These several contingencies in the first year are $\frac{S}{amsr} \times \left(\frac{\overline{a-b}.\overline{m-n}.\overline{s-t}}{2} + \overline{a-b}.\overline{m-n}.t + \overline{a-b}.nt + \frac{\overline{a-b}.\overline{s-t}.n}{2}\right)$

In the second year $\frac{S}{amsr^2} \times \left(\frac{\overline{b-c}.\overline{n-o}.\overline{t-u}}{2} + \overline{b-c}.\overline{n-o}.u + \overline{b-c}.ou + \frac{\overline{b-c}.\overline{t-u}.o}{2} + \frac{\overline{m-n}.\overline{b-c}.\overline{t-u}}{2} + \overline{m-n}.\overline{b-c}.u\right)$.

In the third year $\frac{S}{amsr^3} \times \left(\frac{\overline{c-d}.\overline{o-p}.\overline{u-w}}{2} + \overline{c-d}.\overline{o-p}.w + \overline{c-d}.pw + \frac{\overline{c-d}.\overline{u-w}.p}{2} + \frac{\overline{m-o}.\overline{c-d}.\overline{u-w}}{2} + \overline{m-o}.\overline{c-d}.w\right)$, and so on. These fractions being expanded, &c. will at last be found $= \frac{S}{2ams} \times \left(\frac{ams}{r} + \frac{bmt}{r^2} + \frac{cmu}{r^3}\ \&c.\right) - \frac{S}{2ams} \times \left(\frac{bms}{r} + \frac{cmt}{r^2} + \frac{dmu}{r^3}\ \&c.\right) + \frac{S}{2ams} \times \left(\frac{amt}{r} + \frac{bmu}{r^2} + \frac{cmw}{r^3}\ \&c.\right) - \frac{S}{2ams} \times \left(\frac{bmt}{r} + \frac{mcu}{r^2} + \frac{mdw}{r^3}\ \&c.\right) = \frac{S}{2r} \times \left(1 - \overline{r-1}.\,AC + \frac{t.\overline{1+AT}}{s} - \frac{b.\overline{1+HC}}{a}\right)$ which by Note XVII is the value of S depending on the contingency of C surviving A, which may be proved from other principles.

The value of the given sum in this case depends either on the contingency of A's life being the *first* that fails of the three lives, or on the contingency of C's living to the decease of A after the extinction of the life of B. If the algebraical fractions in Notes XIX and XX be added

together, changing $m$, $n$, B, P in the former Note for $a$, $b$, A, H, and *vice versa*, the sum of the terms will be reduced to R, or to the value of the given sum depending on the contingency of C surviving A.

## NOTE XLI. (Prob. LI.)

In the first year the value of the given sum depends on either of three events:—1st. On the extinction of the three lives, B having died first, A next, and C last, or B having died first, C next, and A last; 3dly. On the extinction of the life of A after B, C having lived to the end of the year. In the second and following years the value will depend on either of five events:—1st. On the extinction of the three lives in the year in the order mentioned above; 2dly. On the extinction of the life of A after B in the year, C surviving that year; 3dly. On the extinction of the lives of A and C in the year, B having died in either of the preceding years; 4thly. On the death of A in the year, B having died in one or other of the preceding years, and C lived to the end of the year; 5thly. On the death of A in the year, B having died before C in either of the preceding years. The value therefore of the given sum in the first year will be $= \frac{S}{amsr} \times \left(\frac{\overline{a-b}.\overline{m-n}.\overline{s-t}}{3} + \frac{\overline{a-b}.\overline{m-n}.t}{2}\right)$; in the second year $\frac{S}{amsr^2} \times \left(\frac{\overline{b-c}.\overline{n-o}.\overline{t-u}}{3} + \frac{\overline{b-c}.\overline{n-o}.u}{2} + \overline{b-c}.\overline{t-u}.\overline{m-n} + \overline{b-c}.\overline{m-n}.u + \frac{\overline{b-c}.\overline{m-n}.\overline{s-t}}{2}\right)$; in the third year $\frac{S}{amsr^3} \times \left(\frac{\overline{c-d}.\overline{o-p}.\overline{u-w}}{3} + \frac{\overline{c-d}.\overline{o-p}.w}{2} + \overline{c-d}.\overline{u-w}.\overline{m-o} + \overline{c-d}.\overline{m-o}.w + \frac{\overline{c-d}.\overline{m-o}.\overline{s-u}}{2}\right)$, and so on. Having expanded these several fractions, &c. we

have $\frac{S}{3ams} \times \left(\frac{ams}{r} + \frac{bnt}{r^2} + \frac{cou}{r^3} + \&c.\right) = \frac{S}{3r} \times \overline{1 + ABC}$ $- \frac{S}{3ams} \times \left(\frac{bms}{r} + \frac{nct}{r^2} + \frac{odu}{r^3} + \&c.\right) = -\frac{S}{3r} \times \frac{b.\overline{1+HBC}}{a}$ $- \frac{S}{3ams} \times \left(\frac{ans}{r} + \frac{bot}{r^2} + \frac{cpu}{r^3} + \&c.\right) = -\frac{S}{3r} \times \frac{n.\overline{1+ACP}}{m}$ $+ \frac{S}{3ams} \times \left(\frac{bns}{r} + \frac{cot}{r^2} + \frac{dpu}{r^3} + \&c.\right) = \frac{S}{3r} \times \frac{nb.\overline{1+HPC}}{am}$ $+ \frac{S}{6ams} \times \left(\frac{amt}{r} + \frac{bnu}{r^2} + \frac{cow}{r^3} + \&c.\right) = \frac{S}{6r} \times \frac{t.\overline{1+ABT}}{s}$ $- \frac{S}{6ams} \times \left(\frac{bmt}{r} + \frac{cnu}{r^2} + \frac{dow}{r^3} + \&c.\right) = -\frac{S}{6r} \times \frac{bt.\overline{1+HBT}}{as}$ $- \frac{S}{6ams} \times \left(\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3} + \&c.\right) = -\frac{S}{6r} \times \frac{nt.\overline{1+APT}}{ms}$ $+ \frac{S}{6ams} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} + \&c.\right) = \frac{S \times ABC}{6}$ $+ \frac{S}{2asr} \times \left(\frac{bt}{r} + \frac{cu}{r^2} + \frac{dw}{r^3} + \&c.\right) = \frac{S + AC}{2r}$ $- \frac{S}{2amsr} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} \&c.\right) = -\frac{S.\,ABC}{2r}$ $- \frac{S}{2nsr} \times \left(\frac{ct}{r} + \frac{du}{r^2} + \frac{ew}{r^3} + \&c.\right) = -\frac{S \times b.\,HC}{2ar}$ $+ \frac{S}{2amsr} \times \left(\frac{nct}{r} + \frac{odu}{r^2} + \frac{pew}{r^3} + \&c.\right) = \frac{S.\,b.\,HBC}{2ar}$ $+ \frac{S}{2ar} \times \left(\frac{b}{r} + \frac{c}{r^2} + \frac{d}{r^3} + \&c.\right) = \frac{S.\,A}{2r}$ $- \frac{S}{2amr} \times \left(\frac{bn}{r} + \frac{co}{r^2} + \frac{dp}{r^3} + \&c.\right) = -\frac{S.\,AB}{2r}$ $- \frac{S}{2ar} \times \left(\frac{c}{r} + \frac{d}{r^2} + \frac{e}{r^3} + \&c.\right) = \frac{S}{2r} \times \overline{A - \frac{b}{a}}$ $+ \frac{S}{2amr} \times \left(\frac{nc}{r} + \frac{od}{r^2} + \frac{pe}{r^3} + \&c.\right) = \frac{S.\,b.\,HB}{2ar}$. The whole value therefore will be, when *A is the oldest of the three lives* $=$ $\frac{S}{r}$ into $\frac{2 + \overline{r-1}.\,ABC}{6} - \frac{\overline{r-1}.\,A}{2} + \frac{AC - AB}{2} + \frac{b.\,\overline{HB - HC}}{2a} + \frac{nb.\,\overline{1+HPC}}{3am} - \frac{bt.\,\overline{1+HBT}}{6as} - \frac{nt.\,\overline{1+APT}}{6ms} + \frac{b.\,\overline{1+HBC}}{6a} + \frac{t.\,\overline{1+ABT}}{6s} - \frac{n.\,\overline{1+APC}}{3m}$.

*When B is the oldest of the three lives,* let $x$ be the number of years between the age of B and of the oldest

person in the table; $\tau$ the number of persons living opposite the age of C and $\alpha$ the same number opposite the age of A at the end of $x$ years; A′, A′C′, and H′C′ the values of annuities on those single and joint lives for $x$ years, and $A^x$ the value of a life $x$ years older than A. The given sum may be received after the first $x$ years on the death of A, provided C shall have survived that term, or that he shall have died *after* B in the mean time. Let $\pi$ denote the last-mentioned probability, then will the value of this part of the reversion be S into $\overline{\pi + \frac{\tau}{s}} \times \frac{\alpha.\overline{r-1}.\overline{V-A^x}}{ar^x+1}$ and the whole value of the reversion $= \frac{S}{r}$ into

$$\frac{2+\overline{r-1}.ABC}{6} - \frac{\overline{r-1}.A'}{2} + \frac{A'C'-AB}{2} + \frac{b.\overline{HB-H'C'}}{2a} + \frac{nb.\overline{1+HPC}}{3am} - \frac{bt.\overline{1+HBT}}{6as} - \frac{nt.\overline{1+APT}}{6ms} + \frac{b.\overline{1+HBC}}{6a} + \frac{t.\overline{1+ABT}}{6s} - \frac{n.\overline{1+APC}}{3m} + \overline{\pi + \frac{\tau}{s}} \times \frac{\alpha.\overline{r-1}.\overline{V-A^x}}{ar^x}.$$

*When C is the oldest of the three lives,* let $z$ be the difference between the age of C and of the oldest person in the table, A′, A′B′, and H′B′ the values of annuities on those single and joint lives for $z$ years, $A^z$ the value of a life $z$ years older than A, $\mu$ the number of persons living opposite the age of A at the end of $z$ years, and $\varphi$ the probability that C dies after B, then will the value of the reversion be $\frac{S}{r}$ into

$$\frac{2+\overline{r-1}.ABC}{6} - \frac{\overline{r-1}.A'}{2} + \frac{AC-A'B'}{2} + \frac{b.\overline{H'B'-HC}}{2a} + \frac{nb.\overline{1+HPC}}{3am} - \frac{bt.\overline{1+HBT}}{6a} - \frac{nt.\overline{1+APT}}{6ms} + \frac{b.\overline{1+HBC}}{6a} + \frac{t.\overline{1+ABT}}{6s} - \frac{n.\overline{1+APC}}{3m} + \overline{\varphi + \frac{\mu}{m}} \times \frac{\overline{r-1}.\overline{V-A^z}}{ar^z}.$$

*When the three lives are of equal age,* the value will be

$$\frac{S.\overline{r-1}}{6r} \times \overline{2V-3\overline{C-CCC}}.$$

As the reversion in this Problem consists of two parts:—1st. Of the contingency of receiving the given sum on the death of A, provided B should be then dead and C living; and 2dly. Of the contingency of receiving it on the death of A, provided B should be the first, C the second, and A the third that fails;—it follows that the present solution may be obtained from those of the twenty-seventh and forty-sixth Problems. But the computations derived from the addition of those of two Problems would be more tedious and complicated, and therefore the preceding rules are to be preferred.

## NOTE XLII. (Prob. lii.)

In the first year the value of the given sum depends upon either of four events:—1st. On the extinction of the three lives, A having been the first or last that failed; 2dly. On A's having been the only life that failed; 3dly. On A's having failed before B, and C's having lived; 4thly. On A's having failed before C and B's having lived; which several contingencies are expressed by the fractions

$$\frac{\overline{a-b}\cdot\overline{m-n}\cdot\overline{s-t}}{2ams} + \frac{\overline{a-b}\cdot nt}{ams} + \frac{\overline{a-b}\cdot\overline{m-n}\cdot t}{2ams} + \frac{\overline{a-b}\cdot\overline{s-t}\cdot n}{2ams}$$

In the second and following years the value will depend on either of six events:—1st. On the extinction of the three lives in the year, A having been the first or last that failed; 2dly. On A's having been the only life that failed in the year; 3dly. On A's having failed before B in the year, and C's having lived; 4thly. On A's having failed before C in the year, and B's having lived; 5thly. On B's having failed in either of the preceding years, and A's having died *after* C in the year; 6thly. On B's having failed *before* C in either of the preceding years, and A's

having died in the year. The contingencies therefore in the second year will be expressed by the fractions $\frac{\overline{b-c}.\overline{n-o}.\overline{t-u}}{2ams} + \frac{\overline{b-c}.ou}{ams} + \frac{\overline{b-c}.\overline{n-o}.u}{ams} + \frac{\overline{b-c}.\overline{t-u}.o}{2ams} + \frac{\overline{b-c}.\overline{m-n}.\overline{t-u}}{2ams} + \frac{\overline{b-c}.\overline{s-t}.\overline{m-n}}{2ams}$, in the third year by the fractions $\frac{\overline{c-d}.\overline{o-p}.\overline{u-w}}{2ams} + \frac{\overline{c-d}.pw}{ams} + \frac{\overline{c-d}.\overline{o-p}.w}{2ams} + \frac{\overline{c-d}.\overline{u-w}.p}{2ams} + \frac{\overline{c-d}.\overline{m-o}.\overline{u-w}}{2ams} + \frac{\overline{c-d}.\overline{s-u}.\overline{m-o}}{2ams}$, and so on. These fractions may be reduced into the twelve following series: $\frac{1}{2ams} \times \left(\frac{ams}{r} + \frac{bnt}{r^2} + \frac{cou}{r^3} + \frac{dpw}{r^3} \&c.\right) + \frac{1}{2ams} \times \left(\frac{ant}{r} + \frac{bou}{r^2} + \frac{cpw}{r^3} + \&c.\right) - \frac{1}{2ams} \times \left(\frac{bms}{r} + \frac{cnt}{r^2} + \frac{dou}{r^3} + \&c.\right) - \frac{1}{2ams} \times \left(\frac{bnt}{r} + \frac{cou}{r^2} + \frac{dpw}{r^3} + \frac{eqx}{r^4} \&c.\right) + \frac{1}{2a} \times \left(\frac{b}{r^2} + \frac{c}{r^3} + \frac{d}{r^4} + \&c.\right) - \frac{1}{2am} \times \left(\frac{bn}{r^2} + \frac{co}{r^3} + \frac{dp}{r^4} + \&c.\right) - \frac{1}{2a} \times \left(\frac{c}{r^2} + \frac{d}{r^3} + \frac{e}{r^4} + \&c.\right) + \frac{1}{2am} \times \left(\frac{cn}{r^2} + \frac{do}{r^3} + \frac{ep}{r^4} + \&c.\right) - \frac{1}{2as} \times \left(\frac{bu}{r^2} + \frac{cw}{r^3} + \frac{dx}{r^4} + \&c.\right) + \frac{1}{2ams} \times \left(\frac{bnu}{r^2} + \frac{cow}{r^3} + \frac{dpx}{r^4} + \&c.\right) + \frac{1}{2as} \times \left(\frac{cu}{r^2} + \frac{dw}{r^3} + \frac{ex}{r^4} \&c.\right) - \frac{1}{2ams} \times \left(\frac{cnu}{r^2} + \frac{dow}{r^3} + \frac{epx}{r^4} + \&c.\right)$. From these series the value of the given sum, *when A is the oldest of the three lives*, may be found equal to S into $\frac{\overline{1-r-1}.\overline{ABC+A}}{2r} + \frac{AC}{2} - \frac{AB}{2r} + \frac{b.\overline{HB-HBC}}{2ar} - \frac{t.\overline{AT-ABT}}{2sr} + \frac{nt.\overline{1+APT}}{2msr} - \frac{bt.\overline{1+HBT}}{2asr}$.

*When C is the oldest of the three lives*, let $\pi$ denote the probability that C dies after B, let $z$ be the difference between the age of C and that of the oldest life in the table, A′, A′B′ and H′B′, the value of these single and joint lives for $z$ years, and $A^z$ the value of a life $z$ years older than A, then will the value be S into

$$\frac{\overline{1-\overline{r-1}.\,\overline{ABC+A'}}}{2r}+\frac{AC}{2}-\frac{A'B'}{2r}+\frac{b.\,\overline{H'B'-HBC}}{2ar}-\frac{t.\,\overline{AT-ABT}}{2sr}+\frac{nt.\,\overline{1+APT}}{2msr}-\frac{bt.\,\overline{1+HBT}}{2asr}+\frac{\pi.\,\alpha.\,\overline{r-1}.\,\overline{V-Az}}{ar^{z+1}}.$$ $\alpha$ denoting the number of persons living in the table opposite to the age of A at the end of $z$ years.

*If B be the oldest of the three lives,* the value of the reversion after the necessary extinction of the life of B will depend on either of three events:—1st. On the contingency of C's having died *after* him in the first $x$ years, and A's having died in the $\overline{x+1}^{st}$. $\overline{x+2}^{d}$. &c. years; 2dly. On A's having died after C in either of those years; 3dly. On C's having lived and A only having died in either of those years, $x$ representing the difference between the age of B and the oldest life in the table. Let $\alpha$, $\beta$, $\gamma$, $\delta$, &c. represent the number of persons living opposite the age of A at the end of $x$, $\overline{x+1}$, $\overline{x+2}$, &c. years, and $\sigma$, $\tau$, $\upsilon$, &c. the same numbers opposite the age of C: Let $\phi$ denote the probability that C dies *after* B in $x$ years, and $A^x$ be the value of a life $x$ years older than A; then will the value on the first of the above mentioned contingencies be $\frac{\phi.\,\alpha.\,\overline{r-1}.\,\overline{V-A^x}}{ar^{x+1}}$, and the value on the 2d and 3d contingencies will be expressed by the fractions $\frac{\overline{\sigma-\tau}.\,\overline{\alpha-\beta}}{2asr^{x+1}}+\frac{\overline{\tau-\upsilon}.\,\overline{\beta-\gamma}}{2asr^{x+2}}+$ &c. and $\frac{\tau.\,\overline{\alpha-\beta}}{asr^{x+}}+\frac{\upsilon.\,\overline{\beta-\gamma}}{asr^{x+2}}+$ &c. $=\frac{1}{2asr^x}\left(\frac{\alpha\tau}{r}+\frac{\beta\upsilon}{r^2}+\text{\&c.}\right)-\frac{1}{2asr^x}\times\left(\frac{\beta\tau}{r}+\frac{\gamma\upsilon}{r^2}+\text{\&c.}\right)+\frac{1}{2asrx}\times\left(\frac{\alpha\sigma}{r}+\frac{\beta\tau}{r^2}+\frac{\gamma\upsilon}{r^3}+\text{\&c.}\right)-\frac{1}{2asr^x}\times\left(\frac{\beta\sigma}{r}+\frac{\gamma\tau}{r^2}+\frac{\delta\upsilon}{r^3}+\text{\&c.}\right)$ The 2d and third of these series are $=\frac{\alpha\sigma}{2asr^{x+1}}$ The 1st and 4th are nearly of equal value, and therefore may be considered as destroying each other; hence the

whole value of the given sum, supposing $A'$ and $A'T'$ to be the values of the single and joint lives for $x$ years, $A''C''$ the value of the joint lives, AC for $x+1$ years, and $AC^x$ to be the value of two joint lives $x$ years older than AC, will be

$$S \text{ into } \frac{1-\overline{r-1}.\overline{ABC+A'}}{2r} + \frac{A''C''}{2} - \frac{AB}{2r} + \frac{b.\overline{HB-HBC}}{2ar} -$$
$$\frac{t.\overline{A'T'-ABT}}{2sr} + \frac{nt.\overline{1+APT}}{2msr} - \frac{bt.\overline{1+HBT}}{2asr} + \frac{\phi.a.\overline{r-1}.\overline{V-A^x}}{ar^{x+1}}.$$

*If the three lives be of equal age* the value becomes

$$\frac{S.\overline{r-1}}{2r} \times \overline{V-C+CC-CCC}.$$

The solution of this Problem may also be derived from the twenty-eighth and forty-sixth Problems, by "finding "the value on the death of A, should his life be the first "that failed; and also the value should B be the first, C "the second, and A the third, that failed;" but the foregoing rules, in being more simple, are preferable.

In the particular case of the *equality of the three lives*, the value by these Problems will be $\frac{S.\overline{r-1}.\overline{V-CCC}}{3r} +$ $\frac{S.\overline{r-1}.\overline{V-3C+3CC-CCC}}{6r} = \frac{S.\overline{r-1}.\overline{V-C+CC-CCC}}{2r}$, agreeing with the value given above, and therefore proving the truth of the solution.

## NOTE XLIII. (Prob. liii.)

The value of the given sum in the first year will depend on the event of the three lives failing, C having died *after* A, and will be expressed by the fraction $\frac{S.\overline{a-b}.\overline{m-n}.\overline{s-t}}{2amsr}$

In the second and following years it will depend on either of five events:—1st. On the failure of the three lives in the year, C having died after A; 2dly. On the failure of the life of A in the preceding years, and of both the lives of B and C in the year; 3dly. On the failure of the life of C *after* A in the preceding years, and of the life of

B only in the year; 4thly. On the failure of the life of B in the preceding years, and of the life of C *after* A in the year; 5thly. On the failure of the lives of A and B in the preceding years, and of the life of C only in the year. The value of the reversion therefore for the second year will be

$$\frac{S.\overline{b-c}.\overline{n-o}.\overline{t-u}}{2amsr^2}+\frac{S.\overline{a-b}.\overline{n-o}.\overline{t-u}}{amsr^2}+\frac{S.\overline{a-b}.\overline{s-t}.\overline{n-o}}{2amsr^2}+$$

$$\frac{S.\overline{b-c}.\overline{t-u}.\overline{m-n}.}{2amsr^2}+\frac{S.\overline{m-n}.\overline{a-b}.\overline{t-u}}{amsr^2};$$ for the third year

$$\frac{S.\overline{c-d}.\overline{o-p}.\overline{u-w}}{2amsr^3}+\frac{S.\overline{a-c}.\overline{o-p}.\overline{u-w}}{amsr^3}+\frac{S.\overline{a-c}.\overline{s-u}.\overline{o-p}}{2amsr^3}+$$

$$\frac{S.\overline{c-d}.\overline{u-w}.\overline{m-o}}{2amsr^3}+\frac{S.\overline{m-o}.\overline{a-c}.\overline{u-w}}{amsr^3},$$ and so on for the other years. These fractions being expanded will form twenty-six different series, the sum of which, by proceeding as in the foregoing notes, may at last be found $=$ S into

$$\frac{nb.\overline{1+\text{HPC}}.}{2amr}-\frac{nt.\overline{1+\text{APT}}.}{2msr}+\frac{\text{BC}.}{2}-\frac{\text{AB}.}{2r}+\frac{n.\overline{\text{AP}-\text{CP}}.}{2mr}-$$

$$\frac{\overline{r-1}.\overline{\text{ABC}-\text{BC}+\text{B}}.}{2r}+\frac{1-\overline{r-1}.\,2\overline{\text{C}-\text{AC}}.}{2r}+\frac{t.\overline{1+\text{AT}}.}{2sr}+\frac{b.\overline{1+\text{HC}}}{2ar}.$$

The last three fractions express the value of S by Prob. XXVI. on the contingency of C dying *after* A; which being denoted by R, and the first two fractions being denoted by E, the value, when *B is the oldest of the three lives*, will be

$$\text{E}+\text{R}+\frac{\text{BC}.}{2}+\frac{n.\overline{\text{AP}-\text{CP}}.}{2mr}-\frac{\overline{r-1}.\overline{\text{ABC}+\text{B}-\text{BC}}.}{2r}-\frac{\text{AB}}{2r}.$$

*When C is the oldest of the three lives*, the general rule will be $\text{E}+\text{R}+\frac{n.\overline{\text{A}'\text{P}'-\text{CP}}.}{2mr}+\frac{\text{BC}.}{2}-\frac{\text{A}'\text{B}'}{2r}-\frac{\overline{r-1}.\overline{\text{ABC}+\text{B}'-\text{BC}}.}{2r}$ $+\frac{\mu\pi.\overline{r-1}.\overline{\text{V}-\text{B}^x}}{mr^{x+1}}$; $x$ denoting the difference between the age of C and of the oldest person in the table; $\mu$ the number of persons living at the age of B after $x$ years; B′, A′P′, and A′B′ the values of annuities on those single and joint lives for $x$ years; and $\pi$ the probability that C dies *after* A.

*When A is the oldest of the three lives*, let $z$ be the difference between the age of A and of the oldest person in the table, and C′P′ and B′C′ be the values of those joint lives for $z$ years. After the extinction of A's life the given sum may be received on either of three events:—1st. On the death of B in the $\overline{z+1}$, $\overline{z+2}$, $\overline{z+3}$, &c. year, C having died *after* A in the first $z$ years; 2dly. On the death of C in the $\overline{z+1}$, $\overline{z+2}$, &c. year, B having died in the first $z$ years; 3dly. On the extinction of both the lives of B and C after the first $z$ years. Let $\phi$ denote the probability that C dies *after* A in $z$ years, $\mu$ and $\sigma$ respectively the number of persons living opposite the ages of B and C at the end of $z$ years, and the value on the first and second of the above contingencies will be $=$ S. into $\frac{\phi\mu.\overline{r-1}.\overline{V-B^z}}{mr^z+1} + \frac{\overline{m-\mu}.\sigma.\overline{r-1}.\overline{V-C^z}}{msr^z+1}$. Let $\nu$, $\xi$, $o$, &c. be the number of persons living at the age of B, and $\tau$, $\upsilon$, $\chi$, &c. the number of persons living at the age of C at the end of $\overline{z+1}$, $\overline{z+2}$, $\overline{z+3}$, &c. years, and the value on the third contingency will be $\frac{S.\mu\sigma}{msr^2}$ into $\frac{\nu\tau}{\mu\sigma r} + \frac{\upsilon\xi}{\mu\sigma r^2} + \frac{o\chi}{\mu\sigma r^3}$, &c. $+ \frac{\tau}{\sigma r} + \frac{\upsilon}{\sigma r^2} + \frac{\chi}{\sigma r^3}$, &c. $- \frac{\upsilon}{\sigma r} - \frac{\chi}{\sigma r^2} - \frac{\psi}{\sigma r^3} -$, &c. $+ \frac{\nu}{\mu r} + \frac{\xi}{\mu r^2} + \frac{o}{\mu r^3} +$, &c. $- \frac{\xi}{\mu r} - \frac{o}{\mu r^2} -$, &c. $- \frac{\nu\tau}{\mu\sigma r} - \frac{\xi\upsilon}{\mu\sigma r^2} - \frac{o\chi}{\mu\sigma r^3}$, &c. $= \frac{S.\mu\sigma}{msr^2} \times \frac{1-\overline{r-1}.\overline{B^z+C^z-BC^z}}{r}$, and the whole value in this case will be E + R + S into $\frac{n.\overline{AP-C'P'}}{2mr} - \frac{\overline{r-1}.\overline{ABC+B'-B'C'}}{2r} - \frac{AB.}{2r} + \frac{B'C'}{2} + \frac{\phi\mu.\overline{r-1}.\overline{V-B^z}}{mr^z+1} + \frac{\overline{m-\mu}.\sigma.\overline{r-1}.\overline{V-C^z}}{msr^z+1} + \frac{\mu\sigma.\overline{r-1}.\overline{V-L^z}.}{msr^z+1}$; $L^z$ being the value of the longest of two lives $z$ years older than B and C.

*When the three lives are of equal age*, the value of the given sum will be $\frac{S.\overline{r-1}.}{2r} \times \overline{V-3C+3CC-CCC}$, or "half the value of the reversion after the extinction of the three lives," which from self-evident principles is known to be the true value.

The solution of this Problem being also derived from the solutions of the 41st and 26th Problems, the accuracy of the foregoing theorems may be satisfactorily proved in the particular case of the equality of the three lives: thus, the value by Prob. XLI. will be $\frac{S.\overline{r-1}.\overline{C-2CC+CCC}}{2r}$, and the value by Prob. XXVI. will be $\frac{S.\overline{r-1}.\overline{V-2C+CC}}{2r}$. The former of these values being subtracted from the latter will leave $\frac{S.\overline{r-1}}{2r} \times \overline{V-3C+3CC-CCC}$, as given above. The rules derived from the foregoing solution are nearly as simple as those derived from the above-mentioned Problems, though not capable of being so concisely expressed in words, and therefore not so well adapted for general use.

# TABLE I.*

Shewing the Probabilities of the Duration of Human Life at all Ages, formed from the Register of Mortality at *Northampton*, for 46 Years, from 1735 to 1780.

| Age. | Living. | Decr. | Age. | Living. | Decr. | Age. | Living. | Decr. |
|---|---|---|---|---|---|---|---|---|
| 0 | 11650 | 1340 | 31 | 4310 | 75 | 65 | 1632 | 80 |
| 3 months | 10310 | 554 | 32 | 4235 | 75 | 66 | 1552 | 80 |
| 6 months | 9756 | 553 | 33 | 4160 | 75 | 67 | 1472 | 80 |
| 9 months | 9203 | 553 | 34 | 4085 | 75 | 68 | 1392 | 80 |
| 1 year | 8650 | 1367 | 35 | 4010 | 75 | 69 | 1312 | 80 |
| 2 years | 7283 | 502 | 36 | 3935 | 75 | 70 | 1232 | 80 |
| 3 | 6781 | 335 | 37 | 3860 | 75 | 71 | 1152 | 80 |
| 4 | 6446 | 197 | 38 | 3785 | 75 | 72 | 1072 | 80 |
| 5 | 6249 | 184 | 39 | 3710 | 75 | 73 | 992 | 80 |
| 6 | 6065 | 140 | 40 | 3635 | 76 | 74 | 912 | 80 |
| 7 | 5925 | 110 | 41 | 3559 | 77 | 75 | 832 | 80 |
| 8 | 5815 | 80 | 42 | 3482 | 78 | 76 | 752 | 77 |
| 9 | 5735 | 60 | 43 | 3404 | 78 | 77 | 675 | 73 |
| 10 | 5675 | 52 | 44 | 3326 | 78 | 78 | 602 | 68 |
| 11 | 5623 | 50 | 45 | 3248 | 78 | 79 | 534 | 65 |
| 12 | 5573 | 50 | 46 | 3170 | 78 | 80 | 469 | 63 |
| 13 | 5523 | 50 | 47 | 3092 | 78 | 81 | 406 | 60 |
| 14 | 5473 | 50 | 48 | 3014 | 78 | 82 | 346 | 57 |
| 15 | 5423 | 50 | 49 | 2936 | 79 | 83 | 289 | 55 |
| 16 | 5373 | 53 | 50 | 2857 | 81 | 84 | 234 | 48 |
| 17 | 5320 | 58 | 51 | 2776 | 82 | 85 | 186 | 41 |
| 18 | 5262 | 63 | 52 | 2694 | 82 | 86 | 145 | 34 |
| 19 | 5199 | 67 | 53 | 2612 | 82 | 87 | 111 | 28 |
| 20 | 5132 | 72 | 54 | 2530 | 82 | 88 | 83 | 21 |
| 21 | 5060 | 75 | 55 | 2448 | 82 | 89 | 62 | 16 |
| 22 | 4985 | 75 | 56 | 2366 | 82 | 90 | 46 | 12 |
| 23 | 4910 | 75 | 57 | 2284 | 82 | 91 | 34 | 10 |
| 24 | 4835 | 75 | 58 | 2202 | 82 | 92 | 24 | 8 |
| 25 | 4760 | 75 | 59 | 2120 | 82 | 93 | 16 | 7 |
| 26 | 4685 | 75 | 60 | 2038 | 82 | 94 | 9 | 5 |
| 27 | 4610 | 75 | 61 | 1956 | 82 | 95 | 4 | 3 |
| 28 | 4535 | 75 | 62 | 1874 | 81 | 96 | 1 | 1 |
| 29 | 4460 | 75 | 63 | 1793 | 81 | | | |
| 30 | 4385 | 75 | 64 | 1712 | 80 | Total | 299,198 | 11650 |

* This and the five following Tables have been taken from Dr. *Price*'s Treatise on Reversionary Payments, &c.

## TABLE II.

Shewing the EXPECTATIONS of Human Life at every Age, deduced from the *Northampton* Table of Observations.

| Ages. | Expectat. | Ages. | Expectat. | Ages. | Expectat. | Ages. | Expectat. |
|---|---|---|---|---|---|---|---|
| 0 | 25·18 | 25 | 30.85 | 50 | 17.99 | 75 | 6.54 |
| 1 | 32.74 | 26 | 30.33 | 51 | 17.50 | 76 | 6.18 |
| 2 | 37.79 | 27 | 29.82 | 52 | 17.02 | 77 | 5.83 |
| 3 | 39.55 | 28 | 29.30 | 53 | 16.54 | 78 | 5.48 |
| 4 | 40.58 | 29 | 28.79 | 54 | 16.06 | 79 | 5.11 |
| 5 | 40.84 | 30 | 28.27 | 55 | 15.58 | 80 | 4.75 |
| 6 | 41.07 | 31 | 27.76 | 56 | 15.10 | 81 | 4.41 |
| 7 | 41.03 | 32 | 27.24 | 57 | 14.63 | 82 | 4.09 |
| 8 | 40.79 | 33 | 26.72 | 58 | 14.15 | 83 | 3.80 |
| 9 | 40.36 | 34 | 26.20 | 59 | 13.68 | 84 | 3.58 |
| 10 | 39.78 | 35 | 25.68 | 60 | 13.21 | 85 | 3.37 |
| 11 | 39.14 | 36 | 25.16 | 61 | 12.75 | 86 | 3.19 |
| 12 | 38.49 | 37 | 24.64 | 62 | 12.28 | 87 | 3.01 |
| 13 | 37.83 | 38 | 24.12 | 63 | 11.81 | 88 | 2.86 |
| 14 | 37.17 | 39 | 23.60 | 64 | 11.35 | 89 | 2.66 |
| 15 | 36.51 | 40 | 23.08 | 65 | 10.88 | 90 | 2.41 |
| 16 | 35.85 | 41 | 22.56 | 66 | 10.42 | 91 | 2.09 |
| 17 | 35.20 | 42 | 22.04 | 67 | 9.96 | 92 | 1.75 |
| 18 | 34.58 | 43 | 21.54 | 68 | 9.50 | 93 | 1.37 |
| 19 | 33.99 | 44 | 21.03 | 69 | 9.05 | 94 | 1.05 |
| 20 | 33.43 | 45 | 20.52 | 70 | 8.60 | 95 | 0.75 |
| 21 | 32.90 | 46 | 20.02 | 71 | 8.17 | 96 | 0.50 |
| 22 | 32.39 | 47 | 19.51 | 72 | 7.74 | | |
| 23 | 31.88 | 48 | 19.00 | 73 | 7.33 | | |
| 24 | 31.36 | 49 | 18.49 | 74 | 6.92 | | |

# TABLE III.

ving the Value of an Annuity on a Single Life at ery Age according to the Probabilities of the Duration Human Life at *Northampton*.

| s. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. | Value at 7 per Cent. | Value at 8 per Cent. |
|---|---|---|---|---|---|---|
| :h | | 10.327 | 8.863 | | | |
| ar | | 13.008 | 11.274 | | | |
| | 16.021 | 13.465 | 11.563 | 10.107 | 8.963 | 8.046 |
| | 18.599 | 15.633 | 13.420 | 11.724 | 10.391 | 9.321 |
| | 19.575 | 16.462 | 14.135 | 12.348 | 10.941 | 9.812 |
| | 20.210 | 17.010 | 14.613 | 12.769 | 11.315 | 10.147 |
| | 20.473 | 17.248 | 14.827 | 12.962 | 11.489 | 10.304 |
| | 20.727 | 17.482 | 15.041 | 13.150 | 11.666 | 10.466 |
| | 20.853 | 17.611 | 15.166 | 13.275 | 11.777 | 10.570 |
| | 20.885 | 17.662 | 15.226 | 13.337 | 11.840 | 10.631 |
| | 20.812 | 17.625 | 15.210 | 13.335 | 11.846 | 10.641 |
| | 20.663 | 17.523 | 15.139 | 13.285 | 11.809 | 10.614 |
| | 20.480 | 17.393 | 15.043 | 13.212 | 11.752 | 10.569 |
| | 20.283 | 17.251 | 14.937 | 13.130 | 11.687 | 10.517 |
| | 20.081 | 17.103 | 14.826 | 13.044 | 11.618 | 10.461 |
| | 19.872 | 16.950 | 14.710 | 12.953 | 11.545 | 10.401 |
| | 19.657 | 16.791 | 14.588 | 12.857 | 11.467 | 10.337 |
| | 19.435 | 16.625 | 14.460 | 12.755 | 11.384 | 10.268 |
| | 19.218 | 16.462 | 14.334 | 12.655 | 11.302 | 10.200 |
| | 19.013 | 16.309 | 14.217 | 12.562 | 11.226 | 10.137 |
| | 18.820 | 16.167 | 14.108 | 12.477 | 11.157 | 10.081 |
| | 18.638 | 16.033 | 14.007 | 12.398 | 11.094 | 10.030 |
| | 18.470 | 15.912 | 13.917 | 12.329 | 11.042 | 9.986 |
| | 18.311 | 15.797 | 13.833 | 12.265 | 10.993 | 9.947 |
| | 18.148 | 15.680 | 13.746 | 12.200 | 10.942 | 9.907 |
| | 17.983 | 15.560 | 13.658 | 12.132 | 10.890 | 9.865 |
| | 17.814 | 15.438 | 13.567 | 12.063 | 10.836 | 9.823 |
| | 17.642 | 15.312 | 13.473 | 11.992 | 10.780 | 9.778 |
| | 17.467 | 15.184 | 13.377 | 11.917 | 10.723 | 9.732 |
| | 17.289 | 15.053 | 13.278 | 11.841 | 10.663 | 9.685 |
| | 17.107 | 14.918 | 13.177 | 11.763 | 10.602 | 9.635 |
| | 16.922 | 14.781 | 13.072 | 11.682 | 10.539 | 9.584 |
| | 16.732 | 14.639 | 12.965 | 11.598 | 10.473 | 9.531 |
| | 16.540 | 14.495 | 12.854 | 11.512 | 10.404 | 9.476 |

## Table III.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. | Value at 7 per Cent. | Value at 8 per Cent. |
|---|---|---|---|---|---|---|
| 33 | 16.343 | 14.347 | 12.740 | 11.423 | 10.333 | 9.418 |
| 34 | 16.142 | 14.195 | 12.623 | 11.331 | 10.260 | 9.359 |
| 35 | 15.938 | 14.039 | 12.502 | 11.236 | 10.183 | 9.296 |
| 36 | 15.729 | 13.880 | 12.377 | 11.137 | 10.104 | 9.231 |
| 37 | 15.515 | 13.716 | 12.249 | 11.035 | 10.021 | 9.164 |
| 38 | 15.298 | 13.548 | 12.116 | 10.929 | 9.935 | 9.093 |
| 39 | 15.075 | 13.375 | 11.979 | 10.819 | 9.845 | 9.019 |
| 40 | 14.848 | 13.197 | 11.837 | 10.705 | 9.752 | 8.941 |
| 41 | 14.620 | 13.018 | 11.695 | 10.589 | 9.657 | 8.863 |
| 42 | 14.391 | 12.838 | 11.551 | 10.473 | 9.562 | 8.783 |
| 43 | 14.162 | 12.657 | 11.407 | 10.356 | 9.466 | 8.703 |
| 44 | 13.929 | 12.472 | 11.258 | 10.235 | 9.366 | 8.620 |
| 45 | 13.692 | 12.283 | 11.105 | 10.110 | 9.262 | 8.533 |
| 46 | 13.450 | 12.089 | 10.947 | 9.980 | 9.154 | 8.443 |
| 47 | 13.203 | 11.890 | 10.784 | 9.846 | 9.042 | 8.348 |
| 48 | 12.951 | 11.685 | 10.616 | 9.707 | 8.925 | 8.249 |
| 49 | 12.693 | 11.475 | 10.443 | 9.563 | 8.804 | 8.146 |
| 50 | 12.436 | 11.264 | 10.269 | 9.417 | 8.681 | 8.041 |
| 51 | 12.183 | 11.057 | 10.097 | 9.273 | 8.559 | 7.937 |
| 52 | 11.930 | 10.849 | 9.925 | 9.129 | 8.437 | 7.833 |
| 53 | 11.674 | 10.637 | 9.748 | 8.980 | 8.311 | 7.725 |
| 54 | 11.414 | 10.421 | 9.567 | 8.827 | 8.181 | 7.614 |
| 55 | 11.150 | 10.201 | 9.382 | 8.670 | 8.047 | 7.499 |
| 56 | 10.882 | 9.977 | 9.193 | 8.509 | 7.909 | 7.379 |
| 57 | 10.611 | 9.749 | 8.999 | 8.343 | 7.766 | 7.256 |
| 58 | 10.337 | 9.516 | 8.801 | 8.173 | 7.619 | 7.128 |
| 59 | 10.058 | 9.280 | 8.599 | 7.999 | 7.468 | 6.996 |
| 60 | 9.777 | 9.039 | 8.392 | 7.820 | 7.312 | 6.860 |
| 61 | 9.493 | 8.795 | 8.181 | 7.637 | 7.152 | 6.719 |
| 62 | 9.205 | 8.547 | 7.966 | 7.449 | 6.988 | 6.574 |
| 63 | 8.910 | 8.291 | 7.742 | 7.253 | 6.815 | 6.421 |
| 64 | 8.611 | 8.030 | 7.514 | 7.052 | 6.637 | 6.262 |
| 65 | 8.304 | 7.761 | 7.276 | 6.841 | 6.449 | 6.095 |
| 66 | 7.994 | 7.488 | 7.034 | 6.625 | 6.256 | 5.922 |
| 67 | 7.682 | 7.211 | 6.787 | 6.405 | 6.058 | 5.743 |
| 68 | 7.367 | 6.930 | 6.536 | 6.179 | 5.855 | 5.559 |
| 69 | 7.051 | 6.647 | 6.281 | 5.949 | 5.646 | 5.370 |
| 70 | 6.734 | 6.361 | 6.023 | 5.716 | 5.434 | 5.176 |
| 71 | 6.418 | 6.075 | 5.764 | 5.479 | 5.218 | 4.978 |
| 72 | 6.103 | 5.790 | 5.504 | 5.241 | 5.000 | 4.778 |

## Table III.—*continued.*

| s. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. | Value at 7 per Cent. | Value at 8 per Cent. |
|---|---|---|---|---|---|---|
| | 5.794 | 5.507 | 5.245 | 5.004 | 4.781 | 4.576 |
| | 5.491 | 5.230 | 4.990 | 4.769 | 4.565 | 4.375 |
| | 5.199 | 4.962 | 4.744 | 4.542 | 4.354 | 4.180 |
| | 4.925 | 4.710 | 4.511 | 4.326 | 4.154 | 3.994 |
| | 4.652 | 4.457 | 4.277 | 4.109 | 3.952 | 3.806 |
| | 4.372 | 4.197 | 4.035 | 3.884 | 3.742 | 3.609 |
| | 4.077 | 3.921 | 3.776 | 3.641 | 3.514 | 3.394 |
| | 3.781 | 3.643 | 3.515 | 3.394 | 3.281 | 3.174 |
| | 3.499 | 3.377 | 3.263 | 3.156 | 3.055 | 2.960 |
| | 3.229 | 3.122 | 3.020 | 2.926 | 2.836 | 2.751 |
| | 2.982 | 2.887 | 2.797 | 2.713 | 2.632 | 2.557 |
| | 2.793 | 2.708 | 2.627 | 2.551 | 2.479 | 2.410 |
| | 2.620 | 2.543 | 2.471 | 2.402 | 2.337 | 2.275 |
| | 2.462 | 2.393 | 2.328 | 2.266 | 2.207 | 2.151 |
| | 2.312 | 2.251 | 2.193 | 2.138 | 2.085 | 2.035 |
| | 2.185 | 2.131 | 2.080 | 2.031 | 1.984 | 1.939 |
| | 2.013 | 1.967 | 1.924 | 1.882 | 1.842 | 1.803 |
| | 1.794 | 1.758 | 1.723 | 1.689 | 1.656 | 1.625 |
| 1 | 1.501 | 1.474 | 1.447 | 1.422 | 1.398 | 1.374 |
| 2 | 1.190 | 1.171 | 1.153 | 1.136 | 1.118 | 1.102 |
| 3 | 0.839 | 0.827 | 0.816 | 0.806 | 0.795 | 0.785 |
| 4 | 0.536 | 0.530 | 0.524 | 0.518 | 0.512 | 0.507 |
| 5 | 0.242 | 0.240 | 0.238 | 0.236 | 0.234 | 0.232 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

# TABLE IV.

Shewing the Value of an Annuity on the *joint Continuance* of Two Lives, having the same common Age, according to the *Northampton* Table of Observations.

Difference of Age 0.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1- 1 | 9.491 | 8.252 | 7.287 | 6.515 |
| 2- 2 | 12.789 | 11.107 | 9.793 | 8.741 |
| 3- 3 | 14.196 | 12.325 | 10.862 | 9.689 |
| 4- 4 | 15.181 | 13.185 | 11.621 | 10.365 |
| 5- 5 | 15.638 | 13.591 | 11.984 | 10.691 |
| 6- 6 | 16.099 | 14.005 | 12.358 | 11.031 |
| 7- 7 | 16.375 | 14.224 | 12.596 | 11.251 |
| 8- 8 | 16.510 | 14.399 | 12.731 | 11.382 |
| 9- 9 | 16.483 | 14.396 | 12.744 | 11.404 |
| 10-10 | 16.339 | 14.277 | 12.665 | 11.345 |
| 11-11 | 16.142 | 14.133 | 12.546 | 11.249 |
| 12-12 | 15.926 | 13.966 | 12.411 | 11.139 |
| 13-13 | 15.702 | 13.789 | 12.268 | 11.023 |
| 14-14 | 15.470 | 13.604 | 12.118 | 10.899 |
| 15-15 | 15.229 | 13.411 | 11.960 | 10.767 |
| 16-16 | 14.979 | 13.212 | 11.793 | 10.626 |
| 17-17 | 14.737 | 13.019 | 11.630 | 10.489 |
| 18-18 | 14.516 | 12.841 | 11.483 | 10.365 |
| 19-19 | 14.316 | 12.679 | 11.351 | 10.255 |
| 20-20 | 14.133 | 12.535 | 11.232 | 10.156 |
| 21-21 | 13.974 | 12.409 | 11.131 | 10.074 |
| 22-22 | 13.830 | 12.293 | 11.042 | 10.002 |
| 23-23 | 13.688 | 12.179 | 10.951 | 9.928 |
| 24-24 | 13.534 | 12.062 | 10.858 | 9.853 |
| 25-25 | 13.383 | 11.944 | 10.764 | 9.776 |
| 26-26 | 13.230 | 11.822 | 10.667 | 9.697 |
| 27-27 | 13.074 | 11.699 | 10.567 | 9.616 |
| 28-28 | 12.915 | 11.573 | 10.466 | 9.533 |
| 29-29 | 12.754 | 11.445 | 10.362 | 9.448 |
| 30-30 | 12.589 | 11.313 | 10.255 | 9.360 |
| 31-31 | 12.422 | 11.179 | 10.146 | 9.270 |
| 32-32 | 12.252 | 11.042 | 10.034 | 9.178 |
| 33-33 | 12.079 | 10.902 | 9.919 | 9.082 |

## TABLE IV.—*continued.*

| s. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 34 | 11.902 | 10.759 | 9.801 | 8.984 |
| 35 | 11.722 | 10.612 | 9.680 | 8.883 |
| 36 | 11.539 | 10.462 | 9.555 | 8.778 |
| 37 | 11.351 | 10.307 | 9.427 | 8.670 |
| 38 | 11.160 | 10.149 | 9.294 | 8.558 |
| 39 | 10.964 | 9.986 | 9.158 | 8.442 |
| 40 | 10.764 | 9.820 | 9.016 | 8.322 |
| 41 | 10.565 | 9.654 | 8.876 | 8.202 |
| 42 | 10.369 | 9.491 | 8.737 | 8.083 |
| 43 | 10.175 | 9.326 | 8.599 | 7.965 |
| 44 | 9.978 | 9.160 | 8.457 | 7.843 |
| 45 | 9.776 | 8.990 | 8.312 | 7.718 |
| 46 | 9.571 | 8.815 | 8.162 | 7.589 |
| 47 | 9.362 | 8.637 | 8.008 | 7.455 |
| 48 | 9.149 | 8.453 | 7.849 | 7.316 |
| 49 | 8.931 | 8.266 | 7.686 | 7.173 |
| 50 | 8.714 | 8.081 | 7.522 | 7.030 |
| 51 | 8.507 | 7.900 | 7.366 | 6.893 |
| 52 | 8.304 | 7.723 | 7.213 | 6.758 |
| 53 | 8.099 | 7.544 | 7.056 | 6.620 |
| 54 | 7.891 | 7.362 | 6.897 | 6.480 |
| 55 | 7.681 | 7.179 | 6.735 | 6.336 |
| 56 | 7.470 | 6.993 | 6.571 | 6.190 |
| 57 | 7.256 | 6.805 | 6.404 | 6.041 |
| 58 | 7.041 | 6.614 | 6.234 | 5.890 |
| 59 | 6.824 | 6.421 | 6.062 | 5.735 |
| 60 | 6.606 | 6.226 | 5.888 | 5.579 |
| 61 | 6.387 | 6.030 | 5.712 | 5.420 |
| 62 | 6.166 | 5.831 | 5.533 | 5.259 |
| 63 | 5.938 | 5.626 | 5.347 | 5.089 |
| 64 | 5.709 | 5.417 | 5.158 | 4.917 |
| 65 | 5.471 | 5.201 | 4.960 | 4.736 |
| 66 | 5.231 | 4.982 | 4.759 | 4.551 |
| 67 | 4.990 | 4.760 | 4.565 | 4.363 |
| 68 | 4.747 | 4.537 | 4.348 | 4.171 |
| 69 | 4.504 | 4.312 | 4.140 | 3.977 |
| 70 | 4.261 | 4.087 | 3.930 | 3.781 |
| 71 | 4.020 | 3.862 | 3.719 | 3.584 |
| 72 | 3.781 | 3.639 | 3.510 | 3.387 |
| 73 | 3.548 | 3.421 | 3.304 | 3.193 |
| 74 | 3.324 | 3.211 | 3.105 | 3.005 |

## TABLE IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 75-75 | 3.114 | 3.015 | 2.917 | 2.827 |
| 76-76 | 2.920 | 2.833 | 2.750 | 2.668 |
| 77-77 | 2.741 | 2.656 | 2.583 | 2.511 |
| 78-78 | 2.550 | 2.470 | 2.410 | 2.346 |
| 79-79 | 2.338 | 2.271 | 2.217 | 2.161 |
| 80-80 | 2.122 | 2.068 | 2.018 | 1.969 |
| 81-81 | 1.917 | 1.869 | 1.827 | 1.786 |
| 82-82 | 1.719 | 1.681 | 1.642 | 1.606 |
| 83-83 | 1.538 | 1.510 | 1.472 | 1.441 |
| 84-84 | 1.416 | 1.387 | 1.357 | 1.330 |
| 85-85 | 1.309 | 1.339 | 1.256 | 1.232 |
| 86-86 | 1.218 | 1.195 | 1.171 | 1.149 |
| 87-87 | 1.141 | 1.124 | 1.098 | 1.078 |
| 88-88 | 1.103 | 1.030 | 1.063 | 1.044 |
| 89-89 | 1.036 | 1.015 | 1.001 | 0.984 |
| 90-90 | 0.938 | 0.922 | 0.909 | 0.895 |
| 91-91 | 0.769 | 0.756 | 0.748 | 0.737 |
| 92-92 | 0.591 | 0.583 | 0.576 | 0.569 |
| 93-93 | 0.369 | 0.365 | 0.361 | 0.357 |
| 94-94 | 0.203 | 0.201 | 0.199 | 0.197 |
| 95-95 | 0.060 | 0.060 | 0.059 | 0.058 |
| 96-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *five* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1- 6 | 12.347 | 10.741 | 9.479 | 8.467 |
| 2- 7 | 14.461 | 12.581 | 11.100 | 9.911 |
| 3- 8 | 15.300 | 13.319 | 11.755 | 10.498 |
| 4- 9 | 15.809 | 13.775 | 12.165 | 10.869 |
| 5-10 | 15.974 | 13.983 | 12.315 | 11.010 |
| 6-11 | 16.110 | 14.068 | 12.447 | 11.136 |
| 7-12 | 16.137 | 14.111 | 12.498 | 11.192 |
| 8-13 | 16.089 | 14.089 | 12.492 | 11.197 |
| 9-14 | 15.957 | 13.992 | 12.421 | 11.144 |
| 10-15 | 15.762 | 13.841 | 12.302 | 11.048 |

## TABLE IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 11-16 | 15.538 | 13.664 | 12.158 | 10.929 |
| 12-17 | 15.308 | 13.480 | 12.009 | 10.805 |
| 13-18 | 15.086 | 13.303 | 11.864 | 10.685 |
| 14-19 | 14.870 | 13.130 | 11.723 | 10.568 |
| 15-20 | 14.660 | 12.961 | 11.585 | 10.453 |
| 16-21 | 14.457 | 12.799 | 11.452 | 10.342 |
| 17-22 | 14.265 | 12.646 | 11.327 | 10.239 |
| 18-23 | 14.082 | 12.500 | 11.209 | 10.140 |
| 19-24 | 13.908 | 12.361 | 11.096 | 10.048 |
| 20-25 | 13.741 | 12.229 | 10.989 | 9.960 |
| 21-26 | 13.584 | 12.105 | 10.890 | 9.879 |
| 22-27 | 13.433 | 11.987 | 10.796 | 9.803 |
| 23-28 | 13.280 | 11.866 | 10.699 | 9.724 |
| 24-29 | 13.124 | 11.743 | 10.600 | 9.643 |
| 25-30 | 12.966 | 11.618 | 10.499 | 9.561 |
| 26-31 | 12.805 | 11.489 | 10.396 | 9.476 |
| 27-32 | 12.641 | 11.359 | 10.289 | 9.389 |
| 28-33 | 12.474 | 11.225 | 10.181 | 9.299 |
| 29-34 | 12.304 | 11.088 | 10.069 | 9.207 |
| 30-35 | 12.131 | 10.948 | 9.954 | 9.112 |
| 31-36 | 11.955 | 10.805 | 9.837 | 9.014 |
| 32-37 | 11.775 | 10.659 | 9.716 | 8.913 |
| 33-38 | 11.592 | 10.508 | 9.591 | 8.808 |
| 34-39 | 11.404 | 10.354 | 9.463 | 8.701 |
| 35-40 | 11.213 | 10.196 | 9.331 | 8.589 |
| 36-41 | 11.021 | 10.037 | 9.198 | 8.476 |
| 37-42 | 10.828 | 9.877 | 9.062 | 8.362 |
| 38-43 | 10.635 | 9.716 | 8.927 | 8.246 |
| 39-44 | 10.437 | 9.550 | 8.787 | 8.127 |
| 40-45 | 10.236 | 9.381 | 8.643 | 8.003 |
| 41-46 | 10.033 | 9.210 | 8.497 | 7.878 |
| 42-47 | 9.829 | 9.037 | 8.350 | 7.751 |
| 43-48 | 9.624 | 8.862 | 8.200 | 7.621 |
| 44-49 | 9.414 | 8.683 | 8.046 | 7.488 |
| 45-50 | 9.204 | 8.503 | 7.891 | 7.353 |
| 46-51 | 8.997 | 8.326 | 7.737 | 7.219 |
| 47-52 | 8.790 | 8.147 | 7.582 | 7.084 |
| 48-53 | 8.579 | 7.965 | 7.424 | 6.945 |
| 49-54 | 8.366 | 7.780 | 7.262 | 6.802 |
| 50-55 | 8.152 | 7.593 | 7.098 | 6.658 |
| 51-56 | 7.941 | 7.409 | 6.936 | 6.515 |

Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 52-57 | 7.730 | 7.225 | 6.774 | 6.371 |
| 53-58 | 7.518 | 7.039 | 6.609 | 6.225 |
| 54-59 | 7.304 | 6.850 | 6.442 | 6.076 |
| 55-60 | 7.088 | 6.659 | 6.272 | 5.924 |
| 56-61 | 6.870 | 6.465 | 6.100 | 5.770 |
| 57-62 | 6.651 | 6.270 | 5.925 | 5.613 |
| 58-63 | 6.427 | 6.070 | 5.744 | 5.450 |
| 59-64 | 6.201 | 5.867 | 5.561 | 5.284 |
| 60-65 | 5.970 | 5.658 | 5.372 | 5.112 |
| 61-66 | 5.737 | 5.447 | 5.180 | 4.938 |
| 62-67 | 5.503 | 5.285 | 4.986 | 4.760 |
| 63-68 | 5.265 | 5.017 | 4.786 | 4.576 |
| 64-69 | 5.025 | 4.798 | 4.585 | 4.390 |
| 65-70 | 4.783 | 4.573 | 4.378 | 4.199 |
| 66-71 | 4.540 | 4.349 | 4.169 | 4.005 |
| 67-72 | 4.298 | 4.124 | 3.960 | 3.811 |
| 68-73 | 4.059 | 3.901 | 3.752 | 3.616 |
| 69-74 | 3.825 | 3.683 | 3.547 | 3.423 |
| 70-75 | 3.599 | 3.471 | 3.347 | 3.236 |
| 71-76 | 3.386 | 3.270 | 3.159 | 3.059 |
| 72-77 | 3.176 | 3.070 | 2.971 | 2.882 |
| 73-78 | 2.963 | 2.869 | 2.780 | 2.701 |
| 74-79 | 2.743 | 2.659 | 2.580 | 2.511 |
| 75-80 | 2.526 | 2.448 | 2.381 | 2.323 |
| 76-81 | 2.325 | 2.258 | 2.195 | 2.147 |
| 77-82 | 2.131 | 2.077 | 2.013 | 1.975 |
| 78-83 | 1.947 | 1.899 | 1.838 | 1.810 |
| 79-84 | 1.793 | 1.751 | 1.750 | 1.672 |
| 80-85 | 1.645 | 1.608 | 1.573 | 1.539 |
| 81-86 | 1.511 | 1.478 | 1.447 | 1.417 |
| 82-87 | 1.385 | 1.356 | 1.329 | 1.303 |
| 83-88 | 1.284 | 1.259 | 1.235 | 1.212 |
| 84-89 | 1.188 | 1.164 | 1.145 | 1.124 |
| 85-90 | 1.074 | 1.054 | 1.038 | 1.021 |
| 86-91 | 0.921 | 0.902 | 0.892 | 0.879 |
| 87-92 | 0.756 | 0.738 | 0.734 | 0.725 |
| 88-93 | 0.562 | 0.554 | 0.547 | 0.541 |
| 89-94 | 0.377 | 0.373 | 0.369 | 0.365 |
| 90-95 | 0.179 | 0.177 | 0.175 | 0.174 |
| 91-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Table IV.—*continued.*

### Difference of Age *ten* Years.

| es. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 11 | 12.346 | 10.782 | 9.544 | 8.547 |
| 12 | 14.239 | 12.438 | 11.010 | 9.857 |
| 13 | 14.895 | 13.019 | 11.528 | 10.324 |
| 14 | 15.287 | 13.374 | 11.850 | 10.617 |
| 15 | 15.391 | 13.479 | 11.954 | 10.716 |
| 16 | 15.486 | 13.578 | 12.052 | 10.812 |
| 17 | 15.490 | 13.599 | 12.083 | 10.849 |
| 18 | 15.436 | 13.569 | 12.070 | 10.847 |
| 19 | 15.316 | 13.482 | 12.006 | 10.799 |
| 20 | 15.151 | 13.355 | 11.906 | 10.719 |
| 21 | 14.974 | 13.217 | 11.797 | 10.631 |
| 22 | 14.795 | 13.078 | 11.686 | 10.541 |
| 23 | 14.612 | 12.934 | 11.570 | 10.446 |
| 24 | 14.424 | 12.784 | 11.450 | 10.348 |
| 25 | 14.230 | 12.630 | 11.324 | 10.244 |
| 26 | 14.030 | 12.470 | 11.193 | 10.135 |
| 27 | 13.832 | 12.311 | 11.063 | 10.027 |
| 28 | 13.642 | 12.158 | 10.939 | 9.924 |
| 29 | 13.461 | 12.013 | 10.820 | 9.826 |
| 30 | 13.286 | 11.873 | 10.707 | 9.732 |
| 31 | 13.121 | 11.742 | 10.600 | 9.644 |
| 32 | 12.961 | 11.615 | 10.498 | 9.561 |
| 33 | 12.798 | 11.485 | 10.393 | 9.474 |
| 34 | 12.632 | 11.352 | 10.285 | 9.386 |
| 35 | 12.463 | 11.217 | 10.175 | 9.295 |
| 36 | 12.291 | 11.078 | 10.062 | 9.201 |
| 37 | 12.116 | 10.936 | 9.946 | 9.105 |
| 38 | 11.937 | 10.791 | 9.826 | 9.005 |
| 39 | 11.755 | 10.642 | 9.703 | 8.902 |
| 40 | 11.568 | 10.490 | 9.576 | 8.795 |
| 41 | 11.382 | 10.336 | 9.448 | 8.688 |
| 42 | 11.195 | 10.182 | 9.320 | 8.560 |
| 43 | 11.007 | 10.027 | 9.190 | 8.471 |
| 44 | 10.817 | 9.869 | 9.058 | 8.358 |
| 45 | 10.622 | 9.706 | 8.921 | 8.242 |
| 46 | 10.424 | 9.540 | 8.781 | 8.122 |
| 47 | 10.221 | 9.370 | 8.636 | 7.998 |
| 48 | 10.014 | 9.195 | 8.487 | 7.870 |
| 49 | 9.803 | 9.015 | 8.333 | 7.737 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 40-50 | 9.590 | 8.834 | 8.177 | 7.602 |
| 41-51 | 9.383 | 8.658 | 8.025 | 7.470 |
| 42-52 | 9.179 | 8.483 | 7.875 | 7.340 |
| 43-53 | 8.975 | 8.308 | 7.724 | 7.208 |
| 44-54 | 8.767 | 8.130 | 7.569 | 7.073 |
| 45-55 | 8.557 | 7.948 | 7.411 | 6.935 |
| 46-56 | 8.344 | 7.763 | 7.249 | 6.793 |
| 47-57 | 8.127 | 7.574 | 7.084 | 6.648 |
| 48-58 | 7.907 | 7.382 | 6.915 | 6.498 |
| 49-59 | 7.684 | 7.186 | 6.742 | 6.344 |
| 50-60 | 7.461 | 6.989 | 6.568 | 6.189 |
| 51-61 | 7.240 | 6.795 | 6.395 | 6.035 |
| 52-62 | 7.021 | 6.600 | 6.222 | 5.880 |
| 53-63 | 6.795 | 6.399 | 6.042 | 5.719 |
| 54-64 | 6.568 | 6.196 | 5.860 | 5.555 |
| 55-65 | 6.334 | 5.986 | 5.671 | 5.384 |
| 56-66 | 6.098 | 5.774 | 5.479 | 5.209 |
| 57-67 | 5.860 | 5.559 | 5.283 | 5.031 |
| 58-68 | 5.621 | 5.341 | 5.084 | 4.849 |
| 59-69 | 5.380 | 5.121 | 4.883 | 4.665 |
| 60-70 | 5.139 | 4.900 | 4.680 | 4.478 |
| 61-71 | 4.898 | 4.679 | 4.476 | 4.289 |
| 62-72 | 4.659 | 4.458 | 4.272 | 4.099 |
| 63-73 | 4.420 | 4.236 | 4.066 | 3.908 |
| 64-74 | 4.186 | 4.019 | 3.864 | 3.719 |
| 65-75 | 3.958 | 3.806 | 3.665 | 3.533 |
| 66-76 | 3.743 | 3.606 | 3.477 | 3.357 |
| 67-77 | 3.529 | 3.405 | 3.289 | 3.180 |
| 68-78 | 3.310 | 3.199 | 3.095 | 2.996 |
| 69-79 | 3.077 | 2.979 | 2.887 | 2.799 |
| 70-80 | 2.843 | 2.757 | 2.675 | 2.598 |
| 71-81 | 2.618 | 2.542 | 2.470 | 2.402 |
| 72-82 | 2.401 | 2.334 | 2.271 | 2.211 |
| 73-83 | 2.199 | 2.141 | 2.085 | 2.032 |
| 74-84 | 2.043 | 1.991 | 1.941 | 1.894 |
| 75-85 | 1.903 | 1.856 | 1.811 | 1.769 |
| 76-86 | 1.781 | 1.739 | 1.699 | 1.661 |
| 77-87 | 1.670 | 1.633 | 1.597 | 1.562 |
| 78-88 | 1.580 | 1.546 | 1.514 | 1.483 |
| 79-89 | 1.456 | 1.427 | 1.400 | 1.373 |
| 80-90 | 1.302 | 1.278 | 1.255 | 1.234 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 81-91 | 1.096 | 1.078 | 1.061 | 1.044 |
| 82-92 | 0.877 | 0.864 | 0.852 | 0.840 |
| 83-93 | 0.622 | 0.614 | 0.606 | 0.599 |
| 84-94 | 0.408 | 0.403 | 0.398 | 0.394 |
| 85-95 | 0.189 | 0.187 | 0.185 | 0.183 |
| 86-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *fifteen* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-16 | 11.864 | 10.406 | 9.243 | 8.301 |
| 2-17 | 13.659 | 11.981 | 10.642 | 9.555 |
| 3-18 | 14.277 | 12.531 | 11.134 | 9.998 |
| 4-19 | 14.657 | 12.876 | 11.447 | 10.284 |
| 5-20 | 14.776 | 12.993 | 11.561 | 10.391 |
| 6-21 | 14.904 | 13.121 | 11.685 | 10.510 |
| 7-22 | 14.950 | 13.178 | 11.748 | 10.576 |
| 8-23 | 14.929 | 13.178 | 11.761 | 10.597 |
| 9-24 | 14.834 | 13.112 | 11.715 | 10.566 |
| 10-25 | 14.683 | 12.998 | 11.627 | 10.497 |
| 11-26 | 14.508 | 12.861 | 11.519 | 10.410 |
| 12-27 | 14.323 | 12.715 | 11.402 | 10.314 |
| 13-28 | 14.132 | 12.564 | 11.280 | 10.215 |
| 14-29 | 13.936 | 12.408 | 11.153 | 10.110 |
| 15-30 | 13.734 | 12.246 | 11.021 | 10.001 |
| 16-31 | 13.527 | 12.078 | 10.883 | 9.886 |
| 17-32 | 13.320 | 11.911 | 10.746 | 9.771 |
| 18-33 | 13.121 | 11.750 | 10.613 | 9.660 |
| 19-34 | 12.930 | 11.595 | 10.486 | 9.554 |
| 20-35 | 12.744 | 11.445 | 10.363 | 9.451 |
| 21-36 | 12.567 | 11.302 | 10.246 | 9.354 |
| 22-37 | 12.394 | 11.163 | 10.132 | 9.260 |
| 23-38 | 12.218 | 11.020 | 10.015 | 9.163 |
| 24-39 | 12.038 | 10.874 | 9.895 | 9.063 |
| 25-40 | 11.854 | 10.725 | 9.771 | 8.960 |
| 26-41 | 11.670 | 10.574 | 9.647 | 8.855 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 27-42 | 11.486 | 10.423 | 9.522 | 8.751 |
| 28-43 | 11.302 | 10.272 | 9.396 | 8.645 |
| 29-44 | 11.114 | 10.117 | 9.267 | 8.536 |
| 30-45 | 10.923 | 9.959 | 9.135 | 8.424 |
| 31-46 | 10.728 | 9.797 | 8.998 | 8.309 |
| 32-47 | 10.530 | 9.631 | 8.858 | 8.189 |
| 33-48 | 10.327 | 9.461 | 8.714 | 8.066 |
| 34-49 | 10.120 | 9.286 | 8.565 | 7.938 |
| 35-50 | 9.912 | 9.110 | 8.415 | 7.809 |
| 36-51 | 9.707 | 8.937 | 8.267 | 7.681 |
| 37-52 | 9.503 | 8.763 | 8.119 | 7.553 |
| 38-53 | 9.296 | 8.586 | 7.966 | 7.421 |
| 39-54 | 9.085 | 8.406 | 7.810 | 7.286 |
| 40-55 | 8.870 | 8.221 | 7.651 | 7.146 |
| 41-56 | 8.655 | 8.035 | 7.489 | 7.005 |
| 42-57 | 8.439 | 7.848 | 7.326 | 6.862 |
| 43-58 | 8.222 | 7.660 | 7.162 | 6.718 |
| 44-59 | 8.003 | 7.469 | 6.994 | 6.570 |
| 45-60 | 7.781 | 7.274 | 6.822 | 6.418 |
| 46-61 | 7.556 | 7.076 | 6.648 | 6.263 |
| 47-62 | 7.328 | 6.875 | 6.469 | 6.104 |
| 48-63 | 7.093 | 6.667 | 6.283 | 5.937 |
| 49-64 | 6.854 | 6.454 | 6.093 | 5.767 |
| 50-65 | 6.611 | 6.236 | 5.897 | 5.590 |
| 51-66 | 6.369 | 6.019 | 5.701 | 5.412 |
| 52-67 | 6.127 | 5.801 | 5.504 | 5.233 |
| 53-68 | 5.884 | 5.580 | 5.303 | 5.050 |
| 54-69 | 5.638 | 5.357 | 5.100 | 4.864 |
| 55-70 | 5.391 | 5.132 | 4.893 | 4.674 |
| 56-71 | 5.145 | 4.905 | 4.685 | 4.482 |
| 57-72 | 4.899 | 4.679 | 4.477 | 4.289 |
| 58-73 | 4.656 | 4.455 | 4.269 | 4.096 |
| 59-74 | 4.418 | 4.234 | 4.064 | 3.906 |
| 60-75 | 4.189 | 4.021 | 3.866 | 3.721 |
| 61-76 | 3.974 | 3.821 | 3.679 | 3.546 |
| 62-77 | 3.760 | 3.621 | 3.492 | 3.371 |
| 63-78 | 3.538 | 3.414 | 3.297 | 3.188 |
| 64-79 | 3.303 | 3.192 | 3.088 | 2.990 |
| 65-80 | 3.063 | 2.965 | 2.873 | 2.786 |
| 66-81 | 2.833 | 2.746 | 2.664 | 2.587 |
| 67-82 | 2.610 | 2.533 | 2.461 | 2.393 |

Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 68-83 | 2.403 | 2.336 | 2.272 | 2.211 |
| 69-84 | 2.244 | 2.183 | 2.126 | 2.071 |
| 70-85 | 2.097 | 2.042 | 1.991 | 1.941 |
| 71-86 | 1.963 | 1.914 | 1.867 | 1.823 |
| 72-87 | 1.838 | 1.794 | 1.753 | 1.713 |
| 73-88 | 1.736 | 1.697 | 1.660 | 1.625 |
| 74-89 | 1.603 | 1.570 | 1.538 | 1.508 |
| 75-90 | 1.440 | 1.413 | 1.387 | 1.361 |
| 76-91 | 1.221 | 1.200 | 1.180 | 1.160 |
| 77-92 | 0.985 | 0.970 | 0.955 | 0.942 |
| 78-93 | 0.706 | 0.697 | 0.688 | 0.679 |
| 79-94 | 0.458 | 0.453 | 0.448 | 0.443 |
| 80-95 | 0.210 | 0.208 | 0.206 | 0.204 |
| 81-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *twenty* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-21 | 11.413 | 10.053 | 8.961 | 8.070 |
| 2-22 | 13.172 | 11.605 | 10.344 | 9.313 |
| 3-23 | 13.794 | 12.161 | 10.843 | 9.764 |
| 4-24 | 14.178 | 12.511 | 11.163 | 10.057 |
| 5-25 | 14.301 | 12.633 | 11.281 | 10.170 |
| 6-26 | 14.420 | 12.754 | 11.400 | 10.285 |
| 7-27 | 14.451 | 12.798 | 11.452 | 10.341 |
| 8-28 | 14.417 | 12.786 | 11.455 | 10.354 |
| 9-29 | 14.310 | 12.710 | 11.401 | 10.315 |
| 10-30 | 14.150 | 12.586 | 11.304 | 10.239 |
| 11-31 | 13.965 | 12.441 | 11.188 | 10.144 |
| 12-32 | 13.770 | 12.286 | 11.062 | 10.042 |
| 13-33 | 13.570 | 12.125 | 10.932 | 9.934 |
| 14-34 | 13.363 | 11.959 | 10.796 | 9.822 |
| 15-35 | 13.151 | 11.787 | 10.655 | 9.703 |
| 16-36 | 12.932 | 11.609 | 10.507 | 9.579 |
| 17-37 | 12.714 | 11.430 | 10.358 | 9.454 |
| 18-38 | 12.502 | 11.257 | 10.214 | 9.333 |
| 19-39 | 12.297 | 11.089 | 10.074 | 9.215 |
| 20-40 | 12.096 | 10.924 | 9.937 | 9.100 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 21-41 | 11.906 | 10.768 | 9.809 | 8.992 |
| 22-42 | 11.723 | 10.619 | 9.685 | 8.889 |
| 23-43 | 11.540 | 10.470 | 9.562 | 8.785 |
| 24-44 | 11.354 | 10.317 | 9.435 | 8.679 |
| 25-45 | 11.164 | 10.160 | 9.304 | 8.569 |
| 26-46 | 10.970 | 10.000 | 9.170 | 8.455 |
| 27-47 | 10.773 | 9.836 | 9.032 | 8.338 |
| 28-48 | 10.572 | 9.667 | 8.890 | 8.217 |
| 29-49 | 10.366 | 9.495 | 8.744 | 8.092 |
| 30-50 | 10.160 | 9.321 | 8.596 | 7.966 |
| 31-51 | 9.957 | 9.151 | 8.451 | 7.841 |
| 32-52 | 9.756 | 8.980 | 8.306 | 7.716 |
| 33-53 | 9.550 | 8.806 | 8.157 | 7.588 |
| 34-54 | 9.342 | 8.629 | 8.005 | 7.457 |
| 35-55 | 9.131 | 8.448 | 7.849 | 7.322 |
| 36-56 | 8.916 | 8.264 | 7.690 | 7.183 |
| 37-57 | 8.699 | 8.076 | 7.527 | 7.041 |
| 38-58 | 8.477 | 7.884 | 7.360 | 6.894 |
| 39-59 | 8.253 | 7.689 | 7.189 | 6.744 |
| 40-60 | 8.025 | 7.490 | 7.015 | 6.590 |
| 41-61 | 7.796 | 7.290 | 6.838 | 6.434 |
| 42-62 | 7.567 | 7.088 | 6.660 | 6.276 |
| 43-63 | 7.332 | 6.881 | 6.477 | 6.112 |
| 44-64 | 7.095 | 6.671 | 6.289 | 5.944 |
| 45-65 | 6.850 | 6.453 | 6.094 | 5.769 |
| 46-66 | 6.602 | 6.230 | 5.894 | 5.588 |
| 47-67 | 6.351 | 6.004 | 5.690 | 5.403 |
| 48-68 | 6.096 | 5.774 | 5.481 | 5.213 |
| 49-69 | 5.839 | 5.541 | 5.268 | 5.019 |
| 50-70 | 5.582 | 5.306 | 5.054 | 4.822 |
| 51-71 | 5.328 | 5.074 | 4.841 | 4.626 |
| 52-72 | 5.077 | 4.845 | 4.630 | 4.430 |
| 53-73 | 4.829 | 4.614 | 4.417 | 4.234 |
| 54-74 | 4.585 | 4.389 | 4.208 | 4.040 |
| 55-75 | 4.350 | 4.171 | 4.006 | 3.852 |
| 56-76 | 4.129 | 3.966 | 3.815 | 3.674 |
| 57-77 | 3.908 | 3.761 | 3.623 | 3.494 |
| 58-78 | 3.682 | 3.549 | 3.424 | 3.308 |
| 59-79 | 3.440 | 3.322 | 3.210 | 3.105 |
| 60-80 | 3.197 | 3.092 | 2.992 | 2.899 |
| 61-81 | 2.964 | 2.870 | 2.782 | 2.699 |

### Table IV.—continued.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 62-82 | 2.739 | 2.656 | 2.578 | 2.504 |
| 63-83 | 2.530 | 2.457 | 2.387 | 2.321 |
| 64-84 | 2.371 | 2.305 | 2.242 | 2.182 |
| 65-85 | 2.228 | 2.163 | 2.107 | 2.053 |
| 66-86 | 2.089 | 2.035 | 1.984 | 1.936 |
| 67-87 | 1.963 | 1.915 | 1.870. | 1.826 |
| 68-88 | 1.860 | 1.817 | 1.777 | 1.737 |
| 69-89 | 1.722 | 1.685 | 1.650 | 1.616 |
| 70-90 | 1.545 | 1.515 | 1.486 | 1.459 |
| 71-91 | 1.303 | 1.280 | 1.259 | 1.238 |
| 72-92 | 1.044 | 1.028 | 1.012 | 0.997 |
| 73-93 | 0.743 | 0.733 | 0.723 | 0.714 |
| 74-94 | 0.480 | 0.474 | 0.469 | 0.464 |
| 75-95 | 0.219 | 0.217 | 0.215 | 0.213 |
| 76-96 | 0.000 | 0.000 | 0.000 | 0.000 |

### Difference of Age *twenty-five* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-26 | 11.037 | 9.770 | 8.742 | 7.897 |
| 2-27 | 12.722 | 11.264 | 10.080 | 9.104 |
| 3-28 | 13.307 | 11.790 | 10.555 | 9.537 |
| 4-29 | 13.661 | 12.116 | 10.855 | 9.813 |
| 5-30 | 13.762 | 12.220 | 10.959 | 9.913 |
| 6-31 | 13.859 | 12.322 | 11.062 | 10.015 |
| 7-32 | 13.871 | 12.350 | 11.100 | 10.060 |
| 8-33 | 13.820 | 12.323 | 11.090 | 10.061 |
| 9-34 | 13.698 | 12.234 | 11.024 | 10.012 |
| 10-35 | 13.525 | 12.098 | 10.916 | 9.925 |
| 11-36 | 13.328 | 11.941 | 10.788 | 9.820 |
| 12-37 | 13.120 | 11.773 | 10.651 | 9.707 |
| 13-38 | 12.906 | 11.600 | 10.509 | 9.588 |
| 14-39 | 12.686 | 11.420 | 10.360 | 9.464 |
| 15-40 | 12.459 | 11.234 | 10.205 | 9.333 |
| 16-41 | 12.229 | 11.044 | 10.046 | 9.198 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 17-42 | 12.002 | 10.856 | 9.889 | 9.065 |
| 18-43 | 11.785 | 10.677 | 9.739 | 8.938 |
| 19-44 | 11.574 | 10.502 | 9.592 | 8.814 |
| 20-45 | 11.367 | 10.330 | 9.448 | 8.692 |
| 21-46 | 11.167 | 10.165 | 9.310 | 8.574 |
| 22-47 | 10.969 | 10.001 | 9.173 | 8.458 |
| 23-48 | 10.768 | 9.833 | 9.031 | 8.338 |
| 24-49 | 10.562 | 9.661 | 8.886 | 8.214 |
| 25-50 | 10.356 | 9.488 | 8.739 | 8.089 |
| 26-51 | 10.154 | 9.318 | 8.595 | 7.966 |
| 27-52 | 9.952 | 9.148 | 8.451 | 7.842 |
| 28-53 | 9.748 | 8.975 | 8.304 | 7.716 |
| 29-54 | 9.540 | 8.799 | 8.153 | 7.586 |
| 30-55 | 9.329 | 8.619 | 7.999 | 7.453 |
| 31-56 | 9.115 | 8.436 | 7.841 | 7.316 |
| 32-57 | 8.897 | 8.250 | 7.680 | 7.175 |
| 33-58 | 8.677 | 8.060 | 7.515 | 7.031 |
| 34-59 | 8.454 | 7.866 | 7.346 | 6.884 |
| 35-60 | 8.227 | 7.669 | 7.174 | 6.732 |
| 36-61 | 7.997 | 7.469 | 6.998 | 6.577 |
| 37-62 | 7.765 | 7.265 | 6.819 | 6.418 |
| 38-63 | 7.525 | 7.053 | 6.631 | 6.252 |
| 39-64 | 7.281 | 6.838 | 6.440 | 6.081 |
| 40-65 | 7.030 | 6.614 | 6.240 | 5.901 |
| 41-66 | 6.776 | 6.388 | 6.037 | 5.718 |
| 42-67 | 6.522 | 6.159 | 5.831 | 5.532 |
| 43-68 | 6.266 | 5.929 | 5.622 | 5.343 |
| 44-69 | 6.008 | 5.696 | 5.411 | 5.150 |
| 45-70 | 5.749 | 5.460 | 5.195 | 4.953 |
| 46-71 | 5.488 | 5.222 | 4.978 | 4.753 |
| 47-72 | 5.228 | 4.983 | 4.758 | 4.551 |
| 48-73 | 4.970 | 4.746 | 4.539 | 4.348 |
| 49-74 | 4.716 | 4.511 | 4.322 | 4.146 |
| 50-75 | 4.472 | 4.285 | 4.112 | 3.951 |
| 51-76 | 4.245 | 4.074 | 3.916 | 3.768 |
| 52 77 | 4.019 | 3.864 | 3.720 | 3.586 |
| 53-78 | 3.787 | 3.648 | 3.518 | 3.396 |
| 54-79 | 3.540 | 3.416 | 3.299 | 3.189 |
| 55-80 | 3.291 | 3.180 | 3.076 | 2.978 |
| 56-81 | 3.051 | 2.953 | 2.861 | 2.774 |
| 57-82 | 2.820 | 2.733 | 2.651 | 2.574 |

## Table IV.—*continued.*

| s. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 83 | 2.608 | 2.530 | 2.457 | 2.388 |
| 84 | 2.446 | 2.376 | 2.310 | 2.247 |
| 85 | 2.297 | 2.234 | 2.174 | 2.118 |
| 86 | 2.162 | 2.105 | 2.051 | 2.000 |
| 87 | 2.036 | 1.985 | 1.937 | 1.891 |
| 88 | 1.932 | 1.886 | 1.843 | 1.802 |
| 89 | 1.790 | 1.751 | 1.714 | 1.678 |
| 90 | 1.606 | 1.575 | 1.544 | 1.515 |
| 91 | 1.354 | 1.330 | 1.307 | 1.285 |
| 92 | 1.083 | 1.067 | 1.050 | 1.035 |
| 93 | 0.770 | 0.760 | 0.750 | 0.740 |
| 94 | 0.497 | 0.491 | 0.485 | 0.480 |
| 95 | 0.227 | 0.224 | 0.222 | 0.220 |
| 96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *thirty* Years.

| s. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 31 | 10.605 | 9.438 | 8.483 | 7.691 |
| 32 | 12.203 | 10.865 | 9.767 | 8.855 |
| 33 | 12.743 | 11.355 | 10.213 | 9.263 |
| 34 | 13.061 | 11.651 | 10.488 | 9.518 |
| 35 | 13.136 | 11.732 | 10.572 | 9.602 |
| 36 | 13.207 | 11.812 | 10.656 | 9.687 |
| 37 | 13.195 | 11.819 | 10.676 | 9.715 |
| 38 | 13.122 | 11.772 | 10.648 | 9.701 |
| 39 | 12.981 | 11.665 | 10.565 | 9.637 |
| 40 | 12.791 | 11.513 | 10.442 | 9.537 |
| 41 | 12.580 | 11.342 | 10.302 | 9.420 |
| 42 | 12.363 | 11.165 | 10-156 | 9.298 |
| 43 | 12.144 | 10.985 | 10.007 | 9.173 |
| 44 | 11.918 | 10.799 | 9.852 | 9.042 |
| 45 | 11.687 | 10.607 | 9.690 | 8.905 |
| 46 | 11.448 | 10.408 | 9.522 | 8.762 |
| 47 | 11.210 | 10.208 | 9.353 | 8.617 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 18-48 | 10.975 | 10.011 | 9.186 | 8.473 |
| 19-49 | 10.746 | 9.818 | 9.021 | 8.332 |
| 20-50 | 10.523 | 9.630 | 8.861 | 8.195 |
| 21-51 | 10.313 | 9.454 | 8.712 | 8.067 |
| 22-52 | 10.111 | 9.284 | 8.568 | 7.944 |
| 23-53 | 9.905 | 9.111 | 8.421 | 7.818 |
| 24-54 | 9.696 | 8.934 | 8.270 | 7.688 |
| 25-55 | 9.484 | 8.754 | 8.116 | 7.555 |
| 26-56 | 9.269 | 8.570 | 7.958 | 7.419 |
| 27-57 | 9.051 | 8.383 | 7.797 | 7.279 |
| 28-58 | 8.830 | 8.193 | 7.632 | 7.135 |
| 29-59 | 8.605 | 7.999 | 7.464 | 6.988 |
| 30-60 | 8.378 | 7.802 | 7.292 | 6.837 |
| 31-61 | 8.147 | 7.601 | 7.116 | 6.682 |
| 32-62 | 7.914 | 7.397 | 6.937 | 6.524 |
| 33-63 | 7.673 | 7.186 | 6.750 | 6.359 |
| 34-64 | 7.429 | 6.971 | 6.559 | 6.189 |
| 35-65 | 7.177 | 6.747 | 6.360 | 6.010 |
| 36-66 | 6.922 | 6.520 | 6.156 | 5.827 |
| 37-67 | 6.663 | 6.288 | 5.948 | 5.639 |
| 38-68 | 6.401 | 6.052 | 5.735 | 5.446 |
| 39-69 | 6.137 | 5.813 | 5.518 | 5.249 |
| 40-70 | 5.871 | 5.571 | 5.298 | 5.047 |
| 41-71 | 5.605 | 5.329 | 5.076 | 4.844 |
| 42-72 | 5.341 | 5.087 | 4.854 | 4.640 |
| 43-73 | 5.081 | 4.848 | 4.634 | 4.436 |
| 44-74 | 4.826 | 4.613 | 4.417 | 4.235 |
| 45-75 | 4.580 | 4.386 | 4.206 | 4.040 |
| 46-76 | 4.348 | 4.171 | 4.006 | 3.853 |
| 47-77 | 4.115 | 3.954 | 3.805 | 3.666 |
| 48-78 | 3.875 | 3.731 | 3.596 | 3.469 |
| 49-79 | 3.619 | 3.490 | 3.369 | 3.256 |
| 50-80 | 3.362 | 3.247 | 3.140 | 3.039 |
| 51-81 | 3.117 | 3.015 | 2.920 | 2.829 |
| 52-82 | 2.882 | 2.792 | 2.707 | 2.627 |
| 53-83 | 2.665 | 2.585 | 2.510 | 2.438 |
| 54-84 | 2.501 | 2.428 | 2.360 | 2.295 |
| 55-85 | 2.349 | 2.284 | 2.222 | 2.164 |
| 56-86 | 2.211 | 2.153 | 2.097 | 2.044 |
| 57-87 | 2.082 | 2.030 | 1.980 | 1.932 |
| 58-88 | 1.975 | 1.928 | 1.883 | 1.841 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 59-89 | 1.828 | 1.788 | 1.750 | 1.713 |
| 60-90 | 1.641 | 1.608 | 1.577 | 1.547 |
| 61-91 | 1.382 | 1.358 | 1.334 | 1.311 |
| 62-92 | 1.105 | 1.088 | 1.071 | 1.055 |
| 63-93 | 0.785 | 0.774 | 0.764 | 0.754 |
| 64-94 | 0.506 | 0.500 | 0.494 | 0.489 |
| 65-95 | 0.230 | 0.228 | 0.226 | 0.224 |
| 66-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *thirty-five* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-36 | 10.104 | 9.047 | 8.173 | 7.442 |
| 2-37 | 11.600 | 10.392 | 9.390 | 8.551 |
| 3-38 | 12.087 | 10.838 | 9.800 | 8.928 |
| 4-39 | 12.362 | 11.097 | 10.043 | 9.157 |
| 5-40 | 12.405 | 11.150 | 10.102 | 9.219 |
| 6-41 | 12.446 | 11.203 | 10.163 | 9.283 |
| 7-42 | 12.412 | 11.190 | 10.165 | 9.296 |
| 8-43 | 12.325 | 11.130 | 10.124 | 9.270 |
| 9-44 | 12.174 | 11.012 | 10.031 | 9.197 |
| 10-45 | 11.976 | 10.851 | 9.900 | 9.088 |
| 11-46 | 11.756 | 10.697 | 9.774 | 8.962 |
| 12-47 | 11.525 | 10.481 | 9.592 | 8.827 |
| 13-48 | 11.288 | 10.284 | 9.425 | 8.686 |
| 14-49 | 11.045 | 10.080 | 9.252 | 8.538 |
| 15-50 | 10.799 | 9.872 | 9.076 | 8.386 |
| 16-51 | 10.554 | 9.665 | 8.899 | 8.234 |
| 17-52 | 10.313 | 9.461 | 8.724 | 8.083 |
| 18-53 | 10.076 | 9.260 | 8.552 | 7.934 |
| 19-54 | 9.845 | 9.063 | 8.383 | 7.788 |
| 20-55 | 9.617 | 8.869 | 8.216 | 7.643 |
| 21-56 | 9.394 | 8.679 | 8.053 | 7.502 |
| 22-57 | 9.174 | 8.491 | 7.891 | 7.362 |
| 23-58 | 8.951 | 8.299 | 7.725 | 7.218 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 24-59 | 8.725 | 8.104 | 7.556 | 7.070 |
| 25-60 | 8.495 | 7.906 | 7.383 | 6.919 |
| 26-61 | 8.263 | 7.704 | 7.207 | 6.764 |
| 27-62 | 8.028 | 7.499 | 7.027 | 6.605 |
| 28-63 | 7.785 | 7.286 | 6.839 | 6.439 |
| 29-64 | 7.539 | 7.069 | 6.648 | 6.268 |
| 30-65 | 7.286 | 6.844 | 6.447 | 6.089 |
| 31-66 | 7.028 | 6.615 | 6.243 | 5.905 |
| 32-67 | 6.768 | 6.382 | 6.033 | 5.717 |
| 33-68 | 6.504 | 6.146 | 5.820 | 5.524 |
| 34-69 | 6.239 | 5.906 | 5.603 | 5.326 |
| 35-70 | 5.971 | 5.663 | 5.382 | 5.125 |
| 36-71 | 5.703 | 5.419 | 5.159 | 4.920 |
| 37-72 | 5.435 | 5.174 | 4.934 | 4.714 |
| 38-73 | 5.169 | 4.930 | 4.710 | 4.507 |
| 39-74 | 4.908 | 4.690 | 4.488 | 4.301 |
| 40-75 | 4.656 | 4.457 | 4.272 | 4.101 |
| 41-76 | 4.420 | 4.238 | 4.069 | 3.912 |
| 42-77 | 4.184 | 4.019 | 3.865 | 3.722 |
| 43-78 | 3.942 | 3.794 | 3.655 | 3.525 |
| 44-79 | 3.685 | 3.552 | 3.428 | 3.312 |
| 45-80 | 3.426 | 3.308 | 3.197 | 3.093 |
| 46-81 | 3.176 | 3.072 | 2.973 | 2.881 |
| 47-82 | 2.936 | 2.843 | 2.756 | 2.673 |
| 48-83 | 2.714 | 2.632 | 2.554 | 2.481 |
| 49-84 | 2.544 | 2.470 | 2.400 | 2.334 |
| 50-85 | 2.388 | 2.322 | 2.258 | 2.198 |
| 51-86 | 2.248 | 2.188 | 2.131 | 2.077 |
| 52-87 | 2.117 | 2.063 | 2.012 | 1.963 |
| 53-88 | 2.008 | 1.960 | 1.914 | 1.870 |
| 54-89 | 1.858 | 1.817 | 1.778 | 1.740 |
| 55-90 | 1.666 | 1.633 | 1.601 | 1.570 |
| 56-91 | 1.402 | 1.377 | 1.353 | 1.330 |
| 57-92 | 1.120 | 1.102 | 1.085 | 1.069 |
| 58-93 | 0.794 | 0.784 | 0.773 | 0.763 |
| 59-94 | 0.511 | 0.505 | 0.499 | 0.494 |
| 60-95 | 0.233 | 0.230 | 0.228 | 0.226 |
| 61-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Table IV.—*continued.*

### Difference of Age *forty* Years.

| s. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 41 | 9.523 | 8.585 | 7.800 | 7.135 |
| 42 | 10.907 | 9.839 | 8.942 | 8.182 |
| 43 | 11.343 | 10.242 | 9.315 | 8.528 |
| 44 | 11.578 | 10.468 | 9.531 | 8.733 |
| 45 | 11.597 | 10.500 | 9.571 | 8.778 |
| 46 | 11.610 | 10.528 | 9.609 | 8.823 |
| 47 | 11.550 | 10.491 | 9.589 | 8.815 |
| 48 | 11.435 | 10.404 | 9.524 | 8.767 |
| 49 | 11.260 | 10.263 | 9.409 | 8.673 |
| 0 | 11.044 | 10.085 | 9.260 | 8.548 |
| 1 | 10.816 | 9.894 | 9.100 | 8.411 |
| 2 | 10.582 | 9.698 | 8.934 | 8.270 |
| 3 | 10.344 | 9.497 | 8.763 | 8.123 |
| 4 | 10.100 | 9.290 | 8.586 | 7.970 |
| 5 | 9.851 | 9.077 | 8.403 | 7.812 |
| 6 | 9.595 | 8.858 | 8.214 | 7.648 |
| 7 | 9.340 | 8.639 | 8.024 | 7.481 |
| 8 | 9.089 | 8.422 | 7.835 | 7.316 |
| 9 | 8.841 | 8.207 | 7.648 | 7.153 |
| 0 | 8.597 | 7.995 | 7.463 | 6.990 |
| 1 | 8.357 | 7.787 | 7.281 | 6.830 |
| 2 | 8.119 | 7.580 | 7.100 | 6.670 |
| 3 | 7.874 | 7.365 | 6.910 | 6.503 |
| 4 | 7.626 | 7.147 | 6.717 | 6.331 |
|  | 7.370 | 6.920 | 6.515 | 6.151 |
|  | 7.110 | 6.689 | 6.309 | 5.966 |
|  | 6.847 | 6.454 | 6.098 | 5.776 |
|  | 6.581 | 6.215 | 5.883 | 5.581 |
|  | 6.313 | 5.973 | 5.664 | 5.383 |
|  | 6.043 | 5.729 | 5.442 | 5.180 |
|  | 5.772 | 5.483 | 5.218 | 4.974 |
|  | 5.502 | 5.236 | 4.992 | 4.767 |
|  | 5.235 | 4.991 | 4.766 | 4.559 |
|  | 4.973 | 4.749 | 4.543 | 4.353 |
|  | 4.720 | 4.516 | 4.327 | 4.152 |
|  | 4.481 | 4.295 | 4.123 | 3.962 |
|  | 4.242 | 4.073 | 3.916 | 3.770 |
|  | 3.996 | 3.844 | 3.702 | 3.570 |
|  | 3.734 | 3.598 | 3.471 | 3.352 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 40-80 | 3.469 | 3.349 | 3.236 | 3.130 |
| 41-81 | 3.216 | 3.109 | 3.009 | 2.914 |
| 42-82 | 2.973 | 2.878 | 2.789 | 2.705 |
| 43-83 | 2.750 | 2.666 | 2.587 | 2.511 |
| 44-84 | 2.581 | 2.505 | 2.433 | 2.365 |
| 45-85 | 2.424 | 2.356 | 2.291 | 2.230 |
| 46-86 | 2.282 | 2.221 | 2.162 | 2.107 |
| 47-87 | 2.148 | 2.093 | 2.041 | 1.991 |
| 48-88 | 2.036 | 1.987 | 1.941 | 1.895 |
| 49-89 | 1.882 | 1.840 | 1.800 | 1.761 |
| 50-90 | 1.685 | 1.651 | 1.619 | 1.590 |
| 51-91 | 1.417 | 1.391 | 1.367 | 1.343 |
| 52-92 | 1.130 | 1.113 | 1.095 | 1.079 |
| 53-93 | 0.801 | 0.790 | 0.780 | 0.770 |
| 54-94 | 0.515 | 0.509 | 0.503 | 0.498 |
| 55-95 | 0.234 | 0.232 | 0.230 | 0.228 |
| 56-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *forty-five* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-46 | 8.888 | 8.071 | 7.379 | 6.787 |
| 2-47 | 10.147 | 9.221 | 8.435 | 7.760 |
| 3-48 | 10.515 | 9.566 | 8.759 | 8.063 |
| 4-49 | 10.697 | 9.744 | 8.932 | 8.230 |
| 5-50 | 10.679 | 9.742 | 8.941 | 8.248 |
| 6-51 | 10.664 | 9.745 | 8.956 | 8.271 |
| 7-52 | 10.586 | 9.690 | 8.919 | 8.248 |
| 8-53 | 10.458 | 9.591 | 8.841 | 8.188 |
| 9-54 | 10.276 | 9.442 | 8.718 | 8.085 |
| 10-55 | 10.055 | 9.256 | 8.560 | 7.951 |
| 11-56 | 9.814 | 9.052 | 8.386 | 7.801 |
| 12-57 | 9.566 | 8.839 | 8.203 | 7.643 |
| 13-58 | 9.312 | 8.622 | 8.015 | 7.479 |
| 14-59 | 9.053 | 8.399 | 7.821 | 7.310 |

## Table IV.—*continued.*

| s. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 0 | 8.790 | 8.170 | 7.622 | 7.135 |
| 1 | 8.521 | 7.935 | 7.416 | 6.953 |
| 2 | 8.252 | 7.700 | 7.208 | 6.770 |
| 3 | 7.981 | 7.462 | 6.998 | 6.583 |
| 4 | 7.714 | 7.226 | 6.789 | 6.396 |
| 5 | 7.444 | 6.986 | 6.576 | 6.205 |
| 6 | 7.177 | 6.749 | 6.364 | 6.015 |
| 7 | 6.911 | 6.512 | 6.151 | 5.824 |
| 8 | 6.643 | 6.271 | 5.934 | 5.628 |
| 9 | 6.372 | 6.027 | 5.713 | 5.427 |
| 0 | 6.099 | 5.780 | 5.489 | 5.223 |
| 1 | 5.826 | 5.532 | 5.263 | 5.016 |
| 2 | 5.554 | 5.283 | 5.035 | 4.807 |
| 3 | 5.284 | 5.036 | 4.808 | 4.597 |
| 4 | 5.019 | 4.792 | 4.583 | 4.390 |
| 5 | 4.764 | 4.557 | 4.365 | 4.188 |
| 6 | 4.523 | 4.335 | 4.160 | 3.997 |
| 7 | 4.282 | 4.111 | 3.952 | 3.804 |
| 8 | 4.035 | 3.881 | 3.737 | 3.602 |
| 9 | 3.771 | 3.633 | 3.505 | 3.384 |
| 0 | 3.506 | 3.383 | 3.268 | 3.160 |
| 1 | 3.251 | 3.142 | 3.040 | 2.944 |
| 2 | 3.005 | 2.909 | 2.818 | 2.733 |
| 3 | 2.779 | 2.694 | 2.613 | 2.537 |
| 4 | 2.607 | 2.530 | 2.457 | 2.388 |
| 5 | 2.448 | 2.379 | 2.313 | 2.251 |
| 6 | 2.304 | 2.241 | 2.182 | 2.126 |
| 7 | 2.168 | 2.113 | 2.060 | 2.009 |
| 8 | 2.055 | 2.006 | 1.959 | 1.914 |
| 9 | 1.901 | 1.859 | 1.818 | 1.779 |
| 0 | 1.702 | 1.668 | 1.635 | 1.604 |
| 1 | 1.431 | 1.405 | 1.380 | 1.356 |
| 2 | 1.140 | 1.122 | 1.105 | 1.089 |
| 3 | 0.808 | 0.797 | 0.786 | 0.776 |
| 4 | 0.519 | 0.512 | 0.507 | 0.501 |
| 5 | 0.235 | 0.233 | 0.231 | 0.229 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 |

## Table IV.—*continued.*

### Difference of Age *fifty* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at Cen |
|---|---|---|---|---|
| 1-51 | 8.171 | 7.479 | 6.885 | 6. |
| 2-52 | 9.300 | 8.520 | 7.848 | 7. |
| 3-53 | 9.611 | 8.815 | 8.128 | 7. |
| 4-54 | 9.751 | 8.957 | 8.269 | 7. |
| 5-55 | 9.707 | 8.931 | 8.256 | 7. |
| 6-56 | 9.659 | 8.902 | 8.241 | 7. |
| 7-57 | 9.549 | 8.817 | 8.176 | 7. |
| 8-58 | 9.395 | 8.691 | 8.073 | 7. |
| 9-59 | 9.191 | 8.519 | 7.927 | 7. |
| 10-60 | 8.952 | 8.314 | 7.750 | 7. |
| 11-61 | 8.696 | 8.092 | 7.557 | 7. |
| 12-62 | 8.433 | 7.863 | 7.357 | 6. |
| 13-63 | 8.161 | 7.625 | 7.147 | 6. |
| 14-64 | 7.884 | 7.381 | 6.931 | 6. |
| 15-65 | 7.597 | 7.127 | 6.705 | 6. |
| 16-66 | 7.304 | 6.866 | 6.472 | 6. |
| 17-67 | 7.012 | 6.604 | 6.236 | 5. |
| 18-68 | 6.721 | 6.343 | 6.001 | 5. |
| 19-69 | 6.434 | 6.084 | 5.766 | 5. |
| 20-70 | 6.149 | 5.826 | 5.532 | 5. |
| 21-71 | 5.870 | 5.572 | 5.300 | 5. |
| 22-72 | 5.595 | 5.321 | 5.070 | 4. |
| 23-73 | 5.323 | 5.072 | 4.841 | 4. |
| 24-74 | 5.056 | 4.827 | 4.615 | 4. |
| 25-75 | 4.799 | 4.589 | 4.396 | 4. |
| 26-76 | 4.556 | 4.365 | 4.188 | 4. |
| 27-77 | 4.313 | 4.140 | 3.979 | 3. |
| 28-78 | 4.064 | 3.908 | 3.762 | 3. |
| 29-79 | 3.798 | 3.659 | 3.528 | 3. |
| 30-80 | 3.530 | 3.406 | 3.290 | 3. |
| 31-81 | 3.274 | 3.164 | 3.060 | 2. |
| 32-82 | 3.027 | 2.929 | 2.838 | 2. |
| 33-83 | 2.800 | 2.713 | 2.632 | 2. |
| 34-84 | 2.627 | 2.549 | 2.476 | 2. |
| 35-85 | 2.468 | 2.398 | 2.331 | 2. |
| 36-86 | 2.323 | 2.260 | 2.200 | 2. |
| 37-87 | 2.187 | 2.130 | 2.077 | 2. |
| 38-88 | 2.072 | 2.022 | 1.974 | 1. |
| 39-89 | 1.915 | 1.872 | 1.832 | 1. |

## Table IV.—*continued*.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 40-90 | 1.713 | 1.679 | 1.646 | 1.614 |
| 41-91 | 1.439 | 1.413 | 1.388 | 1.364 |
| 42-92 | 1.146 | 1.128 | 1.111 | 1.094 |
| 43-93 | 0.811 | 0.800 | 0.790 | 0.779 |
| 44-94 | 0.521 | 0.515 | 0.509 | 0.503 |
| 45-95 | 0.236 | 0.234 | 0.232 | 0.230 |
| 46-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *fifty-five* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-56 | 7.412 | 6.843 | 6.346 | 5.911 |
| 2-57 | 8.392 | 7.756 | 7.199 | 6.709 |
| 3-58 | 8.630 | 7.986 | 7.421 | 6.922 |
| 4-59 | 8.712 | 8.075 | 7.514 | 7.017 |
| 5-60 | 8.629 | 8.011 | 7.466 | 6.982 |
| 6-61 | 8.542 | 7.944 | 7.415 | 6.945 |
| 7-62 | 8.400 | 7.828 | 7.319 | 6.865 |
| 8-63 | 8.214 | 7.669 | 7.184 | 6.750 |
| 9-64 | 7.984 | 7.470 | 7.010 | 6.598 |
| 10-65 | 7.718 | 7.236 | 6.803 | 6.414 |
| 11-66 | 7.437 | 6.987 | 6.581 | 6.215 |
| 12-67 | 7.149 | 6.730 | 6.351 | 6.009 |
| 13-68 | 6.857 | 6.468 | 6.116 | 5.796 |
| 14-69 | 6.562 | 6.202 | 5.876 | 5.578 |
| 15-70 | 6.264 | 5.933 | 5.631 | 5.355 |
| 16-71 | 5.964 | 5.660 | 5.382 | 5.127 |
| 17-72 | 5.667 | 5.389 | 5.133 | 4.899 |
| 18-73 | 5.378 | 5.123 | 4.889 | 4.673 |
| 19-74 | 5.098 | 4.866 | 4.651 | 4.453 |
| 20-75 | 4.831 | 4.619 | 4.424 | 4.242 |
| 21-76 | 4.583 | 4.391 | 4.212 | 4.046 |
| 22-77 | 4.339 | 4.164 | 4.001 | 3.850 |
| 23-78 | 4.087 | 3.930 | 3.783 | 3.646 |
| 24-79 | 3.820 | 3.679 | 3.548 | 3.424 |
| 25-80 | 3.550 | 3.425 | 3.308 | 3.198 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 26-81 | 3.292 | 3.181 | 3.077 | 2.979 |
| 27-82 | 3.043 | 2.945 | 2.853 | 2.765 |
| 28-83 | 2.815 | 2.728 | 2.646 | 2.568 |
| 29-84 | 2.641 | 2.563 | 2.489 | 2.418 |
| 30-85 | 2.481 | 2.411 | 2.344 | 2.280 |
| 31-86 | 2.336 | 2.272 | 2.212 | 2.154 |
| 32-87 | 2.198 | 2.142 | 2.088 | 2.036 |
| 33-88 | 2.083 | 2.033 | 1.985 | 1.939 |
| 34-89 | 1.925 | 1.882 | 1.841 | 1.802 |
| 35-90 | 1.723 | 1.688 | 1.654 | 1.622 |
| 36-91 | 1.446 | 1.420 | 1.395 | 1.371 |
| 37-92 | 1.152 | 1.134 | 1.116 | 1.099 |
| 38-93 | 0.815 | 0.804 | 0.793 | 0.783 |
| 39-94 | 0.523 | 0.517 | 0.511 | 0.505 |
| 40-95 | 0.237 | 0.235 | 0.233 | 0.231 |
| 41-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *sixty* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-61 | 6.571 | 6.123 | 5.725 | 5.372 |
| 2-62 | 7.391 | 6.894 | 6.452 | 6.059 |
| 3-63 | 7.545 | 7.048 | 6.605 | 6.209 |
| 4-64 | 7.562 | 7.076 | 6.641 | 6.251 |
| 5-65 | 7.429 | 6.963 | 6.546 | 6.171 |
| 6-66 | 7.290 | 6.846 | 6.447 | 6.087 |
| 7-67 | 7.104 | 6.684 | 6.306 | 5.963 |
| 8-68 | 6.884 | 6.490 | 6.134 | 5.811 |
| 9-69 | 6.628 | 6.262 | 5.929 | 5.626 |
| 10-70 | 6.347 | 6.008 | 5.700 | 5.418 |
| 11-71 | 6.056 | 5.744 | 5.460 | 5.199 |
| 12-72 | 5.763 | 5.478 | 5.216 | 4.976 |
| 13-73 | 5.473 | 5.212 | 4.972 | 4.751 |
| 14-74 | 5.188 | 4.950 | 4.731 | 4.528 |
| 15-75 | 4.911 | 4.695 | 4.495 | 4.310 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 16-76 | 4.649 | 4.452 | 4.270 | 4.101 |
| 17-77 | 4.388 | 4.210 | 4.045 | 3.892 |
| 18-78 | 4.123 | 3.964 | 3.815 | 3.677 |
| 19-79 | 3.846 | 3.704 | 3.571 | 3.447 |
| 20-80 | 3.569 | 3.443 | 3.325 | 3.214 |
| 21-81 | 3.307 | 3.195 | 3.091 | 2.992 |
| 22-82 | 3.057 | 2.958 | 2.865 | 2.777 |
| 23-83 | 2.828 | 2.740 | 2.657 | 2.579 |
| 24-84 | 2.653 | 2.574 | 2.499 | 2.429 |
| 25-85 | 2.492 | 2.421 | 2.354 | 2.290 |
| 26-86 | 2.346 | 2.282 | 2.221 | 2.163 |
| 27-87 | 2.208 | 2.151 | 2.096 | 2.044 |
| 28-88 | 2.091 | 2.041 | 1.992 | 1.946 |
| 29-89 | 1·933 | 1.889 | 1.848 | 1.808 |
| 30-90 | 1.729 | 1.694 | 1.660 | 1.628 |
| 31-91 | 1.451 | 1.425 | 1.400 | 1.376 |
| 32-92 | 1.155 | 1.137 | 1.119 | 1.102 |
| 33-93 | 0.817 | 0.806 | 0.795 | 0.785 |
| 34-94 | 0.524 | 0.518 | 0.512 | 0.506 |
| 35-95 | 0.238 | 0.235 | 0.233 | 0.231 |
| 36-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *sixty-five* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-66 | 5.633 | 5.295 | 4.996 | 4.728 |
| 2-67 | 6.266 | 5.896 | 5.569 | 5.276 |
| 3-68 | 6.330 | 5.965 | 5.641 | 5.352 |
| 4-69 | 6.277 | 5.924 | 5.611 | 5.332 |
| 5-70 | 6.102 | 5.768 | 5.472 | 5.209 |
| 6-71 | 5.925 | 5.610 | 5.331 | 5.084 |
| 7-72 | 5.714 | 5.418 | 5.157 | 4.929 |
| 8-73 | 5.480 | 5.204 | 4.963 | 4.752 |
| 9-74 | 5.225 | 4.969 | 4.747 | 4.556 |
| 10-75 | 4.962 | 4.725 | 4.522 | 4.350 |

## Table IV.—*continued.*

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 11-76 | 4.707 | 4.487 | 4.301 | 4.148 |
| 12-77 | 4.449 | 4.268 | 4.195 | 3.943 |
| 13-78 | 4.185 | 4.022 | 3.871 | 3.729 |
| 14-79 | 3.904 | 3.759 | 3.624 | 3.497 |
| 15-80 | 3.621 | 3.492 | 3.372 | 3.259 |
| 16-81 | 3.348 | 3.235 | 3.128 | 3.028 |
| 17-82 | 3.087 | 2.987 | 2.893 | 2.804 |
| 18-83 | 2.849 | 2.760 | 2.677 | 2.598 |
| 19-84 | 2.668 | 2.589 | 2.513 | 2.442 |
| 20-85 | 2.503 | 2.431 | 2.364 | 2.299 |
| 21-86 | 2.354 | 2.290 | 2.229 | 2.171 |
| 22-87 | 2.216 | 2.158 | 2.104 | 2.051 |
| 23-88 | 2.099 | 2.048 | 1.999 | 1.953 |
| 24-89 | 1.939 | 1.895 | 1.854 | 1.814 |
| 25-90 | 1.734 | 1.699 | 1.665 | 1.633 |
| 26-91 | 1.455 | 1.429 | 1.404 | 1.379 |
| 27-92 | 1.158 | 1.140 | 1.122 | 1.105 |
| 28-93 | 0.819 | 0.808 | 0.797 | 0.786 |
| 29-94 | 0.525 | 0.519 | 0.513 | 0.507 |
| 30-95 | 0.238 | 0.236 | 2.234 | 0.231 |
| 31-96 | 0.000 | 0.000 | 0.000 | 0.000 |

## Difference of Age *seventy* Years.

| Ages. | Value at 3 per Cent. | Value at 4 per Cent. | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 1-71 | 4.611 | 4.380 | 4.169 | 3.976 |
| 2-72 | 5.061 | 4.814 | 4.588 | 4.380 |
| 3-73 | 5.051 | 4.811 | 4.591 | 4.389 |
| 4-74 | 4.953 | 4.726 | 4.516 | 4.323 |
| 5-75 | 4.768 | 4.557 | 4.362 | 4.181 |
| 6-76 | 4.599 | 4.403 | 4.221 | 4.053 |
| 7-77 | 4.402 | 4.222 | 4.055 | 3.899 |
| 8-78 | 4.180 | 4.016 | 3.864 | 3.722 |
| 9-79 | 3.921 | 3.775 | 3.638 | 3.510 |
| 10-80 | 3.647 | 3.517 | 3.395 | 3.281 |

## Table IV.—*continued.*

| Ages | Value at 3 per Cent. | Value at 4 per Centt | Value at 5 per Cent. | Value at 6 per Cent. |
|---|---|---|---|---|
| 11-81 | 3.380 | 3.264 | 3.156 | 3.054 |
| 12-82 | 3.122 | 3.020 | 2.924 | 2.833 |
| 13-83 | 2.884 | 2.794 | 2.709 | 2.628 |
| 14-84 | 2.703 | 2.622 | 2.545 | 2.472 |
| 15-85 | 2.535 | 2.462 | 2.393 | 2.327 |
| 16-86 | 2.380 | 2.315 | 2.253 | 2.194 |
| 17-87 | 2.235 | 2.177 | 2.121 | 2.069 |
| 18-88 | 2.112 | 2.061 | 2.012 | 1.965 |
| 19-89 | 1.948 | 1.904 | 1.862 | 1.822 |
| 20-90 | 1.739 | 1.704 | 1.670 | 1.638 |
| 21-91 | 1.459 | 1.432 | 1.407 | 1.382 |
| 22-92 | 1.160 | 1.142 | 1.124 | 1.107 |
| 23-93 | 0.820 | 0.809 | 0.798 | 0.788 |
| 24-94 | 0.526 | 0.520 | 0.514 | 0.508 |
| 25-95 | 0.238 | 0.236 | 0.234 | 0.232 |
| 26-96 | 0.000 | 0.000 | 0.000 | 0.000 |

# TABLE V.

Shewing the Values of *three equal* joint Lives, according to the *Northampton* Table of Observations, reckoning Interest at 4 per Cent.

| Common Age. | Value at 4 per Cent. | Common Age. | Value at 4 per Cent. | Common Age. | Value at 4 per Cent. |
|---|---|---|---|---|---|
| 1 | 5.309 | 33 | 8.848 | 65 | 3.914 |
| 2 | 8.251 | 34 | 8.718 | 66 | 3.733 |
| 3 | 9.632 | 35 | 8.585 | 67 | 3.550 |
| 4 | 10.661 | 36 | 8.448 | 68 | 3.366 |
| 5 | 11.170 | 37 | 8.309 | 69 | 3.181 |
| 6 | 11.707 | 38 | 8.165 | 70 | 2.995 |
| 7 | 12.058 | 39 | 8.017 | 71 | 2.810 |
| 8 | 12.266 | 40 | 7.865 | 72 | 2.627 |
| 9 | 12.298 | 41 | 7.714 | 73 | 2.448 |
| 10 | 12.200 | 42 | 7.567 | 74 | 2.277 |
| 11 | 12.043 | 43 | 7.423 | 75 | 2.119 |
| 12 | 11.865 | 44 | 7.276 | 76 | 1.985 |
| 13 | 11.678 | 45 | 7.126 | 77 | 1.855 |
| 14 | 11.481 | 46 | 6.972 | 78 | 1.720 |
| 15 | 11.274 | 47 | 6.813 | 79 | 1.563 |
| 16 | 11.056 | 48 | 6.650 | 80 | 1.400 |
| 17 | 10.845 | 49 | 6.482 | 81 | 1.245 |
| 18 | 10.656 | 50 | 6.317 | 82 | 1.092 |
| 19 | 10.490 | 51 | 6.161 | 83 | 0.949 |
| 20 | 10.342 | 52 | 6.011 | 84 | 0.860 |
| 21 | 10.222 | 53 | 5.859 | 85 | 0.782 |
| 22 | 10.118 | 54 | 5.705 | 86 | 0.716 |
| 23 | 10.012 | 55 | 5.550 | 87 | 0.662 |
| 24 | 9.905 | 56 | 5.393 | 88 | 0.646 |
| 25 | 9.796 | 57 | 5.235 | 89 | 0.614 |
| 26 | 9.685 | 58 | 5.076 | 90 | 0.563 |
| 27 | 9.572 | 59 | 4.916 | 91 | 0.452 |
| 28 | 9.457 | 60 | 4.755 | 92 | 0.337 |
| 29 | 9.340 | 61 | 4.593 | 93 | 0.185 |
| 30 | 9.221 | 62 | 4.432 | 94 | 0.085 |
| 31 | 9.099 | 63 | 4.263 | 95 | 0.015 |
| 32 | 8.975 | 64 | 4.093 | | |

# TABLE VI.

Shewing the Values of *three joint* Lives, whose Differences of Age are 10 and 20 Years, according to the *Northampton* Table of Observations, reckoning Interest at 4 per Cent.

## Differences of Age 10 and 20 Years.

| Ages. | | | Value at 4 per Cent. | Ages. | | | Value at 4 per Cent. |
|---|---|---|---|---|---|---|---|
| 1 | 11 | 21 | 8.627 | 39 | 49 | 59 | 6.164 |
| 2 | 12 | 22 | 9.914 | 40 | 50 | 60 | 5.994 |
| 3 | 13 | 23 | 10.344 | 41 | 51 | 61 | 5.827 |
| 4 | 14 | 24 | 10.598 | 42 | 52 | 62 | 5.662 |
| 5 | 15 | 25 | 10.655 | 43 | 53 | 63 | 5.494 |
| 6 | 16 | 26 | 10.708 | 44 | 54 | 64 | 5.322 |
| 7 | 17 | 27 | 10.700 | 45 | 55 | 65 | 5.145 |
| 8 | 18 | 28 | 10.654 | 46 | 56 | 66 | 4.965 |
| 9 | 19 | 29 | 10.562 | 47 | 57 | 67 | 4.782 |
| 10 | 20 | 30 | 10.438 | 48 | 58 | 68 | 4.597 |
| 11 | 21 | 31 | 10.305 | 49 | 59 | 69 | 4.408 |
| 12 | 22 | 32 | 10.170 | 50 | 60 | 70 | 4.219 |
| 13 | 23 | 33 | 10.031 | 51 | 61 | 71 | 4.032 |
| 14 | 24 | 34 | 9.887 | 52 | 62 | 72 | 3.847 |
| 15 | 25 | 35 | 9.738 | 53 | 63 | 73 | 3.660 |
| 16 | 26 | 36 | 9.584 | 54 | 64 | 74 | 3.477 |
| 17 | 27 | 37 | 9.429 | 55 | 65 | 75 | 3.298 |
| 18 | 28 | 38 | 9.278 | 56 | 66 | 76 | 3.128 |
| 19 | 29 | 39 | 9.131 | 57 | 67 | 77 | 2.959 |
| 20 | 30 | 40 | 8.986 | 58 | 68 | 78 | 2.785 |
| 21 | 31 | 41 | 8.850 | 59 | 69 | 79 | 2.598 |
| 22 | 32 | 42 | 8.718 | 60 | 70 | 80 | 2.408 |
| 23 | 33 | 43 | 8.586 | 61 | 71 | 81 | 2.224 |
| 24 | 34 | 44 | 8.451 | 62 | 72 | 82 | 2.044 |
| 25 | 35 | 45 | 8.313 | 63 | 73 | 83 | 1.875 |
| 26 | 36 | 46 | 8.171 | 64 | 74 | 84 | 1.743 |
| 27 | 37 | 47 | 8.027 | 65 | 75 | 85 | 1.623 |
| 28 | 38 | 48 | 7.878 | 66 | 76 | 86 | 1.519 |
| 29 | 39 | 49 | 7.725 | 67 | 77 | 87 | 1.425 |
| 30 | 40 | 50 | 7.571 | 68 | 78 | 88 | 1.350 |
| 31 | 41 | 51 | 7.420 | 69 | 79 | 89 | 1.248 |
| 32 | 42 | 52 | 7.272 | 70 | 80 | 90 | 1.122 |
| 33 | 43 | 53 | 7.123 | 71 | 81 | 91 | 0.951 |
| 34 | 44 | 54 | 6.971 | 72 | 82 | 92 | 0.767 |
| 35 | 45 | 55 | 6.816 | 73 | 83 | 93 | 0.548 |
| 36 | 46 | 56 | 6.658 | 74 | 84 | 94 | 0.362 |
| 37 | 47 | 57 | 6.497 | 75 | 85 | 95 | 0.169 |
| 38 | 48 | 58 | 6.332 | | | | |

## TABLE VII.

Shewing the Sum to which an Annuity of £1 forborne and improved at 4 per Cent., Compound Interest, will amount on the Extinction of a given Life, computed from the *Northampton* Table of Observations.

| Age. | Amount. | Age. | Amount. | Age. | Amount. | Age. | Amount. | Age. | Amount. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 137.793 | 19 | 98.005 | 37 | 50.542 | 55 | 23.597 | 73 | 8.354 |
| 2 | 156.400 | 20 | 94.504 | 38 | 48.599 | 56 | 22.514 | 74 | 7.776 |
| 3 | 160.555 | 21 | 91.201 | 39 | 46.713 | 57 | 21.464 | 75 | 7.234 |
| 4 | 161.441 | 22 | 88.051 | 40 | 44.882 | 58 | 20.445 | 76 | 6.735 |
| 5 | 159.164 | 23 | 84.996 | 41 | 43.115 | 59 | 19.458 | 77 | 6.253 |
| 6 | 156.727 | 24 | 82.033 | 42 | 41.412 | 60 | 18.501 | 78 | 5.780 |
| 7 | 153.298 | 25 | 79.159 | 43 | 39.770 | 61 | 17.573 | 79 | 5.304 |
| 8 | 149.228 | 26 | 76.372 | 44 | 38.176 | 62 | 16.675 | 80 | 4.845 |
| 9 | 144.529 | 27 | 73.667 | 45 | 36.628 | 63 | 15.797 | 81 | 4.420 |
| 10 | 137.870 | 28 | 71.044 | 46 | 35.124 | 64 | 14.946 | 82 | 4.025 |
| 11 | 132.832 | 29 | 68.499 | 47 | 33.663 | 65 | 14.115 | 83 | 3.672 |
| 12 | 127.908 | 30 | 66.029 | 48 | 32.245 | 66 | 13.271 | 84 | 3.399 |
| 13 | 123.140 | 31 | 63.633 | 49 | 30.867 | 67 | 12.454 | 85 | 3.151 |
| 14 | 118.524 | 32 | 61.307 | 50 | 29.539 | 68 | 11.702 | 86 | 2.925 |
| 15 | 114.055 | 33 | 59.051 | 51 | 28.270 | 69 | 10.976 | 87 | 2.712 |
| 16 | 109.727 | 34 | 56.727 | 52 | 27.048 | 70 | 10.278 | 88 | 2.526 |
| 17 | 105.596 | 35 | 54.603 | 53 | 25.863 | 71 | 9.607 | 89 | 2.290 |
| 18 | 101.697 | 36 | 54.542 | 54 | 24.713 | 72 | 8.965 | 90 | 2.006 |

Shewing the Value of an Annuity increasing £1 annually during any given Life, computed at 4 per Cent., from the *Northampton* Table of Observations.

| Age. | Value. | Age. | Value. | Age. | Value. | Age. | Value. | Age. | Value. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 227.229 | 19 | 241.804 | 37 | 168.354 | 55 | 91.585 | 73 | 28.628 |
| 2 | 264.241 | 20 | 237.726 | 38 | 164.010 | 56 | 87.573 | 74 | 26.155 |
| 3 | 277.694 | 21 | 233.841 | 39 | 159.644 | 57 | 83.497 | 75 | 23.855 |
| 4 | 285.801 | 22 | 230.056 | 40 | 155.258 | 58 | 79.555 | 76 | 21.738 |
| 5 | 288.356 | 23 | 226.233 | 41 | 150.897 | 59 | 75.658 | 77 | 19.730 |
| 6 | 290.507 | 24 | 222.372 | 42 | 146.565 | 60 | 71.811 | 78 | 17.811 |
| 7 | 290.655 | 25 | 218.473 | 43 | 142.262 | 61 | 68.019 | 79 | 15.962 |
| 8 | 289.337 | 26 | 214.537 | 44 | 137.953 | 62 | 64.288 | 80 | 13.828 |
| 9 | 286.483 | 27 | 210.564 | 45 | 133.634 | 63 | 60.589 | 81 | 12.736 |
| 10 | 282.570 | 28 | 206.555 | 46 | 129.310 | 64 | 56.964 | 82 | 10.810 |
| 11 | 278.197 | 29 | 202.512 | 47 | 124.985 | 65 | 53.386 | 83 | 9.603 |
| 12 | 273.670 | 30 | 198.434 | 48 | 120.663 | 66 | 49.895 | 84 | 8.627 |
| 13 | 269.091 | 31 | 193.841 | 49 | 116.349 | 67 | 46.500 | 85 | 7.735 |
| 14 | 264.461 | 32 | 189.670 | 50 | 112.085 | 68 | 43.209 | 86 | 6.926 |
| 15 | 259.784 | 33 | 185.466 | 51 | 107.913 | 69 | 40.030 | 87 | 6.158 |
| 16 | 255.065 | 34 | 181.229 | 52 | 103.797 | 70 | 36.974 | 88 | 5.434 |
| 17 | 250.441 | 35 | 176.967 | 53 | 99.701 | 71 | 34.048 | 89 | 4.599 |
| 18 | 246.029 | 36 | 172.673 | 54 | 95.629 | 72 | 31.263 | 90 | 3.689 |

## TABLE VIII. (B.)

**Shewing the Value of an Annuity increasing £1 annually during Two joint Lives, whose Difference of Age is Five Years, computed from the *Northampton* Table at 4 per Cent.**

| Ages. | | Value. | Ages. | | Value. | Ages. | | Value. | Ages. | | Value. | Ages. | | Value. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 142.800 | 18 | 23 | 147.872 | 35 | 40 | 97.294 | 52 | 57 | 49.569 | 69 | 74 | 14.293 |
| 2 | 7 | 172.450 | 19 | 24 | 144.704 | 36 | 41 | 94.279 | 53 | 58 | 47.111 | 70 | 75 | 12.881 |
| 3 | 8 | 181.950 | 20 | 25 | 141.630 | 37 | 42 | 91.279 | 54 | 59 | 44.690 | 71 | 76 | 11.581 |
| 4 | 9 | 187.063 | 21 | 26 | 138.676 | 38 | 43 | 88.313 | 55 | 60 | 42.309 | 72 | 77 | 10.416 |
| 5 | 10 | 187.868 | 22 | 27 | 135.790 | 39 | 44 | 85.350 | 56 | 61 | 39.970 | 73 | 78 | 9.257 |
| 6 | 11 | 188.104 | 23 | 28 | 132.884 | 40 | 45 | 82.390 | 57 | 62 | 37.676 | 74 | 79 | 8.224 |
| 7 | 12 | 186.934 | 24 | 29 | 129.960 | 41 | 46 | 79.458 | 58 | 63 | 35.408 | 75 | 80 | 7.227 |
| 8 | 13 | 184.793 | 25 | 30 | 127.017 | 42 | 47 | 76.557 | 59 | 64 | 33.191 | 76 | 81 | 6.348 |
| 9 | 14 | 181.654 | 26 | 31 | 124.059 | 43 | 48 | 73.690 | 60 | 65 | 31.010 | 77 | 82 | 5.554 |
| 10 | 15 | 177.837 | 27 | 32 | 121.095 | 44 | 49 | 70.827 | 61 | 66 | 28.888 | 78 | 83 | 4.835 |
| 11 | 16 | 173.734 | 28 | 33 | 118.105 | 45 | 50 | 68.012 | 62 | 67 | 26.778 | 79 | 84 | 4.250 |
| 12 | 17 | 169.654 | 29 | 34 | 115.100 | 46 | 51 | 65.262 | 63 | 68 | 24.763 | 80 | 85 | 3.723 |
| 13 | 18 | 165.697 | 30 | 35 | 112.414 | 47 | 52 | 62.556 | 64 | 69 | 22.819 | 81 | 86 | 3.334 |
| 14 | 19 | 161.877 | 31 | 36 | 109.406 | 48 | 53 | 59.872 | 65 | 70 | 20.938 | 82 | 87 | 2.890 |
| 15 | 20 | 158.162 | 32 | 37 | 106.388 | 49 | 54 | 57.213 | 66 | 71 | 19.139 | 83 | 88 | 2.554 |
| 16 | 21 | 154.583 | 33 | 38 | 103.365 | 50 | 55 | 54.602 | 67 | 72 | 17.428 | 84 | 89 | 2.227 |
| 17 | 22 | 151.165 | 34 | 39 | 100.332 | 51 | 56 | 52.060 | 68 | 73 | 15.811 | 85 | 90 | 1.874 |

## TABLE IX.

**Shewing the Probabilities of Survivorship between two Persons of all Ages, whose common Difference of Age is not less than ten Years, computed from the *Northampton* Table of Observations.**

| Ten years difference. | | | | Twenty years difference. | | | | Thirty years difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Probabilities. | | Ages. | | Probabilities. | | Ages. | | Probabilities. | |
| 11 | 1 | .5859 | .4141 | 21 | 1 | .5246 | .4754 | 31 | 1 | .4823 | .5177 |
| 12 | 2 | .5136 | .4864 | 22 | 2 | .4433 | .5567 | 32 | 2 | .3936 | .6064 |
| 13 | 3 | .4823 | .5177 | 23 | 3 | .4088 | .5912 | 33 | 3 | .3557 | .6443 |
| 14 | 4 | .4598 | .5402 | 24 | 4 | .3900 | .6100 | 34 | 4 | .3286 | .6714 |
| 15 | 5 | .4470 | .5530 | 25 | 5 | .3769 | .6231 | 35 | 5 | .3135 | .6865 |
| 16 | 6 | .4343 | .5657 | 26 | 6 | .3640 | .6360 | 36 | 6 | .2985 | .7015 |
| 17 | 7 | .4253 | .5747 | 27 | 7 | .3548 | .6452 | 37 | 7 | .2883 | .7117 |
| 18 | 8 | .4191 | .5809 | 28 | 8 | .3484 | .6516 | 38 | 8 | .2798 | .7202 |
| 19 | 9 | .4160 | .5840 | 29 | 9 | .3452 | .6548 | 39 | 9 | .2753 | .7247 |
| 20 | 10 | .4153 | .5847 | 30 | 10 | .3441 | .6559 | 40 | 10 | .2733 | .7267 |
| 21 | 11 | .4158 | .5842 | 31 | 11 | .3440 | .6560 | 41 | 11 | .2724 | .7276 |
| 22 | 12 | .4168 | .5832 | 32 | 12 | .3442 | .6558 | 42 | 12 | .2718 | .7282 |
| 23 | 13 | .4178 | .5822 | 33 | 13 | .3444 | .6556 | 43 | 13 | .2714 | .7286 |
| 24 | 14 | .4190 | .5810 | 34 | 14 | .3447 | .6553 | 44 | 14 | .2711 | .7289 |
| 25 | 15 | .4202 | .5798 | 35 | 15 | .3451 | .6549 | 45 | 15 | .2708 | .7292 |
| 26 | 16 | .4216 | .5784 | 36 | 16 | .3456 | .6544 | 46 | 16 | .2706 | .7294 |
| 27 | 17 | .4226 | .5774 | 37 | 17 | .3458 | .6542 | 47 | 17 | .2701 | .7299 |
| 28 | 18 | .4232 | .5768 | 38 | 18 | .3454 | .6546 | 48 | 18 | .2690 | .7310 |
| 29 | 19 | .4234 | .5766 | 39 | 19 | .3444 | .6556 | 49 | 19 | .2672 | .7328 |
| 30 | 20 | .4231 | .5769 | 40 | 20 | .3429 | .6571 | 50 | 20 | .2650 | .7350 |
| 31 | 21 | .4222 | .5778 | 41 | 21 | .3408 | .6592 | 51 | 21 | .2621 | .7379 |
| 32 | 22 | .4209 | .5791 | 42 | 22 | .3384 | .6616 | 52 | 22 | .2589 | .7411 |
| 33 | 23 | .4197 | .5803 | 43 | 23 | .3359 | .6641 | 53 | 23 | .2556 | .7444 |
| 34 | 24 | .4183 | .5817 | 44 | 24 | .3335 | .6665 | 54 | 24 | .2522 | .7478 |
| 35 | 25 | .4170 | .5830 | 45 | 25 | .3309 | .6691 | 55 | 25 | .2487 | .7513 |
| 36 | 26 | .4156 | .5844 | 46 | 26 | .3283 | .6717 | 56 | 26 | .2452 | .7548 |
| 37 | 27 | .4141 | .5859 | 47 | 27 | .3256 | .6744 | 57 | 27 | .2416 | .7584 |
| 38 | 28 | .4126 | .5874 | 48 | 28 | .3228 | .6772 | 58 | 28 | .2379 | .7621 |
| 39 | 29 | .4110 | .5890 | 49 | 29 | .3199 | .6801 | 59 | 29 | .2341 | .7659 |
| 40 | 30 | .4094 | .5906 | 50 | 30 | .3170 | .6830 | 60 | 30 | .2302 | .7698 |
| 41 | 31 | .4078 | .5922 | 51 | 31 | .3143 | .6857 | 61 | 31 | .2263 | .7737 |
| 42 | 32 | .4063 | .5937 | 52 | 32 | .3116 | .6884 | 62 | 32 | .2223 | .7777 |
| 43 | 33 | .4049 | .5951 | 53 | 33 | .3088 | .6912 | 63 | 33 | .2181 | .7819 |
| 44 | 34 | .4034 | .5966 | 54 | 34 | .3060 | .6940 | 64 | 34 | .2138 | .7862 |
| 45 | 35 | .4019 | .5981 | 55 | 35 | .3032 | .6968 | 65 | 35 | .2093 | .7907 |

## Table IX.—*continued.*

| Ten years difference. | | | | Twenty years difference. | | | | Thirty years difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Probabilities. | | Ages. | | Probabilities. | | Ages. | | Probabilities. | |
| 46 | 36 | .4004 | .5996 | 56 | 36 | .3003 | .6997 | 66 | 36 | .2047 | .7953 |
| 47 | 37 | .3988 | .6012 | 57 | 37 | .2973 | .7027 | 67 | 37 | .2001 | .7999 |
| 48 | 38 | .3971 | .6029 | 58 | 38 | .2943 | .7057 | 68 | 38 | .1954 | .8046 |
| 49 | 39 | .3954 | .6046 | 59 | 39 | .2913 | .7087 | 69 | 39 | .1906 | .8094 |
| 50 | 40 | .3938 | .6062 | 60 | 40 | .2882 | .7118 | 70 | 40 | .1859 | .8141 |
| 51 | 41 | .3923 | .6077 | 61 | 41 | .2849 | .7151 | 71 | 41 | .1810 | .8190 |
| 52 | 42 | .3907 | .6093 | 62 | 42 | .2814 | .7186 | 72 | 42 | .1758 | .8242 |
| 53 | 43 | .3890 | .6110 | 63 | 43 | .2774 | .7226 | 73 | 43 | .1705 | .8295 |
| 54 | 44 | .3881 | .6119 | 64 | 44 | .2733 | .7267 | 74 | 44 | .1653 | .8347 |
| 55 | 45 | .3853 | .6147 | 65 | 45 | .2690 | .7310 | 75 | 45 | .1604 | .8396 |
| 56 | 46 | .3835 | .6165 | 66 | 46 | .2646 | .7354 | 76 | 46 | .1560 | .8440 |
| 57 | 47 | .3816 | .6184 | 67 | 47 | .2601 | .7399 | 77 | 47 | .1515 | .8485 |
| 58 | 48 | .3797 | .6203 | 68 | 48 | .2555 | .7445 | 78 | 48 | .1468 | .8532 |
| 59 | 49 | .3777 | .6223 | 69 | 49 | .2509 | .7491 | 79 | 49 | .1416 | .8584 |
| 60 | 50 | .3756 | .6244 | 70 | 50 | .2461 | .7539 | 80 | 50 | .1361 | .8639 |
| 61 | 51 | .3730 | .6270 | 71 | 51 | .2406 | .7594 | 81 | 51 | .1304 | .8696 |
| 62 | 52 | .3700 | .6300 | 72 | 52 | .2349 | .7651 | 82 | 52 | .1246 | .8754 |
| 63 | 53 | .3668 | .6332 | 73 | 53 | .2291 | .7709 | 83 | 53 | .1194 | .8806 |
| 64 | 54 | .3634 | .6366 | 74 | 54 | .2235 | .7765 | 84 | 54 | .1160 | .8840 |
| 65 | 55 | .3597 | .6403 | 75 | 55 | .2181 | 7819 | 85 | 55 | .1130 | .8870 |
| 66 | 56 | .3558 | .6442 | 76 | 56 | .2131 | .7869 | 86 | 56 | .1104 | .8896 |
| 67 | 57 | .3517 | .6483 | 77 | 57 | .2080 | .7920 | 87 | 57 | .1080 | .8920 |
| 68 | 58 | .3474 | .6526 | 78 | 58 | .2024 | .7976 | 88 | 58 | .1063 | .8937 |
| 69 | 59 | .3430 | .6570 | 79 | 59 | .1959 | .8041 | 89 | 59 | .1025 | .8975 |
| 70 | 60 | .3385 | .6615 | 80 | 60 | .1890 | 8110 | 90 | 60 | .965 | .9035 |
| 71 | 61 | .3338 | .6662 | 81 | 61 | .1823 | .8177 | 91 | 61 | .868 | .9132 |
| 72 | 62 | .3290 | .6710 | 82 | 62 | .1757 | .8243 | 92 | 62 | .754 | .9246 |
| 73 | 63 | .3246 | .6754 | 83 | 63 | .1703 | .8297 | 93 | 63 | .618 | .9382 |
| 74 | 64 | .3204 | .6796 | 84 | 64 | .1673 | .8327 | 94 | 64 | .493 | .9507 |
| 75 | 65 | .3170 | .6830 | 85 | 65 | .1655 | .8345 | 95 | 65 | .368 | .9632 |
| 76 | 66 | .3145 | .6855 | 86 | 66 | .1629 | .8371 | 96 | 66 | .258 | .9742 |
| 77 | 67 | .3120 | .6880 | 87 | 67 | .1638 | .8362 | | | | |
| 78 | 68 | .3090 | .6910 | 88 | 68 | .1644 | .8356 | | | | |
| 79 | 69 | .3047 | .6953 | 89 | 69 | .1623 | .8377 | | | | |
| 80 | 70 | .3000 | .7000 | 90 | 70 | .1567 | .8433 | | | | |
| 81 | 71 | .2958 | .7042 | 91 | 71 | .1450 | .8550 | | | | |
| 82 | 72 | .2919 | .7081 | 92 | 72 | .1305 | .8695 | | | | |
| 83 | 73 | .2890 | .7110 | 93 | 73 | .1108 | .8892 | | | | |
| 84 | 74 | .2902 | .7098 | 94 | 74 | .924 | .9076 | | | | |
| 85 | 75 | .2917 | .7083 | 95 | 75 | .717 | .9283 | | | | |
| 86 | 76 | .2925 | .7075 | 96 | 76 | .512 | .9488 | | | | |

## Table IX.—*continued.*

| Ten years difference. | | | |
|---|---|---|---|
| Ages. | | Probabilities. | |
| 87 | 77 | .2942 | .7058 |
| 88 | 78 | .2994 | .7006 |
| 89 | 79 | .3029 | .6971 |
| 90 | 80 | .3022 | .6978 |
| 91 | 81 | .2897 | .7103 |
| 92 | 82 | .2720 | .7280 |
| 93 | 83 | .2420 | .7580 |
| 94 | 84 | .2049 | .7951 |
| 95 | 85 | .1606 | .8394 |
| 96 | 86 | .1173 | .8827 |

| Forty years difference. | | | | Fifty years difference. | | | | Sixty years difference, | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Probabilities. | | Ages. | | Probabilities. | | Ages. | | Probabilities. | |
| 41 | 1 | .4357 | .5643 | 51 | 1 | .3916 | .6084 | 61 | 1 | .3494 | .6506 |
| 42 | 2 | .3398 | .6602 | 52 | 2 | .2887 | .7113 | 62 | 2 | .2413 | .7587 |
| 43 | 3 | .2985 | .7015 | 53 | 3 | .2446 | .7554 | 63 | 3 | .1952 | .8048 |
| 44 | 4 | .2688 | .7312 | 54 | 4 | .2128 | .7872 | 64 | 4 | .1619 | .8381 |
| 45 | 5 | .2520 | .7480 | 55 | 5 | .1948 | .8052 | 65 | 5 | .1428 | .8572 |
| 46 | 6 | .2353 | .7647 | 56 | 6 | .1768 | .8232 | 66 | 6 | .1237 | .8763 |
| 47 | 7 | .2230 | .7770 | 57 | 7 | .1635 | .8365 | 67 | 7 | .1092 | .8908 |
| 48 | 8 | .2140 | .7860 | 58 | 8 | .1535 | .8465 | 68 | 8 | .0982 | .9018 |
| 49 | 9 | .2086 | .7914 | 59 | 9 | .1474 | .8526 | 69 | 9 | .0912 | .9088 |
| 50 | 10 | .2059 | .7941 | 60 | 10 | .1442 | .8558 | 70 | 10 | .0873 | .9127 |
| 51 | 11 | .2045 | .7955 | 61 | 11 | .1422 | .8578 | 71 | 11 | .0846 | .9154 |
| 52 | 12 | .2035 | .7965 | 62 | 12 | .1406 | .8594 | 72 | 12 | .0825 | .9175 |
| 53 | 13 | .2026 | .7974 | 63 | 13 | .1390 | .8610 | 73 | 13 | .0805 | .9195 |
| 54 | 14 | .2018 | .7982 | 64 | 14 | .1376 | .8624 | 74 | 14 | .0788 | .9212 |
| 55 | 15 | .2011 | .7989 | 65 | 15 | .1362 | .8638 | 75 | 15 | .0775 | .9225 |
| 56 | 16 | .2005 | .7995 | 66 | 16 | .1350 | .8650 | 76 | 16 | .0768 | .9232 |
| 57 | 17 | .1996 | .8004 | 67 | 17 | .1335 | .8665 | 77 | 17 | .0759 | .9241 |
| 58 | 18 | .1981 | .8019 | 68 | 18 | .1314 | .8686 | 78 | 18 | .0743 | .9257 |
| 59 | 19 | .1959 | .8041 | 69 | 19 | .1286 | .8714 | 79 | 19 | .0719 | .9281 |
| 60 | 20 | .1931 | .8069 | 70 | 20 | .1253 | .8747 | 80 | 20 | .0690 | .9310 |

## Table IX.—*continued.*

| Forty years difference. | | | | Fifty years difference. | | | | Sixty years difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Probabilities. | | Ages. | | Probabilities. | | Ages. | | Probabilities. | |
| 61 | 21 | .1896 | .8104 | 71 | 21 | .1211 | .8789 | 81 | 21 | .0655 | .9345 |
| 62 | 22 | .1855 | .8145 | 72 | 22 | .1165 | .8835 | 82 | 22 | .0616 | .9384 |
| 63 | 23 | .1812 | .8188 | 73 | 23 | .1120 | .8880 | 83 | 23 | .0581 | .9419 |
| 64 | 24 | .1768 | .8232 | 74 | 24 | .1075 | .8925 | 84 | 24 | .0556 | .9444 |
| 65 | 25 | .1722 | .8278 | 75 | 25 | .1032 | .8968 | 85 | 25 | .0532 | .9468 |
| 66 | 26 | .1676 | .8324 | 76 | 26 | .0991 | .9009 | 86 | 26 | .0511 | .9489 |
| 67 | 27 | .1629 | .8371 | 77 | 27 | .0950 | .9050 | 87 | 27 | .0490 | .9510 |
| 68 | 28 | .1581 | .8419 | 78 | 28 | .0907 | .9093 | 88 | 28 | .0473 | .9527 |
| 69 | 29 | .1532 | .8468 | 79 | 29 | .0861 | .9139 | 89 | 29 | .0448 | .9552 |
| 70 | 30 | .1483 | .8517 | 80 | 30 | .0815 | .9185 | 90 | 30 | .0413 | .9587 |
| 71 | 31 | .1434 | .8566 | 81 | 31 | .0770 | .9230 | 91 | 31 | .0363 | .9637 |
| 72 | 32 | .1384 | .8616 | 82 | 32 | .0727 | .9273 | 92 | 32 | .0310 | .9690 |
| 73 | 33 | .1335 | .8665 | 83 | 33 | .0688 | .9312 | 93 | 33 | .0248 | .9752 |
| 74 | 34 | .1287 | .8713 | 84 | 34 | .0660 | .9340 | 94 | 34 | .0194 | .9806 |
| 75 | 35 | .1242 | .8758 | 85 | 35 | .0635 | .9365 | 95 | 35 | .0140 | .9860 |
| 76 | 36 | .1199 | .8801 | 86 | 36 | .0612 | .9388 | 96 | 36 | .0095 | .9905 |
| 77 | 37 | .1156 | .8844 | 87 | 37 | .0591 | .9409 | | | | |
| 78 | 38 | .1112 | .8888 | 88 | 38 | .0574 | .9426 | | | | |
| 79 | 39 | .1064 | .8936 | 89 | 39 | .0548 | .9452 | | | | |
| 80 | 40 | .1021 | .8979 | 90 | 40 | .0511 | .9489 | | | | |
| 81 | 41 | .0968 | .9032 | 91 | 41 | .0455 | .9545 | | | | |
| 82 | 42 | .0919 | .9081 | 92 | 42 | .0392 | .9608 | | | | |
| 83 | 43 | .0876 | .9124 | 93 | 43 | .0315 | .9685 | | | | |
| 84 | 44 | .0846 | .9154 | 94 | 44 | .0247 | .9753 | | | | |
| 85 | 45 | .0818 | .9182 | 95 | 45 | .0180 | .9820 | | | | |
| 86 | 46 | .0795 | .9205 | 96 | 46 | .0123 | .9877 | | | | |
| 87 | 47 | .0774 | .9226 | | | | | | | | |
| 88 | 48 | .0759 | .9241 | | | | | | | | |
| 89 | 49 | .0732 | .9268 | | | | | | | | |
| 90 | 50 | .0690 | .9310 | | | | | | | | |
| 91 | 51 | .0617 | .9383 | | | | | | | | |
| 92 | 52 | .0533 | .9467 | | | | | | | | |
| 93 | 53 | .0432 | .9568 | | | | | | | | |
| 94 | 54 | .0342 | .9658 | | | | | | | | |
| 95 | 55 | .0251 | .9749 | | | | | | | | |
| 96 | 56 | .0173 | .9827 | | | | | | | | |

## Table IX.—*continued.*

| Seventy years difference. | | | | Eighty years difference. | | | | Ninety years difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Probabilities. | | Ages. | | Probabilities. | | Ages. | | Probabilities. | |
| 71 | 1 | .3049 | .6951 | 81 | 1 | .2495 | .7505 | 91 | 1 | .1893 | .8107 |
| 72 | 2 | .1945 | .8055 | 82 | 2 | .1437 | .8563 | 92 | 2 | .0918 | .9082 |
| 73 | 3 | .1487 | .8513 | 83 | 3 | .1034 | .8966 | 93 | 3 | .0553 | .9446 |
| 74 | 4 | .1159 | .8841 | 84 | 4 | .0763 | .9237 | 94 | 4 | .0312 | .9688 |
| 75 | 5 | .0980 | .9020 | 85 | 5 | .0634 | .9366 | 95 | 5 | .0212 | .9788 |
| 76 | 6 | .0798 | .9202 | 86 | 6 | .0492 | .9508 | 96 | 6 | .0115 | .9885 |
| 77 | 7 | .0660 | .9340 | 87 | 7 | .0385 | .9615 | | | | |
| 78 | 8 | .0553 | .9447 | 88 | 8 | .0304 | .9696 | | | | |
| 79 | 9 | .0484 | .9516 | 89 | 9 | .0250 | .9750 | | | | |
| 80 | 10 | .0444 | .9556 | 90 | 10 | .0216 | .9784 | | | | |
| 81 | 11 | .0418 | .9582 | 91 | 11 | .0186 | .9814 | | | | |
| 82 | 12 | .0397 | .9603 | 92 | 12 | .0157 | .9843 | | | | |
| 83 | 13 | .0380 | .9620 | 93 | 13 | .0125 | .9875 | | | | |
| 84 | 14 | .0372 | .9628 | 94 | 14 | .0097 | .9903 | | | | |
| 85 | 15 | .0368 | .9632 | 95 | 15 | .0070 | .9930 | | | | |
| 86 | 16 | .0370 | .9630 | 96 | 16 | .0049 | .9951 | | | | |
| 87 | 17 | .0373 | .9627 | | | | | | | | |
| 88 | 18 | .0375 | .9625 | | | | | | | | |
| 89 | 19 | .0367 | .9633 | | | | | | | | |
| 90 | 20 | .0345 | .9655 | | | | | | | | |
| 91 | 21 | .0309 | .9691 | | | | | | | | |
| 92 | 22 | .0263 | .9737 | | | | | | | | |
| 93 | 23 | .0210 | .9790 | | | | | | | | |
| 94 | 24 | .0164 | .9836 | | | | | | | | |
| 95 | 25 | .0118 | .9882 | | | | | | | | |
| 96 | 26 | .0080 | .9920 | | | | | | | | |

# TABLE X.

## Shewing the Probability of one Life's dying after another. *

| Ten Years Difference. | | | | Twenty Years Difference. | | | | Thirty Years Difference. | | | | Forty Years Difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. |
| 1 | 11 | .3974 | .5857 | 1 | 21 | .3885 | .5244 | 1 | 31 | .3384 | .4821 | 1 | 41 | .2908 | .4355 |
| 2 | 12 | .4666 | .5134 | 2 | 22 | .4536 | .4431 | 2 | 32 | .3934 | .3934 | 2 | 42 | .3355 | .3396 |
| 3 | 13 | .4963 | .4821 | 3 | 23 | .4803 | .4086 | 3 | 33 | .4155 | .3555 | 3 | 43 | .3526 | .2983 |
| 4 | 14 | .5177 | .4596 | 4 | 24 | .4934 | .3898 | 4 | 34 | .4307 | .3284 | 4 | 44 | .3642 | .2686 |
| 5 | 15 | .5298 | .4468 | 5 | 25 | .5028 | .3767 | 5 | 35 | .4382 | .3133 | 5 | 45 | .3694 | .2518 |
| 6 | 16 | .5418 | .4341 | 6 | 26 | .5120 | .3638 | 6 | 36 | .4456 | .2983 | 6 | 46 | .3746 | .2351 |
| 7 | 17 | .5502 | .4251 | 7 | 27 | .5183 | .3546 | 7 | 37 | .4498 | .2881 | 7 | 47 | .3777 | .2228 |
| 8 | 18 | .5560 | .4189 | 8 | 28 | .5223 | .3482 | 8 | 38 | .4533 | .2796 | 8 | 48 | .3791 | .2138 |
| 9 | 19 | .5587 | .4158 | 9 | 29 | .5237 | .3450 | 9 | 39 | .4541 | .2751 | 9 | 49 | .3788 | .2084 |
| 10 | 20 | .5592 | .4151 | 10 | 30 | .5232 | .3439 | 10 | 40 | .4532 | .2731 | 10 | 50 | .3772 | .2057 |
| 11 | 21 | .5584 | .4156 | 11 | 31 | .5223 | .3438 | 11 | 41 | .4516 | .2722 | 11 | 51 | .3747 | .2043 |
| 12 | 22 | .5572 | .4166 | 12 | 32 | .5209 | .3440 | 12 | 42 | .4497 | .2716 | 12 | 52 | .3719 | .2033 |
| 13 | 23 | .5560 | .4176 | 13 | 33 | .5187 | .3449 | 13 | 43 | .4476 | .2712 | 13 | 53 | .3690 | .2024 |
| 14 | 24 | .5545 | .4188 | 14 | 34 | .5179 | .3445 | 14 | 44 | .4453 | .2709 | 14 | 54 | .3659 | .2016 |
| 15 | 25 | .5531 | .4200 | 15 | 35 | .5162 | .3449 | 15 | 45 | .4430 | .2706 | 15 | 55 | .3626 | .2009 |
| 16 | 26 | .5514 | .4214 | 16 | 36 | .5144 | .3454 | 16 | 46 | .4405 | .2704 | 16 | 56 | .3591 | .2003 |
| 17 | 27 | .5501 | .4224 | 17 | 37 | .5128 | .3456 | 17 | 47 | .4381 | .2699 | 17 | 57 | .3551 | .1994 |
| 18 | 28 | .5492 | .4230 | 18 | 38 | .5117 | .3452 | 18 | 48 | .4360 | .2688 | 18 | 58 | .3523 | .1978 |
| 19 | 29 | .5487 | .4232 | 19 | 39 | .5109 | .3442 | 19 | 49 | .4342 | .2670 | 19 | 59 | .3491 | .1956 |

* It may not be improper to observe, that though this and the 9th Table have been deduced from the decrements of life at Northampton, they may be safely used, even when the values of the life annuities are derived from a different source, as the probabilities they express are nearly the same, from whatever Table of Observations they are computed.

Table X.—*continued.*

| Ten Years Difference. | | | | Twenty Years Difference. | | | | Thirty Years Difference. | | | | Forty Years Difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. |
| 20 | 30 | .5486 | .4229 | 20 | 40 | .5105 | .3427 | 20 | 50 | .4325 | .2648 | 20 | 60 | .3459 | .1928 |
| 21 | 31 | .5491 | .4220 | 21 | 41 | .5105 | .3406 | 21 | 51 | .4312 | .2618 | 21 | 61 | .3428 | .1898 |
| 22 | 32 | .5550 | .4207 | 22 | 42 | .5107 | .3382 | 22 | 52 | .4298 | .2586 | 22 | 62 | .3399 | .1852 |
| 23 | 33 | .5507 | .4195 | 23 | 43 | .5110 | .3356 | 23 | 53 | .4284 | .2553 | 23 | 63 | .3244 | .1809 |
| 24 | 34 | .5517 | .4189 | 24 | 44 | .5110 | .3332 | 24 | 54 | .4268 | .2519 | 24 | 64 | .3839 | .1765 |
| 25 | 35 | .5525 | .4195 | 25 | 45 | .5112 | .3306 | 25 | 55 | .4253 | .2484 | 25 | 65 | .3307 | .1719 |
| 26 | 36 | .5534 | .4154 | 26 | 46 | .5112 | .3280 | 26 | 56 | .4235 | .2449 | 26 | 66 | .3274 | .1673 |
| 27 | 37 | .5544 | .4139 | 27 | 47 | .5118 | .3258 | 27 | 57 | .4212 | .2413 | 27 | 67 | .3238 | .1626 |
| 28 | 38 | .5555 | .4123 | 28 | 48 | .5114 | .3225 | 28 | 58 | .4199 | .2376 | 28 | 68 | .3201 | .1578 |
| 29 | 39 | .5565 | .4107 | 29 | 49 | .5115 | .3196 | 29 | 59 | .4179 | .2338 | 29 | 69 | .3164 | .1528 |
| 30 | 40 | .5576 | .4091 | 30 | 50 | .5115 | .3167 | 30 | 60 | .4159 | .2299 | 30 | 70 | .8122 | .1479 |
| 31 | 41 | .5586 | .4075 | 31 | 51 | .5112 | .3140 | 31 | 61 | .4136 | .2260 | 31 | 71 | .3077 | .1430 |
| 32 | 42 | .5595 | .4060 | 32 | 52 | .5108 | .3113 | 32 | 62 | .4112 | .2220 | 32 | 72 | .3029 | .1380 |
| 33 | 43 | .5603 | .4046 | 33 | 53 | .5104 | .3085 | 33 | 63 | .4089 | .2177 | 33 | 73 | .2977 | .1331 |
| 34 | 44 | .5613 | .4031 | 34 | 54 | .5099 | .3057 | 34 | 64 | .4066 | .2134 | 34 | 74 | .2921 | .1282 |
| 35 | 45 | .5619 | .4016 | 35 | 55 | .5092 | .3029 | 35 | 65 | .4037 | .2089 | 35 | 75 | .2858 | .1237 |
| 36 | 46 | .5627 | .4001 | 36 | 56 | .5086 | .3000 | 36 | 66 | .4009 | .2043 | 36 | 76 | .2788 | .1194 |
| 37 | 47 | .5636 | .3985 | 37 | 57 | .5072 | .2969 | 37 | 67 | .3978 | .1997 | 37 | 77 | .2715 | .1150 |
| 38 | 48 | .5646 | .3968 | 38 | 58 | .5071 | .2939 | 38 | 68 | .3945 | .1950 | 38 | 78 | .2637 | .1106 |
| 39 | 49 | .5655 | .3951 | 39 | 59 | .5051 | .2909 | 39 | 69 | .3910 | .1902 | 39 | 79 | .2559 | .1057 |
| 40 | 50 | .5664 | .3934 | 40 | 60 | .5049 | .2878 | 40 | 70 | .3872 | .1854 | 40 | 80 | .2470 | .1014 |
| 41 | 51 | .5669 | .3920 | 41 | 61 | .5038 | .2845 | 41 | 71 | .3829 | .1805 | 41 | 81 | .2384 | .0960 |

## Table X.—*continued.*

| Ten Years Difference. | | | | Twenty Years Difference. | | | | Thirty Years Difference. | | | | Forty Years Difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. |
| 42 | 52 | .5676 | .3904 | 42 | 62 | .5026 | .2810 | 42 | 72 | .3785 | .1753 | 42 | 82 | .2286 | .0910 |
| 43 | 53 | .5684 | .3886 | 43 | 63 | .5017 | .2770 | 43 | 73 | .3736 | .1699 | 43 | 83 | .2174 | .0865 |
| 44 | 54 | .5692 | .3868 | 44 | 64 | .5007 | .2728 | 44 | 74 | .3681 | .1647 | 44 | 84 | .2040 | .0834 |
| 45 | 55 | .5701 | .3849 | 45 | 65 | .4995 | .2685 | 45 | 75 | .3617 | .1598 | 45 | 85 | .1898 | .0803 |
| 46 | 56 | .5709 | .3830 | 46 | 66 | .4982 | .2641 | 46 | 76 | .3544 | .1553 | 46 | 86 | .1743 | .0776 |
| 47 | 57 | .5716 | .3811 | 47 | 67 | .4967 | .2596 | 47 | 77 | .3465 | .1508 | 47 | 87 | .1576 | .0751 |
| 48 | 58 | .5723 | .3792 | 48 | 68 | .4950 | .2550 | 48 | 78 | .3383 | .1460 | 48 | 88 | .1393 | .0731 |
| 49 | 59 | .5729 | .3773 | 49 | 69 | .4945 | .2503 | 49 | 79 | .3308 | .1407 | 49 | 89 | .1214 | .0700 |
| 50 | 60 | .5737 | .3751 | 50 | 70 | .4907 | .2455 | 50 | 80 | .3207 | .1351 | 50 | 90 | .1032 | .0649 |
| 51 | 61 | .5748 | .3725 | 51 | 71 | .4885 | .2400 | 51 | 81 | .3105 | .1293 | 51 | 91 | .0865 | .0569 |
| 52 | 62 | .5762 | .3695 | 52 | 72 | .4860 | .2342 | 52 | 82 | .2993 | .1234 | 52 | 92 | .0691 | .0476 |
| 53 | 63 | .5778 | .3662 | 53 | 73 | .4830 | .2284 | 53 | 83 | .2864 | .1180 | 53 | 93 | .0520 | .0363 |
| 54 | 64 | .5792 | .3629 | 54 | 74 | .4793 | .2227 | 54 | 84 | .2706 | .1143 | 54 | 94 | .0324 | .0252 |
| 55 | 65 | .5811 | .3591 | 55 | 75 | .4747 | .2173 | 55 | 85 | .2530 | .1110 | 55 | 95 | .0126 | .0126 |
| 56 | 66 | .5830 | .3551 | 56 | 76 | .4691 | .2122 | 56 | 86 | .2338 | .1079 | | | | |
| 57 | 67 | .5848 | .3511 | 57 | 77 | .4628 | .2070 | 57 | 87 | .2127 | .1049 | | | | |
| 58 | 68 | .5868 | .3467 | 58 | 78 | .4561 | .2013 | 58 | 88 | .1891 | .1025 | | | | |
| 59 | 69 | .5886 | .3423 | 59 | 79 | .4494 | .1947 | 59 | 89 | .1657 | .0979 | | | | |
| 60 | 70 | .5904 | .3377 | 60 | 80 | .4421 | .1876 | 60 | 90 | .1424 | .0909 | | | | |
| 61 | 71 | .5921 | .3330 | 61 | 81 | .4332 | .1808 | 61 | 91 | .1204 | .0801 | | | | |
| 62 | 72 | .5936 | .3282 | 62 | 82 | .4175 | .1740 | 62 | 92 | .0973 | .0674 | | | | |
| 63 | 73 | .5945 | .3237 | 63 | 83 | .4108 | .1683 | 63 | 93 | .0740 | .0520 | | | | |

# TABLE X.—*continued.*

| Ten Years Difference. | | | | Twenty Years Difference. | | | | Thirty Years Difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. |
| 64 | 74 | .5950 | .3195 | 64 | 84 | .3935 | .1649 | 64 | 94 | .0467 | .0364 |
| 65 | 75 | .5942 | .3159 | 65 | 85 | .3737 | .1626 | 65 | 95 | .0184 | .0184 |
| 66 | 76 | .5921 | .3133 | 66 | 86 | .3512 | .1607 | | | | |
| 67 | 77 | .5895 | .3107 | 67 | 87 | .3256 | .1591 | | | | |
| 68 | 78 | .5868 | .3075 | 68 | 88 | .2956 | .1586 | | | | |
| 69 | 79 | .5848 | .3030 | 69 | 89 | .2651 | .1549 | | | | |
| 70 | 80 | .5822 | .2981 | 70 | 90 | .2338 | .1473 | | | | |
| 71 | 81 | .5784 | .2936 | 71 | 91 | .2032 | .1338 | | | | |
| 72 | 82 | .5728 | .2894 | 72 | 92 | .1695 | .1166 | | | | |
| 73 | 83 | .5649 | .2860 | 73 | 93 | .1335 | .0933 | | | | |
| 74 | 84 | .5508 | .2866 | 74 | 94 | .0878 | .0682 | | | | |
| 75 | 85 | .5340 | .2873 | 75 | 95 | .0361 | .0361 | | | | |
| 76 | 86 | .5147 | .2869 | | | | | | | | |
| 77 | 87 | .4910 | .2871 | | | | | | | | |
| 78 | 88 | .4598 | .2902 | | | | | | | | |
| 79 | 89 | .4256 | .2911 | | | | | | | | |
| 80 | 90 | .3886 | .2872 | | | | | | | | |
| 81 | 91 | .3532 | .2708 | | | | | | | | |
| 82 | 92 | .3089 | .2478 | | | | | | | | |
| 83 | 93 | .2555 | .2116 | | | | | | | | |
| 84 | 94 | .1755 | .1626 | | | | | | | | |
| 85 | 95 | .0827 | .0827 | | | | | | | | |

## Table X.—*continued.*

| Fifty Years Difference. | | | | Sixty Years Difference. | | | | Seventy Years Difference. | | | | Eighty Years Difference. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ages. | | Youngest. | Eldest. | Ages. | | Youngest | Eldest. | Ages. | | Youngest. | Eldest. | Ages. | | Youngest. | Eldest. |
| 1 | 51 | .2419 | .3914 | 1 | 61 | .1957 | .3491 | 1 | 71 | .1544 | .3045 | 1 | 81 | .1293 | .2486 |
| 2 | 52 | .2761 | .2885 | 2 | 62 | .2184 | .2411 | 2 | 72 | .1622 | .1942 | 2 | 82 | .1186 | .1429 |
| 3 | 53 | .2879 | .2444 | 3 | 63 | .2245 | .1950 | 3 | 73 | .1604 | .1484 | 3 | 83 | .1042 | .1027 |
| 4 | 54 | .2954 | .2126 | 4 | 64 | .2276 | .1617 | 4 | 74 | .1573 | .1156 | 4 | 84 | .0901 | .0756 |
| 5 | 55 | .2979 | .1946 | 5 | 65 | .2275 | .1426 | 5 | 75 | .1523 | .0977 | 5 | 85 | .0768 | .0626 |
| 6 | 56 | .3005 | .1766 | 6 | 66 | .2275 | .1235 | 6 | 76 | .1477 | .0795 | 6 | 86 | .0649 | .0484 |
| 7 | 57 | .3011 | .1633 | 7 | 67 | .2266 | .1090 | 7 | 77 | .1433 | .0657 | 7 | 87 | .0546 | .0377 |
| 8 | 58 | .3014 | .1533 | 8 | 68 | .2251 | .0980 | 8 | 78 | .1390 | .0550 | 8 | 88 | .0456 | .0295 |
| 9 | 59 | .2998 | .1472 | 9 | 69 | .2226 | .0910 | 9 | 79 | .1346 | .0481 | 9 | 89 | .0381 | .0240 |
| 10 | 60 | .2972 | .1440 | 10 | 70 | .2194 | .0870 | 10 | 80 | .1301 | .0440 | 10 | 90 | .0317 | .0204 |
| 11 | 61 | .2940 | .1420 | 11 | 71 | .2156 | .0843 | 11 | 81 | .1250 | .0414 | 11 | 91 | .0259 | .0173 |
| 12 | 62 | .2906 | .1404 | 12 | 72 | .2114 | .0822 | 12 | 82 | .1193 | .0392 | 12 | 92 | .0202 | .0142 |
| 13 | 63 | .2870 | .1388 | 13 | 73 | .2070 | .0802 | 13 | 83 | .1137 | .0375 | 13 | 93 | .0146 | .0108 |
| 14 | 64 | .2833 | .1373 | 14 | 74 | .2022 | .0785 | 14 | 84 | .1068 | .0366 | 14 | 94 | .0084 | .0077 |
| 15 | 65 | .2793 | .1359 | 15 | 75 | .1973 | .0768 | 15 | 85 | .0993 | .0361 | 15 | 95 | .0035 | .0035 |
| 16 | 66 | .2751 | .1347 | 16 | 76 | .1909 | .0764 | 16 | 86 | .0912 | .0360 | | | | |
| 17 | 67 | .2707 | .1332 | 17 | 77 | .1845 | .0755 | 17 | 87 | .0821 | .0362 | | | | |
| 18 | 68 | .2662 | .1311 | 18 | 78 | .1780 | .0739 | 18 | 88 | .0723 | .0360 | | | | |
| 19 | 69 | .2617 | .1283 | 19 | 79 | .1713 | .0714 | 19 | 89 | .0623 | .0350 | | | | |
| 20 | 70 | .2570 | .1250 | 20 | 80 | .1643 | .0685 | 20 | 90 | .0525 | .0327 | | | | |
| 21 | 71 | .2524 | .1208 | 21 | 81 | .1569 | .0649 | 21 | 91 | .0433 | .0286 | | | | |
| 22 | 72 | .2476 | .1162 | 22 | 82 | .1490 | .0610 | 22 | 92 | .0342 | .0235 | | | | |

## Table X.—*continued.*

| Fifty Years Difference. | | | |
|---|---|---|---|
| Ages. | | Youngest. | Eldest. |
| 23 | 73 | .2424 | .1116 |
| 24 | 74 | .2369 | .1071 |
| 25 | 75 | .2308 | .1028 |
| 26 | 76 | .2242 | .0987 |
| 27 | 77 | .2174 | .0945 |
| 28 | 78 | .2103 | .0902 |
| 29 | 79 | .2031 | .0856 |
| 30 | 80 | .1956 | .0809 |
| 31 | 81 | .1876 | .0763 |
| 32 | 82 | .1788 | .0720 |
| 33 | 83 | .1692 | .0680 |
| 34 | 84 | .1580 | .0650 |
| 35 | 85 | .1460 | .0624 |
| 36 | 86 | .1333 | .0598 |
| 37 | 87 | .1198 | .0574 |
| 38 | 88 | .1052 | .0553 |
| 39 | 89 | .0909 | .0523 |
| 40 | 90 | .0770 | .0481 |
| 41 | 91 | .0641 | .0420 |
| 42 | 92 | .0509 | .0350 |
| 43 | 93 | .0380 | .0265 |
| 44 | 94 | .0235 | .0182 |
| 45 | 95 | .0090 | .0090 |

| Sixty Years Difference. | | | |
|---|---|---|---|
| Ages. | | Youngest. | Eldest. |
| 23 | 83 | .1405 | .0574 |
| 24 | 84 | .1305 | .0548 |
| 25 | 85 | .1201 | .0523 |
| 26 | 86 | .1091 | .0499 |
| 27 | 87 | .0975 | .0476 |
| 28 | 88 | .0851 | .0456 |
| 29 | 89 | .0731 | .0427 |
| 30 | 90 | .0615 | .0389 |
| 31 | 91 | .0510 | .0335 |
| 32 | 92 | .0402 | .0277 |
| 33 | 93 | .0298 | .0209 |
| 34 | 94 | .0183 | .0143 |
| 35 | 95 | .0059 | .0059 |

| Seventy Years Difference. | | | |
|---|---|---|---|
| Ages. | | Youngest. | Eldest. |
| 23 | 93 | .0253 | .0177 |
| 24 | 94 | .0155 | .0121 |
| 25 | 95 | .0059 | .0059 |

| Ninety Years Difference. | | | |
|---|---|---|---|
| Ages. | | Youngest. | Eldest. |
| 1 | 91 | .1096 | .1805 |
| 2 | 92 | .0755 | .0848 |
| 3 | 93 | .0503 | .0487 |
| 4 | 94 | .0282 | .0246 |
| 5 | 95 | .0110 | .0110 |

# TABLE XI.

Shewing the Probabilities and Expectation of Human Life among Males and Females, separately taken, in the Kingdom of *Sweden*. *

| | Males. Born 10,282—282 born dead. | | | Females. 10,277—217 born dead. | | |
|---|---|---|---|---|---|---|
| Ages. | Living. | Decr. | Expectat. | Living. | Decr. | Expectat. |
| Born alive | 10,000 | 2300 | 33.20 | 10,000 | 2090 | 35.70 |
| 1 year | 7,700 | 500 | 42.45 | 7,910 | 518 | 44.00 |
| 2 | 7,200 | 337 | 43.83 | 7,392 | 350 | 46.05 |
| 3 | 6,863 | 240 | 44.96 | 7,042 | 250 | 47.31 |
| 4 | 6,623 | 150 | 45.57 | 6,792 | 135 | 48.04 |
| 5 | 6,473 | 125 | 45.62 | 6,657 | 120 | 48.00 |
| 6 | 6,348 | 105 | 45.50 | 6,537 | 105 | 47.87 |
| 7 | 6,243 | 90 | 45.26 | 6,432 | 85 | 47.64 |
| 8 | 6,153 | 75 | 44.91 | 6,347 | 70 | 47.28 |
| 9 | 6,078 | 65 | 44.46 | 6,277 | 60 | 46.80 |
| 10 | 6,013 | 55 | 43.94 | 6,217 | 52 | 46.25 |
| 11 | 5,958 | 45 | 43.26 | 6,165 | 46 | 45.55 |
| 12 | 5,913 | 45 | 42.58 | 6,119 | 40 | 44.85 |
| 13 | 5,868 | 40 | 41.91 | 6,079 | 35 | 44.15 |
| 14 | 5,828 | 40 | 41.24 | 6,044 | 35 | 43.46 |
| 15 | 5,788 | 39 | 40.56 | 6,009 | 35 | 42.76 |
| 16 | 5,749 | 39 | 39.83 | 5,974 | 40 | 42.04 |
| 17 | 5,710 | 39 | 39.11 | 5,934 | 40 | 41.31 |
| 18 | 5,671 | 44 | 38.39 | 5,894 | 42 | 40.59 |
| 19 | 5,627 | 44 | 37.67 | 5,852 | 43 | 39.87 |
| 20 | 5,583 | 50 | 36.95 | 5,809 | 43 | 39.15 |
| 21 | 5,533 | 50 | 36.28 | 5,766 | 43 | 38.43 |
| 22 | 5,483 | 50 | 35.62 | 5,723 | 43 | 37.72 |
| 23 | 5,433 | 55 | 34.96 | 5,680 | 44 | 37.01 |
| 24 | 5,378 | 55 | 34.30 | 5,636 | 45 | 36.29 |
| 25 | 5,323 | 55 | 33.63 | 5,591 | 45 | 35.58 |
| 26 | 5,268 | 55 | 32.98 | 5,546 | 50 | 34.90 |
| 27 | 5,213 | 55 | 32.32 | 5,496 | 52 | 34.21 |

* See Dr. Price's Treatise on Reversionary Payments, Vol. II. pag. 410, 7th edition.

Table XI.—*continued.*

| | Males. | | | Females. | | |
|---|---|---|---|---|---|---|
| Ages. | Living. | Decr. | Expectat. | Living. | Decr. | Expectat. |
| 28 | 5,158 | 55 | 31.66 | 5,444 | 55 | 33.53 |
| 29 | 5,103 | 56 | 31.00 | 5,389 | 55 | 32.85 |
| 30 | 5,049 | 59 | 30.34 | 5,334 | 60 | 32.17 |
| 31 | 4,988 | 60 | 29.69 | 5,274 | 60 | 31.54 |
| 32 | 4,928 | 60 | 29.04 | 5,214 | 65 | 30.91 |
| 33 | 4,868 | 60 | 28.39 | 5,149 | 65 | 30.28 |
| 34 | 4,808 | 60 | 27.74 | 5,084 | 65 | 29.66 |
| 35 | 4,748 | 60 | 27.09 | 5,019 | 60 | 29.03 |
| 36 | 4,688 | 60 | 26.43 | 4,959 | 56 | 28.26 |
| 37 | 4,628 | 60 | 25.76 | 4,903 | 56 | 27.50 |
| 38 | 4,568 | 60 | 25.09 | 4,847 | 56 | 26.74 |
| 39 | 4,508 | 60 | 24.42 | 4,791 | 58 | 25.97 |
| 40 | 4,448 | 65 | 23.75 | 4,733 | 65 | 25.21 |
| 41 | 4,383 | 72 | 23.15 | 4,668 | 75 | 24.68 |
| 42 | 4,311 | 80 | 22.54 | 4,593 | 76 | 24.75 |
| 43 | 4,231 | 80 | 21.93 | 4,517 | 76 | 23.62 |
| 44 | 4,151 | 80 | 21.32 | 4,441 | 75 | 23.10 |
| 45 | 4,071 | 80 | 20.71 | 4,366 | 72 | 22.57 |
| 46 | 3,991 | 80 | 20.12 | 4,294 | 67 | 21.91 |
| 47 | 3,911 | 80 | 19.52 | 4,227 | 65 | 21.24 |
| 48 | 3,831 | 80 | 18.92 | 4,162 | 65 | 20.58 |
| 49 | 3,751 | 85 | 18.32 | 4,097 | 70 | 19.92 |
| 50 | 3,666 | 95 | 17.72 | 4,027 | 75 | 19.26 |
| 51 | 3,571 | 95 | 17.17 | 3,952 | 80 | 18.64 |
| 52 | 3,476 | 95 | 16.63 | 3,872 | 85 | 18.01 |
| 53 | 3,381 | 95 | 16.08 | 3,787 | 85 | 17.39 |
| 54 | 3,286 | 95 | 15.53 | 3,702 | 85 | 16.77 |
| 55 | 3,191 | 95 | 14.98 | 3,617 | 85 | 16.15 |
| 56 | 3,096 | 95 | 14.43 | 3,532 | 85 | 15.53 |
| 57 | 3,001 | 100 | 13.87 | 3,447 | 90 | 14.92 |
| 58 | 2,901 | 100 | 13.33 | 3,357 | 90 | 14.31 |
| 59 | 2,801 | 100 | 12.79 | 3,267 | 100 | 13.69 |
| 60 | 2,701 | 105 | 12.24 | 3,167 | 110 | 13.08 |
| 61 | 2,596 | 110 | 11.72 | 3,057 | 118 | 12.56 |
| 62 | 2,486 | 115 | 11.21 | 2,939 | 120 | 12.04 |
| 63 | 2,371 | 115 | 10.73 | 2,819 | 120 | 11.52 |
| 64 | 2,256 | 115 | 10.26 | 2,699 | 120 | 11.01 |
| 65 | 2,141 | 115 | 9.78 | 2,579 | 120 | 10.49 |
| 66 | 2,026 | 115 | 9.30 | 2,459 | 120 | 9.97 |

Table XI.—*continued.*

| | Males. | | | Females. | | |
|---|---|---|---|---|---|---|
| Ages. | Living. | Decr. | Expectat. | Living. | Decr. | Expectat. |
| 67 | 1,911 | 120 | 8.84 | 2,339 | 120 | 9.46 |
| 68 | 1,791 | 125 | 8.40 | 2,219 | 120 | 8.94 |
| 69 | 1,666 | 125 | 7.99 | 2,099 | 120 | 8.42 |
| 70 | 1,541 | 125 | 7.60 | 1,979 | 130 | 7.91 |
| 71 | 1,416 | 125 | 7.22 | 1,849 | 140 | 7.53 |
| 72 | 1,291 | 120 | 6.87 | 1,709 | 150 | 7.16 |
| 73 | 1,171 | 120 | 6.53 | 1,559 | 160 | 6.78 |
| 74 | 1,051 | 110 | 6.22 | 1,399 | 150 | 6.40 |
| 75 | 941 | 105 | 5.89 | 1,249 | 140 | 6.03 |
| 76 | 836 | 100 | 5.56 | 1,109 | 130 | 5.73 |
| 77 | 736 | 90 | 5.25 | 979 | 120 | 5.43 |
| 78 | 646 | 85 | 4.92 | 859 | 110 | 5.11 |
| 79 | 561 | 80 | 4.59 | 749 | 100 | 4.79 |
| 80 | 481 | 75 | 4.27 | 649 | 95 | 4.47 |
| 81 | 406 | 70 | 3.96 | 554 | 90 | 4.13 |
| 82 | 336 | 65 | 3.69 | 464 | 85 | 3.84 |
| 83 | 271 | 60 | 3.45 | 379 | 80 | 3.59 |
| 84 | 211 | 50 | 3.30 | 299 | 75 | 3.42 |
| 85 | 161 | 40 | 3.16 | 224 | 55 | 3.40 |
| 86 | 121 | 30 | 3.04 | 169 | 40 | 3.34 |
| 87 | 91 | 22 | 2.88 | 129 | 30 | 3.22 |
| 88 | 69 | 17 | 2.64 | 99 | 23 | 3.05 |
| 89 | 52 | 14 | 2.34 | 76 | 18 | 2.82 |
| 90 | 38 | 12 | 2.02 | 58 | 15 | 2.55 |
| 91 | 26 | 9 | | 43 | 12 | |
| 92 | 17 | 7 | | 31 | 10 | |
| 93 | 10 | 6 | | 21 | 8 | |
| 94 | 4 | 3 | | 13 | 6 | |
| 95 | 1 | 1 | | 7 | 4 | |
| 96 | 0 | 0 | | 3 | 2 | |
| 97 | 0 | 0 | | 1 | 1 | |

# TABLE XII.

Shewing the Probabilities of the Duration of Human Life among Males and Females, taken collectively, deduced from the preceding Table.

| Born 10,249—349 born dead. | | | | | | | |
|---|---|---|---|---|---|---|---|
| Age. | Living. | Decr. | Expect. | Age. | Living. | Decr. | Expect. |
| Born alive | 10,000 | 2195 | 34.42 | 33 | 5,010 | 63 | 29.30 |
| 1 year | 7,805 | 509 | 42.95 | 34 | 4,947 | 63 | 28.67 |
| 2 years | 7,296 | 344 | 44.92 | 35 | 4,884 | 59 | 28.03 |
| 3 | 6,952 | 245 | 46.11 | 36 | 4,825 | 58 | 27.31 |
| 4 | 6,707 | 143 | 46.78 | 37 | 4,767 | 58 | 26.68 |
| 5 | 6,564 | 122 | 46.79 | 38 | 4,709 | 58 | 26.01 |
| 6 | 6,442 | 105 | 46.66 | 39 | 4,651 | 60 | 25.33 |
| 7 | 6,337 | 87 | 46.43 | 40 | 4,591 | 65 | 24.66 |
| 8 | 6,250 | 73 | 46.07 | 41 | 4,526 | 73 | 24.05 |
| 9 | 6,177 | 62 | 45.61 | 42 | 4,453 | 78 | 23.44 |
| 10 | 6,115 | 54 | 45.07 | 43 | 4,375 | 78 | 22.83 |
| 11 | 6,061 | 45 | 44.38 | 44 | 4,297 | 78 | 22.22 |
| 12 | 6,016 | 42 | 43.70 | 45 | 4,219 | 76 | 21.61 |
| 13 | 5,974 | 38 | 43.01 | 46 | 4.143 | 74 | 20.98 |
| 14 | 5,936 | 37 | 42.33 | 47 | 4,069 | 72 | 20.35 |
| 15 | 5,899 | 37 | 41.64 | 48 | 3,997 | 73 | 19.72 |
| 16 | 5,862 | 40 | 40.92 | 49 | 3,924 | 78 | 19.09 |
| 17 | 5,822 | 40 | 40.19 | 50 | 3,846 | 85 | 18.46 |
| 18 | 5,782 | 42 | 39.47 | 51 | 3,761 | 87 | 17.87 |
| 19 | 5,740 | 43 | 38.74 | 52 | 3,674 | 90 | 17.29 |
| 20 | 5,697 | 47 | 38.02 | 53 | 3,584 | 90 | 16.70 |
| 21 | 5,650 | 47 | 37.33 | 54 | 3,494 | 91 | 16.12 |
| 22 | 5,603 | 48 | 36.64 | 55 | 3,403 | 91 | 15.53 |
| 23 | 5,555 | 48 | 35.96 | 56 | 3,312 | 92 | 14.95 |
| 24 | 5,507 | 50 | 35.27 | 57 | 3,220 | 95 | 14.37 |
| 25 | 5,457 | 50 | 34.58 | 58 | 3,125 | 95 | 13.79 |
| 26 | 5,407 | 52 | 33.91 | 59 | 3,030 | 100 | 13.21 |
| 27 | 5,355 | 54 | 33.23 | 60 | 2,930 | 108 | 12.63 |
| 28 | 5,301 | 55 | 32.56 | 61 | 2,822 | 114 | 12.12 |
| 29 | 5,246 | 55 | 31.88 | 62 | 2,708 | 118 | 11.62 |
| 30 | 5,191 | 59 | 31.21 | 63 | 2,590 | 118 | 11.11 |
| 31 | 5,132 | 60 | 30.57 | 64 | 2,472 | 118 | 10.61 |
| 32 | 5,072 | 62 | 29.94 | 65 | 2,354 | 118 | 10.10 |

Table XII.—*continued.*

| Born 10,249—249 born dead. | | | | | | | |
|---|---|---|---|---|---|---|---|
| Age. | Living. | Decr. | Expect. | Age. | Living. | Decr. | Expect. |
| 66 | 2,236 | 118 | 9.62 | 83 | 309 | 65 | 3.57 |
| 67 | 2,118 | 121 | 9.15 | 84 | 244 | 55 | 3.39 |
| 68 | 1,997 | 124 | 8.67 | 85 | 189 | 45 | 3.23 |
| 69 | 1,873 | 124 | 8.20 | 86 | 144 | 35 | 3.09 |
| 70 | 1,749 | 127 | 7.72 | 87 | 109 | 27 | 2.92 |
| 71 | 1,622 | 133 | 7.32 | 88 | 82 | 20 | 2.71 |
| 72 | 1,489 | 135 | 6.89 | 89 | 62 | 15 | 2.43 |
| 73 | 1,354 | 140 | 6.53 | 90 | 47 | 14 | 2.05 |
| 74 | 1,214 | 130 | 6.23 | 91 | 33 | 12 | 1.71 |
| 75 | 1,084 | 121 | 5.91 | 92 | 21 | 10 | 1.40 |
| 76 | 963 | 115 | 5.59 | 93 | 11 | 6 | |
| 77 | 848 | 105 | 5.28 | 94 | 5 | 3 | |
| 78 | 743 | 95 | 4.96 | 95 | 2 | 1 | |
| 79 | 648 | 90 | 4.61 | 96 | 1 | 1 | |
| 80 | 558 | 90 | 4.28 | | | | |
| 81 | 468 | 84 | 4.01 | | | | |
| 82 | 384 | 75 | 3.80 | | | | |

# TABLE XIII.

Shewing the Values of Annuities on Single Lives among Males and Females, according to the Probabilities of the Duration of Life in the Kingdom of *Sweden*. See Table XI.

| | Males. | | Females. | | Lives in general. | |
|---|---|---|---|---|---|---|
| Ages. | 4 per Cent. | 5 per Cent. | 4 per Cent. | 5 per Cent. | 4 per Cent. | 5 per Cent. |
| 1 | 16.503 | 14.051 | 16.820 | 14.271 | 16.661 | 14.161 |
| 2 | 17.355 | 14.778 | 17.719 | 15.034 | 17.537 | 14.906 |
| 3 | 17.935 | 15.279 | 18.344 | 15.571 | 18.139 | 15.425 |
| 4 | 18.328 | 15.624 | 18.780 | 15.951 | 18.554 | 15.787 |
| 5 | 18.503 | 15.786 | 18.927 | 16.088 | 18.715 | 15.937 |
| 6 | 18.622 | 15.901 | 19.045 | 16.203 | 18.833 | 16.052 |
| 7 | 18.693 | 15.977 | 19.131 | 16.291 | 18.912 | 16.134 |
| 8 | 18.725 | 16.021 | 19.162 | 16.335 | 18.943 | 16.178 |
| 9 | 18.715 | 16.030 | 19.151 | 16.343 | 18.933 | 16.186 |
| 10 | 18.674 | 16.014 | 19.109 | 16.325 | 18.891 | 16.169 |
| 11 | 18.600 | 15.970 | 19.041 | 16.286 | 18.820 | 16.128 |
| 12 | 18.491 | 15.896 | 18.952 | 16.229 | 18.721 | 16.062 |
| 13 | 18.378 | 15.819 | 18.840 | 16.153 | 18.609 | 15.986 |
| 14 | 18.246 | 15.724 | 18.707 | 16.059 | 18.476 | 15.891 |
| 15 | 18.105 | 15.624 | 18.568 | 15.960 | 18.336 | 15.792 |
| 16 | 17.958 | 15.517 | 18.424 | 15.856 | 18.191 | 15.686 |
| 17 | 17.803 | 15.404 | 18.290 | 15.761 | 18.046 | 15.582 |
| 18 | 17.643 | 15.285 | 18.151 | 15.662 | 17.897 | 15.473 |
| 19 | 17.492 | 15.175 | 18.013 | 15.563 | 17.752 | 15.369 |
| 20 | 17.335 | 15.059 | 17.872 | 15.462 | 17.603 | 15.260 |
| 21 | 17.192 | 14.955 | 17.725 | 15.356 | 17.458 | 15.155 |
| 22 | 17.042 | 14.846 | 17.573 | 15.245 | 17.307 | 15.045 |
| 23 | 16.887 | 14.732 | 17.414 | 15.129 | 17.150 | 14.930 |
| 24 | 16.742 | 14.627 | 17.252 | 15.009 | 16.997 | 14.818 |
| 25 | 16.592 | 14.517 | 17.087 | 14.886 | 16.839 | 14.701 |
| 26 | 16.436 | 14.402 | 16.915 | 14.757 | 16.675 | 14.579 |
| 27 | 16.274 | 14.282 | 16.751 | 14.636 | 16.512 | 14.459 |
| 28 | 16.105 | 14.156 | 16.588 | 14.515 | 16.346 | 14.335 |
| 29 | 15.930 | 14.024 | 16.427 | 14.396 | 16.178 | 14.210 |
| 30 | 15.751 | 13.889 | 16.261 | 14.272 | 16.006 | 14.080 |
| 31 | 15.575 | 13.756 | 16.104 | 14.156 | 15.839 | 13.956 |
| 32 | 15.395 | 13.619 | 15.941 | 14.035 | 15.668 | 13.827 |

## Table XIII.—*continued.*

| | Males. | | Females. | | Lives in general. | |
|---|---|---|---|---|---|---|
| Ages. | 4 per Cent. | 5 per Cent. | 4 per Cent. | 5 per Cent. | 4 per Cent. | 5 per Cent. |
| 33 | 15.208 | 13.477 | 15.787 | 13.923 | 15.497 | 13.700 |
| 34 | 15.014 | 13.327 | 15.629 | 13.806 | 15.321 | 13.566 |
| 35 | 14.812 | 13.170 | 15.465 | 13.684 | 15.138 | 13.427 |
| 36 | 14.601 | 13.006 | 15.278 | 13.542 | 14.939 | 13.274 |
| 37 | 14.382 | 12.833 | 15.070 | 13.382 | 14.726 | 13.107 |
| 38 | 14.154 | 12.652 | 14.854 | 13.213 | 14.504 | 12.932 |
| 39 | 13.916 | 12.462 | 14.629 | 13.036 | 14.272 | 12.749 |
| 40 | 13.668 | 12.261 | 14.401 | 12.856 | 14.034 | 12.558 |
| 41 | 13.426 | 12.065 | 14.185 | 12.687 | 13.805 | 12.376 |
| 42 | 13.196 | 11.880 | 13.994 | 12.538 | 13.595 | 12.209 |
| 43 | 12.984 | 11.710 | 13.798 | 12.387 | 13.391 | 12.048 |
| 44 | 12.763 | 11.532 | 13.596 | 12.229 | 13.179 | 11.880 |
| 45 | 12.535 | 11.347 | 13.383 | 12.061 | 12.959 | 11.704 |
| 46 | 12.297 | 11.153 | 13.151 | 11.876 | 12.724 | 11.514 |
| 47 | 12.051 | 10.951 | 12.894 | 11.668 | 12.472 | 11.309 |
| 48 | 11.795 | 10.738 | 12.620 | 11.443 | 12.217 | 11.090 |
| 49 | 11.528 | 10.516 | 12.333 | 11.205 | 11.930 | 10.860 |
| 50 | 11.267 | 10.298 | 12.049 | 10.970 | 11.658 | 10.634 |
| 51 | 11.030 | 10.100 | 11.769 | 10.737 | 11.399 | 10.418 |
| 52 | 10.785 | 9.895 | 11.492 | 10.507 | 11.138 | 10.201 |
| 53 | 10.531 | 9.682 | 11.220 | 10.280 | 10.875 | 9.981 |
| 54 | 10.269 | 9.460 | 10.937 | 10.042 | 10.603 | 9.751 |
| 55 | 9.998 | 9.229 | 10.642 | 9.792 | 10.320 | 9.510 |
| 56 | 9.717 | 8.988 | 10.334 | 9.529 | 10.025 | 9.258 |
| 57 | 9.425 | 8.736 | 10.012 | 9.253 | 9.718 | 8.994 |
| 58 | 9.140 | 8.489 | 9.692 | 8.976 | 9.416 | 8.732 |
| 59 | 8.845 | 8.232 | 9.358 | 8.684 | 9.101 | 8.458 |
| 60 | 8.540 | 7.963 | 9.039 | 8.406 | 8.789 | 8.184 |
| 61 | 8.241 | 7.700 | 8.739 | 8.144 | 8.490 | 7.922 |
| 62 | 7.950 | 7.442 | 8.453 | 7.895 | 8.201 | 7.668 |
| 63 | 7.669 | 7.193 | 8.166 | 7.643 | 7.917 | 7.418 |
| 64 | 7.382 | 6.938 | 7.870 | 7.382 | 7.626 | 7.160 |
| 65 | 7.090 | 6.676 | 7.566 | 7.111 | 7.328 | 6.893 |
| 66 | 6.792 | 6.408 | 7.252 | 6.831 | 7.022 | 6.619 |
| 67 | 6.489 | 6.134 | 6.930 | 6.541 | 6.709 | 6.337 |
| 68 | 6.201 | 5.872 | 6.596 | 6.239 | 6.398 | 6.055 |
| 69 | 5.933 | 5.628 | 6.253 | 5.926 | 6.093 | 5.777 |
| 70 | 5.670 | 5.389 | 5.897 | 5.599 | 5.783 | 5.494 |
| 71 | 5.418 | 5.158 | 5.564 | 5.293 | 5.491 | 5.225 |

## Table XIII.—*continued.*

| | Males. | | Females. | | Lives in general. | |
|---|---|---|---|---|---|---|
| Ages. | 4 per Cent. | 5 per Cent. | 4 per Cent. | 5 per Cent. | 4 per Cent. | 5 per Cent. |
| 72 | 5.180 | 4.940 | 5.261 | 5.013 | 5.220 | 4.976 |
| 73 | 4.940 | 4.719 | 4.998 | 4.770 | 4.969 | 4.744 |
| 74 | 4.724 | 4.521 | 4.792 | 4.581 | 4.758 | 4.551 |
| 75 | 4.487 | 4.302 | 4.582 | 4.388 | 4.534 | 4.345 |
| 76 | 4.253 | 4.084 | 4.367 | 4.189 | 4.310 | 4.136 |
| 77 | 4.024 | 3.871 | 4.145 | 3.983 | 4.084 | 3.927 |
| 78 | 3.768 | 3.631 | 3.913 | 3.767 | 3.840 | 3.699 |
| 79 | 3.512 | 3.390 | 3.668 | 3.536 | 3.590 | 3.463 |
| 80 | 3.260 | 3.152 | 3.402 | 3.285 | 3.331 | 3.218 |
| 81 | 3.017 | 2.921 | 3.145 | 3.041 | 3.081 | 2.981 |
| 82 | 2.792 | 2.706 | 2.905 | 2.812 | 2.848 | 2.759 |
| 83 | 2.600 | 2.523 | 2.699 | 2.615 | 2.649 | 2.569 |
| 84 | 2.473 | 2.403 | 2.559 | 2.480 | 2.516 | 2.441 |
| 85 | 2.371 | 2.306 | 2.552 | 2.476 | 2.461 | 2.391 |
| 86 | 2.281 | 2.222 | 2.518 | 2.446 | 2.399 | 2.334 |
| 87 | 2.154 | 2.103 | 2.431 | 2.365 | 2.292 | 2.238 |
| 88 | 1.955 | 1.912 | 2.294 | 2.236 | 2.124 | 2.074 |
| 89 | 1.698 | 1.664 | 2.108 | 2.059 | 1.903 | 1.861 |
| 90 | 1.417 | 1.392 | 1.873 | 1.833 | 1.645 | 1.612 |
| 91 | 1.154 | 1.136 | 1.628 | 1.596 | 1.391 | 1.366 |
| 92 | 0.835 | 0.824 | 1.349 | 1.325 | 1.092 | 1.074 |
| 93 | 0.477 | 0.471 | 1.071 | 1.054 | 0.774 | 0.762 |
| 94 | 0.240 | 0.238 | 0.799 | 0.788 | 0.519 | 0.513 |
| 95 | 0.000 | 0.000 | 0.544 | 0.537 | | |
| 96 | 0.000 | 0.000 | 0.320 | 0.317 | | |

# TABLE XIV.

**Shewing the Values of Annuities on Two *joint* Lives, according to the Probabilities (in Table XII.) of the Duration of Human Life among Males and Females collectively, reckoning Interest at 4 per Cent.**

**Interest 4 per Cent.**

**Differences of Age 0, 6, 12, and 18 Years.**

| Ages. | Values. | Ages. | Values. | Ages. | Values. | Ages. | Values. |
|---|---|---|---|---|---|---|---|
| 1- 1 | 12.252 | 1- 7 | 13.989 | 1-13 | 13.894 | 1-19 | 13.389 |
| 2- 2 | 13.583 | 2- 8 | 14.780 | 2-14 | 14.557 | 2-20 | 14.008 |
| 3- 3 | 14.558 | 3- 9 | 15.323 | 3-15 | 14.988 | 3-21 | 14.417 |
| 4- 4 | 15.267 | 4-10 | 15.685 | 4-16 | 15.259 | 4-22 | 14.671 |
| 5- 5 | 15.577 | 5-11 | 15.817 | 5-17 | 15.326 | 5-23 | 14.725 |
| 6- 6 | 15.820 | 6-12 | 15.887 | 6-18 | 15.354 | 6-24 | 14.740 |
| 7- 7 | 16.003 | 7-13 | 15.914 | 7-19 | 15.351 | 7-25 | 14.727 |
| 8- 8 | 16.109 | 8-14 | 15.888 | 8-20 | 15.310 | 8-26 | 14.673 |
| 9- 9 | 16.152 | 9-15 | 15.824 | 9-21 | 15.244 | 9-27 | 14.590 |
| 10-10 | 16.141 | 10-16 | 15.729 | 10-22 | 15.149 | 10-28 | 14.484 |
| 11-11 | 16.087 | 11-17 | 15.617 | 11-23 | 15.033 | 11-29 | 14.357 |
| 12-12 | 15.982 | 12-18 | 15.477 | 12-24 | 14.889 | 12-30 | 14.202 |
| 13-13 | 15.855 | 13-19 | 15.327 | 13-25 | 14.736 | 13-31 | 14.045 |
| 14-14 | 15.701 | 14-20 | 15.164 | 14-26 | 14.566 | 14-32 | 13.874 |
| 15-15 | 15.535 | 15-21 | 15.001 | 15-27 | 14.392 | 15-33 | 13.700 |
| 16-16 | 15.361 | 16-22 | 14.832 | 16-28 | 14.216 | 16-34 | 13.520 |
| 17-17 | 15.196 | 17-23 | 14.665 | 17-29 | 14.042 | 17-35 | 13.340 |
| 18-18 | 15.023 | 18-24 | 14.491 | 18-30 | 13.860 | 18-36 | 13.141 |
| 19-19 | 14.854 | 19-25 | 14.320 | 19-31 | 13.687 | 19-37 | 12.934 |
| 20-20 | 14.682 | 20-26 | 14.144 | 20-32 | 13.512 | 20-38 | 12.720 |
| 21-21 | 14.525 | 21-27 | 13.976 | 21-33 | 13.345 | 21-39 | 12.505 |
| 22-22 | 14.360 | 22-28 | 13.807 | 22-34 | 13.173 | 22-40 | 12.286 |
| 23-23 | 14.194 | 23-29 | 13.635 | 23-35 | 12.997 | 23-41 | 12.073 |
| 24-24 | 14.020 | 24-30 | 13.455 | 24-36 | 12.801 | 24-42 | 11.873 |
| 25-25 | 13.849 | 25-31 | 13.284 | 25-37 | 12.599 | 25-43 | 11.683 |
| 26-26 | 13.671 | 26-32 | 13.108 | 26-38 | 12.387 | 26-44 | 11.485 |
| 27-27 | 13.495 | 27-33 | 12.935 | 27-39 | 12.170 | 27-45 | 11.284 |
| 28-28 | 13.323 | 28-34 | 12.763 | 28-40 | 11.953 | 28-46 | 11.072 |
| 29-29 | 13.148 | 29-35 | 12.586 | 29-41 | 11.742 | 29-47 | 10.847 |
| 30-30 | 12.965 | 30-36 | 12.390 | 30-42 | 11.543 | 30-48 | 10.606 |
| 31-31 | 12.795 | 31-37 | 12.192 | 31-43 | 11.359 | 31-49 | 10.365 |

## TABLE XIV.—*continued.*

### Interest 4 per Cent.

| Ages. | Values. | Ages. | Values. | Ages. | Values. | Ages. | Values. |
|---|---|---|---|---|---|---|---|
| 32-32 | 12.624 | 32-38 | 11.988 | 32-44 | 11.170 | 32-50 | 10.128 |
| 33-33 | 12.456 | 33-39 | 11.779 | 33-45 | 10.978 | 33-51 | 9.905 |
| 34-34 | 12.286 | 34-40 | 11.568 | 34-46 | 10.775 | 34-52 | 9.679 |
| 35-35 | 12.109 | 35-41 | 11.361 | 35-47 | 10.557 | 35-53 | 9.452 |
| 36-36 | 11.904 | 36-42 | 11.156 | 36-48 | 10.314 | 36-54 | 9.207 |
| 37-37 | 11.683 | 37-43 | 10.953 | 37-49 | 10.059 | 37-55 | 8.951 |
| 38-38 | 11.452 | 38-44 | 10.741 | 38-50 | 9.805 | 38-56 | 8.683 |
| 39-39 | 11.209 | 39-45 | 10.519 | 39-51 | 9.558 | 39-57 | 8.404 |
| 40-40 | 10.964 | 40-46 | 10.286 | 40-52 | 9.308 | 40-58 | 8.124 |
| 41-41 | 10.732 | 41-47 | 10.049 | 41-53 | 9.066 | 41-59 | 7.839 |
| 42-42 | 10.531 | 42-48 | 9.813 | 42-54 | 8.830 | 42-60 | 7.569 |
| 43-43 | 10.346 | 43-49 | 9.581 | 43-55 | 8.597 | 43-61 | 7.318 |
| 44-44 | 10.154 | 44-50 | 9.351 | 44-56 | 8.354 | 44-62 | 7.075 |
| 45-45 | 9.954 | 45-51 | 9.129 | 45-57 | 8.101 | 45-63 | 6.836 |
| 46-46 | 9.736 | 46-52 | 8.897 | 46-58 | 7.841 | 46-64 | 6.586 |
| 47-47 | 9.497 | 47-53 | 8.658 | 47-59 | 7.563 | 47-65 | 6.323 |
| 48-48 | 9.236 | 48-54 | 8.402 | 48-60 | 7.281 | 48-66 | 6.048 |
| 49-49 | 8.966 | 49-55 | 8.139 | 49-61 | 7.008 | 49-67 | 5.764 |
| 50-50 | 8.707 | 50-56 | 7.874 | 50-62 | 6.749 | 50-68 | 5.487 |
| 51-51 | 8.469 | 51-57 | 7.613 | 51-63 | 6.505 | 51-69 | 5.221 |
| 52-52 | 8.230 | 52-58 | 7.351 | 52-64 | 6.256 | 52-70 | 4.953 |
| 53-53 | 7.994 | 53-59 | 7.083 | 53-65 | 6.004 | 53-71 | 4.694 |
| 54-54 | 7.748 | 54-60 | 6.814 | 54-66 | 5.743 | 54-72 | 4.455 |
| 55-55 | 7.495 | 55-61 | 6.555 | 55-67 | 5.474 | 55-73 | 4.231 |
| 56-56 | 7.229 | 56-62 | 6.299 | 56-68 | 5.204 | 56-74 | 4.043 |
| 57-57 | 6.954 | 57-63 | 6.045 | 57-69 | 4.936 | 57-75 | 3.844 |
| 58-58 | 6.678 | 58-64 | 5.788 | 58-70 | 4.664 | 58-76 | 3.637 |
| 59-59 | 6.388 | 59-65 | 5.519 | 59-71 | 4.395 | 59-77 | 3.430 |
| 60-60 | 6.104 | 60-66 | 5.249 | 60-72 | 4.149 | 60-78 | 3.210 |
| 61-61 | 5.844 | 61-67 | 4.984 | 61-73 | 3.927 | 61-79 | 2.974 |
| 62-62 | 5.600 | 62-68 | 4.729 | 62-74 | 3.747 | 62-80 | 2.744 |
| 63-63 | 5.367 | 63-69 | 4.482 | 63-75 | 3.563 | 63-81 | 2.557 |
| 64-64 | 5.128 | 64-70 | 4.231 | 64-76 | 3.370 | 64-82 | 2.396 |
| 65-65 | 4.881 | 65-71 | 3.982 | 65-77 | 3.180 | 65-83 | 2.252 |
| 66-66 | 4.626 | 66-72 | 3.750 | 66-78 | 2.974 | 66-84 | 2.123 |
| 67-67 | 4.362 | 67-73 | 3.527 | 67-79 | 2.743 | 67-85 | 2.010 |
| 68-68 | 4.130 | 68-74 | 3.340 | 68-80 | 2.514 | 68-86 | 1.910 |
| 69-69 | 3.851 | 69-75 | 3.147 | 69-81 | 2.324 | 69-87 | 1.798 |
| 70-70 | 3.593 | 70-76 | 2.946 | 70-82 | 2.155 | 70-88 | 1.661 |
| 71-71 | 3.345 | 71-77 | 2.752 | 71-83 | 2.004 | 71-89 | 1.464 |

## Table XIV.—*continued.*

### Interest 4 per Cent.

| Ages. | Values. | Ages. | Values. | Ages. | Values. | Ages. | Values. |
|---|---|---|---|---|---|---|---|
| 72-72 | 3.128 | 72-78 | 2.558 | 72-84 | 1.875 | 72-90 | 1.189 |
| 73-73 | 2.935 | 73-79 | 2.355 | 73-85 | 1.768 | 73-91 | 0.937 |
| 74-74 | 2.797 | 74-80 | 2.172 | 74-86 | 1.692 | 74-92 | 0.708 |
| 75-75 | 2.648 | 75-81 | 2.017 | 75-87 | 1.605 | 75-93 | 0.575 |
| 76-76 | 2.490 | 76-82 | 1.877 | 76-88 | 1.497 | 76-94 | 0.481 |
| 77-77 | 2.340 | 77-83 | 1.756 | 77-89 | 1.339 | 77-95 | 0.421 |
| 78-78 | 2.170 | 78-84 | 1.639 | 78-90 | 1.097 | | |
| 79-79 | 1.967 | 79-85 | 1.524 | 79-91 | 0.863 | | |
| 80-80 | 1.758 | 80-86 | 1.416 | 80-92 | 0.638 | | |
| 81-81 | 1.600 | 81-87 | 1.320 | 81-93 | 0.511 | | |
| 82-82 | 1.472 | 82-88 | 1.225 | 82-94 | 0.427 | | |
| 83-83 | 1.364 | 83-89 | 1.094 | 83-95 | 0.379 | | |
| 84-84 | 1.276 | 84-90 | 0.902 | | | | |
| 85-85 | 1.212 | 85-91 | 0.725 | | | | |
| 86-86 | 1.172 | 86-92 | 0.556 | | | | |
| 87-87 | 1.127 | 87-93 | 0.459 | | | | |
| 88-88 | 1.071 | 88-94 | 0.396 | | | | |
| 89-89 | 0.949 | 89-95 | 0.364 | | | | |
| 90-90 | 0.718 | | | | | | |
| 91-91 | 0.516 | | | | | | |
| 92-92 | 0.326 | | | | | | |
| 93-93 | 0.236 | | | | | | |
| 94-94 | 0.190 | | | | | | |
| 95-95 | 0.024 | | | | | | |

Table XIV.—*continued.*

Interest 4 per Cent.

Differences of Age 24, 30, 36, and 42 Years.

| Ages. | Values. | Ages. | Values. | Ages. | Values. | Ages. | Values. |
|---|---|---|---|---|---|---|---|
| 1-25 | 12.832 | 1-31 | 12.196 | 1-37 | 11.465 | 1-43 | 10.546 |
| 2-26 | 13.409 | 2-32 | 12.730 | 2-38 | 11.913 | 2-44 | 10.946 |
| 3-27 | 13.778 | 3-33 | 13.066 | 3-39 | 12.164 | 3-45 | 11.168 |
| 4-28 | 14.003 | 4-34 | 13.264 | 4-40 | 12.284 | 4-46 | 11.260 |
| 5-29 | 14.037 | 5-35 | 13.277 | 5-41 | 12.242 | 5-47 | 11.183 |
| 6-30 | 14.033 | 6-36 | 13.242 | 6-42 | 12.185 | 6-48 | 11.064 |
| 7-31 | 14.006 | 7-37 | 13.170 | 7-43 | 12.112 | 7-49 | 10.915 |
| 8-32 | 13.944 | 8-38 | 13.059 | 8-44 | 12.004 | 8-50 | 10.743 |
| 9-33 | 13.855 | 9-39 | 12.913 | 9-45 | 11.865 | 9-51 | 10.560 |
| 10-34 | 13.741 | 10-40 | 12.743 | 10-46 | 11.694 | 10-52 | 10.357 |
| 11-35 | 13.604 | 11-41 | 12.563 | 11-47 | 11.493 | 11-53 | 10.140 |
| 12-36 | 13.428 | 12-42 | 12.379 | 12-48 | 11.259 | 12-54 | 9.898 |
| 13-37 | 13.234 | 13-43 | 12.196 | 13-49 | 11.011 | 13-55 | 9.644 |
| 14-38 | 13.023 | 14-44 | 11.997 | 14-50 | 10.759 | 14-56 | 9.371 |
| 15-39 | 12.798 | 15-45 | 11.787 | 15-51 | 10.514 | 15-57 | 9.087 |
| 16-40 | 12.570 | 16-46 | 11.562 | 16-52 | 10.264 | 16-58 | 8.799 |
| 17-41 | 12.351 | 17-47 | 11.328 | 17-53 | 10.018 | 17-59 | 8.503 |
| 18-42 | 12.146 | 18-48 | 11.076 | 18-54 | 9.761 | 18-60 | 8.208 |
| 19-43 | 11.951 | 19-49 | 10.819 | 19-55 | 9.500 | 19-61 | 7.928 |
| 20-44 | 11.751 | 20-50 | 10.567 | 20-56 | 9.228 | 20-62 | 7.658 |
| 21-45 | 11.550 | 21-51 | 10.332 | 21-57 | 8.953 | 21-63 | 7.396 |
| 22-46 | 11.335 | 22-52 | 10.092 | 22-58 | 8.675 | 22-64 | 7.127 |
| 23-47 | 11.107 | 23-53 | 9.852 | 23-59 | 8.385 | 23-65 | 6.851 |
| 24-48 | 10.862 | 24-54 | 9.602 | 24-60 | 8.097 | 24-66 | 6.566 |
| 25-49 | 10.612 | 25-55 | 9.347 | 25-61 | 7.823 | 25-67 | 6.275 |
| 26-50 | 10.364 | 26-56 | 9.080 | 26-62 | 7.557 | 26-68 | 5.986 |
| 27-51 | 10.130 | 27-57 | 8.807 | 27-63 | 7.297 | 27-69 | 5.702 |
| 28-52 | 9.894 | 28-58 | 8.534 | 28-64 | 7.032 | 28-70 | 5.415 |
| 29-53 | 9.659 | 29-59 | 8.250 | 29-65 | 6.761 | 29-71 | 5.136 |
| 30-54 | 9.413 | 30-60 | 7.967 | 30-66 | 6.481 | 30-72 | 4.881 |
| 31-55 | 9.167 | 31-61 | 7.702 | 31-67 | 6.197 | 31-73 | 4.646 |
| 32-56 | 8.912 | 32-62 | 7.446 | 32-68 | 5.917 | 32-74 | 4.453 |
| 33-57 | 8.651 | 33-63 | 7.196 | 33-69 | 5.642 | 33-75 | 4.251 |
| 34-58 | 8.389 | 34-64 | 6.942 | 34-70 | 5.364 | 34-76 | 4.040 |
| 35-59 | 8.114 | 35-65 | 6.679 | 35-71 | 5.093 | 35-77 | 3.833 |
| 36-60 | 7.833 | 36-66 | 6.402 | 36-72 | 4.840 | 36-78 | 3.605 |
| 37-61 | 7.561 | 37-67 | 6.115 | 37-73 | 4.603 | 37-79 | 3.352 |
| 38-62 | 7.296 | 38-68 | 5.828 | 38-74 | 4.405 | 38-80 | 3.098 |

## Table XIV.—*continued.*

### Interest 4 per Cent.

| Ages. | Values. | Ages. | Values. | Ages. | Values. | Ages. | Values. |
|---|---|---|---|---|---|---|---|
| 39-63 | 7.033 | 39-69 | 5.543 | 39-75 | 4.195 | 39-81 | 2.889 |
| 40-64 | 6.763 | 40-70 | 5.254 | 40-76 | 3.975 | 40-82 | 2.710 |
| 41-65 | 6.492 | 41-71 | 4.977 | 41-77 | 3.762 | 41-83 | 2.553 |
| 42-66 | 6.225 | 42-72 | 4.730 | 42-78 | 3.539 | 42-84 | 2.418 |
| 43-67 | 5.957 | 43-73 | 4.507 | 43-79 | 3.295 | 43-85 | 2.305 |
| 44-68 | 5.689 | 44-74 | 4.322 | 44-80 | 3.052 | 44-86 | 2.203 |
| 45-69 | 5.426 | 45-75 | 4.128 | 45-81 | 2.854 | 45-87 | 2.083 |
| 46-70 | 5.153 | 46-76 | 3.921 | 46-82 | 2.684 | 46-88 | 1.933 |
| 47-71 | 4.884 | 47-77 | 3.715 | 47-83 | 2.533 | 47-89 | 1.708 |
| 48-72 | 4.633 | 48-78 | 3.489 | 48-84 | 2.396 | 48-90 | 1.385 |
| 49-73 | 4.398 | 49-79 | 3.238 | 49-85 | 2.277 | 49-91 | 1.090 |
| 50-74 | 4.205 | 50-80 | 2.990 | 50-86 | 2.171 | 50-92 | 0.818 |
| 51-75 | 4.008 | 51-81 | 2.792 | 51-87 | 2.050 | 51-93 | 0.662 |
| 52-76 | 3.803 | 52-82 | 2.623 | 52-88 | 1.901 | 52-94 | 0.551 |
| 53-77 | 3.605 | 53-83 | 2.475 | 53-89 | 1.681 | 53-95 | 0.468 |
| 54-78 | 3.389 | 54-84 | 2.344 | 54-90 | 1.366 | | |
| 55-79 | 3.150 | 55-85 | 2.232 | 55-91 | 1.078 | | |
| 56-80 | 2.909 | 56-86 | 2.130 | 56-92 | 0.810 | | |
| 57-81 | 2.710 | 57-87 | 2.010 | 57-93 | 0.655 | | |
| 58-82 | 2.539 | 58-88 | 1.864 | 58-94 | 0.546 | | |
| 59-83 | 2.385 | 59-89 | 1.644 | 59-95 | 0.464 | | |
| 60-84 | 2.248 | 60-90 | 1.333 | | | | |
| 61-85 | 2.135 | 61-91 | 1.050 | | | | |
| 62-86 | 2.037 | 62-92 | 0.789 | | | | |
| 63-87 | 1.926 | 63-93 | 0.639 | | | | |
| 64-88 | 1.790 | 64-94 | 0.533 | | | | |
| 65-89 | 1.585 | 65-95 | 0.456 | | | | |
| 66-90 | 1.290 | | | | | | |
| 67-91 | 1.017 | | | | | | |
| 68-92 | 0.764 | | | | | | |
| 69-93 | 0.617 | | | | | | |
| 70-94 | 0.514 | | | | | | |
| 71-95 | 0.441 | | | | | | |

# TABLE XV.

Shewing the Decrements and Expectations of Life according to the Tables of Mortality published by M. de Parcieux.

| Age. | Living. | Decr. | Expect. | Age. | Living. | Decr. | Expect. |
|---|---|---|---|---|---|---|---|
| 0 | 10,000 | 2550 | 34.89 | 35 | 4,740 | 52 | 30.95 |
| 1 | 7,450 | 362 | 45.67 | 36 | 4,688 | 51 | 30.29 |
| 2 | 7,088 | 265 | 47.27 | 37 | 4,637 | 49 | 29.62 |
| 3 | 6,823 | 205 | 47.78 | 38 | 4,587 | 49 | 28.94 |
| 4 | 6,618 | 150 | 48.25 | 39 | 4,538 | 48 | 28.24 |
| 5 | 6,468 | 123 | 48.35 | 40 | 4,490 | 49 | 27.76 |
| 6 | 6,345 | 102 | 48.28 | 41 | 4,441 | 49 | 26.84 |
| 7 | 6,243 | 91 | 48.06 | 42 | 4,392 | 50 | 26.13 |
| 8 | 6,154 | 81 | 47.75 | 43 | 4,342 | 51 | 25.42 |
| 9 | 6,073 | 69 | 47.38 | 44 | 4,291 | 52 | 24.72 |
| 10 | 6,004 | 58 | 46.92 | 45 | 4,239 | 53 | 24.02 |
| 11 | 5,946 | 49 | 46.37 | 46 | 4,186 | 54 | 23.32 |
| 12 | 5,897 | 43 | 45.75 | 47 | 4,132 | 55 | 22.61 |
| 13 | 5,854 | 39 | 45.08 | 48 | 4,077 | 56 | 21.91 |
| 14 | 5,815 | 37 | 44.38 | 49 | 4,021 | 57 | 21.21 |
| 15 | 5,778 | 38 | 43.64 | 50 | 3,964 | 59 | 20.50 |
| 16 | 5,740 | 41 | 42.95 | 51 | 3,905 | 62 | 19.80 |
| 17 | 5,699 | 44 | 42.25 | 52 | 3,843 | 66 | 19.12 |
| 18 | 5,655 | 47 | 41.58 | 53 | 3,777 | 70 | 18.45 |
| 19 | 5,608 | 50 | 40.92 | 54 | 3,707 | 76 | 17.78 |
| 20 | 5,558 | 52 | 40.29 | 55 | 3,631 | 81 | 17.15 |
| 21 | 5,506 | 53 | 39.66 | 56 | 3,550 | 85 | 16.53 |
| 22 | 5,453 | 54 | 39.04 | 57 | 3,465 | 88 | 15.92 |
| 23 | 5,399 | 55 | 38.43 | 58 | 3,377 | 91 | 15.33 |
| 24 | 5,344 | 56 | 37.82 | 59 | 3,286 | 95 | 14.74 |
| 25 | 5,288 | 57 | 37.21 | 60 | 3,191 | 99 | 14.16 |
| 26 | 5,231 | 58 | 36.61 | 61 | 3,092 | 102 | 13.59 |
| 27 | 5,173 | 57 | 35.83 | 62 | 2,990 | 105 | 13.04 |
| 28 | 5,116 | 56 | 35.41 | 63 | 2,885 | 107 | 12.50 |
| 29 | 5,060 | 55 | 35.00 | 64 | 2,778 | 109 | 11.96 |
| 30 | 5,005 | 54 | 34.17 | 65 | 2,669 | 110 | 11.42 |
| 31 | 4,951 | 54 | 33.55 | 66 | 2,559 | 111 | 10.89 |
| 32 | 4,897 | 53 | 32.91 | 67 | 2,448 | 112 | 10.37 |
| 33 | 4,844 | 52 | 32.26 | 68 | 2,336 | 113 | 9.85 |
| 34 | 4,792 | 52 | 31.61 | 69 | 2,223 | 114 | 9.32 |

Table XV.—*continued.*

| Age. | Living. | Decr. | Expect. | Age. | Living. | Decr. | Expect. |
|---|---|---|---|---|---|---|---|
| 70 | 2,109 | 116 | 8.79 | 84 | 404 | 77 | 3.59 |
| 71 | 1,993 | 119 | 8.28 | 85 | 327 | 66 | 3.33 |
| 72 | 1,874 | 125 | 7.77 | 86 | 261 | 55 | 3.04 |
| 73 | 1,749 | 132 | 7.29 | 87 | 206 | 47 | 2.72 |
| 74 | 1,617 | 138 | 6.85 | 88 | 159 | 42 | 2.38 |
| 75 | 1,479 | 142 | 6.43 | 89 | 117 | 37 | 2.05 |
| 76 | 1,337 | 139 | 5.99 | 90 | 80 | 30 | 1.77 |
| 77 | 1,198 | 134 | 5.71 | 91 | 50 | 22 | 1.54 |
| 78 | 1,064 | 128 | 5.37 | 92 | 28 | 14 | 1.36 |
| 79 | 936 | 124 | 5.04 | 93 | 14 | 8 | 1.21 |
| 80 | 812 | 115 | 4.73 | 94 | 6 | 3 | 1.17 |
| 81 | 697 | 107 | 4.43 | 95 | 3 | 2 | 83 |
| 82 | 590 | 98 | 4.14 | 96 | 1 | 1 | 50 |
| 83 | 492 | 88 | 3.86 | 97 | 0 | 0 | 0 |

# TABLE XVI.

Shewing the Values of Single Lives according to the Probabilities of the Duration of Life in Mr. De Parcieux's Table of Mortality. See Mr. Florencourt's Dissertations on Political Arithmetic, p. 288.

Interest 5 per Cent.

| Age. | Value. | Age. | Value. | Age. | Value. | Age. | Value. |
|---|---|---|---|---|---|---|---|
| 0 | 11.083 | 25 | 15.112 | 50 | 11.425 | 75 | 4.674 |
| 1 | 14.620 | 26 | 15.040 | 51 | 11.178 | 76 | 4.429 |
| 2 | 15.135 | 27 | 14.969 | 52 | 10.926 | 77 | 4.190 |
| 3 | 15.509 | 28 | 14.893 | 53 | 10.673 | 78 | 3.953 |
| 4 | 15.750 | 29 | 14.810 | 54 | 10.418 | 79 | 3.719 |
| 5 | 15.924 | 30 | 14.722 | 55 | 10.168 | 80 | 3.501 |
| 6 | 16.041 | 31 | 14.627 | 56 | 9.930 | 81 | 3.283 |
| 7 | 16.118 | 32 | 14.527 | 57 | 9.682 | 82 | 3.072 |
| 8 | 16.169 | 33 | 14.421 | 58 | 9.431 | 83 | 2.868 |
| 9 | 16.204 | 34 | 14.306 | 59 | 9.177 | 84 | 2.668 |
| 10 | 16.210 | 35 | 14.189 | 60 | 8.923 | 85 | 2.461 |
| 11 | 16.194 | 36 | 14.065 | 61 | 8.669 | 86 | 2.237 |
| 12 | 16.145 | 37 | 13.930 | 62 | 8.413 | 87 | 1.976 |
| 13 | 16.077 | 38 | 13.786 | 63 | 8.155 | 88 | 1.688 |
| 14 | 15.994 | 39 | 13.632 | 64 | 7.893 | 89 | 1.409 |
| 15 | 15.901 | 40 | 13.466 | 65 | 7.626 | 90 | 1.164 |
| 16 | 15.807 | 41 | 13.296 | 66 | 7.351 | | |
| 17 | 15.716 | 42 | 13.116 | 67 | 7.069 | | |
| 18 | 15.631 | 43 | 12.931 | 68 | 6.778 | | |
| 19 | 15.550 | 44 | 12.738 | 69 | 6.479 | | |
| 20 | 15.474 | 45 | 12.539 | 70 | 6.171 | | |
| 21 | 15.401 | 46 | 12.333 | 71 | 5.856 | | |
| 22 | 15.328 | 47 | 12.119 | 72 | 5.540 | | |
| 23 | 15.256 | 48 | 11.897 | 73 | 5.232 | | |
| 24 | 15.184 | 49 | 11.666 | 74 | 4.942 | | |

# TABLE XVII.

Shewing the Probabilities and Expectation of Life among Males and Females at *Chester*.

| | Males. | | | Females. | | |
|---|---|---|---|---|---|---|
| Age. | Living. | Decr. | Expect. | Living. | Decr. | Expectat. |
| 0 | 1,927 | 220 | 28.13 | 2,139 | 161 | 33.27 |
| 3 months | - - | 75 | - - | - - | 64 | - - |
| 6 months | - - | 76 | - - | - - | 69 | - - |
| 9 months | - - | 67 | - - | - - | 74 | - - |
| 1 year | 1,489 | 180 | 35.26 | 1,771 | 181 | 39.09 |
| 2 | 1,309 | 107 | 39.04 | 1,580 | 127 | 42.75 |
| 3 | 1,202 | 67 | 41.47 | 1,463 | 77 | 45.13 |
| 4 | 1,135 | 34 | 42.89 | 1,386 | 53 | 46.61 |
| 5 | 1,101 | 30 | 43.20 | 1,333 | 30 | 47.44 |
| 6 | 1,071 | 24 | 43.40 | 1,303 | 18 | 47.52 |
| 7 | 1,047 | 18 | 43.38 | 1,285 | 11 | 47.18 |
| 8 | 1,029 | 11 | 43.03 | 1,274 | 9 | 46.59 |
| 9 | 1,018 | 8 | 42.59 | 1,265 | 7 | 45.44 |
| 10 | 1,010 | 6 | 41.92 | 1,258 | 6 | 45.17 |
| 11 | 1,004 | 5 | 41.16 | 1,252 | 6 | 44.30 |
| 12 | 999 | 5 | 40.37 | 1,246 | 7 | 42.94 |
| 13 | 994 | 6 | 39.57 | 1,239 | 7 | 42.84 |
| 14 | 988 | 6 | 38.81 | 1,232 | 8 | 42.08 |
| 15 | 982 | 7 | 38.05 | 1,224 | 9 | 41.35 |
| 16 | 975 | 9 | 37.32 | 1,215 | 10 | 40.64 |
| 17 | 966 | 10 | 36.66 | 1,205 | 11 | 39.98 |
| 18 | 956 | 11 | 36.04 | 1,194 | 12 | 39.35 |
| 19 | 945 | 11 | 35.34 | 1,182 | 11 | 38.74 |
| 20 | 934 | 11 | 34.86 | 1,171 | 10 | 38.10 |
| 21 | 923 | 11 | 34.27 | 1,161 | 10 | 37.43 |
| 22 | 912 | 12 | 33.68 | 1,151 | 10 | 36.75 |
| 23 | 900 | 12 | 33.12 | 1,141 | 11 | 36.06 |
| 24 | 888 | 12 | 32.56 | 1,130 | 12 | 35.41 |
| 25 | 876 | 13 | 32.00 | 1,118 | 16 | 34.78 |
| 26 | 863 | 13 | 31.48 | 1,102 | 16 | 34.18 |
| 27 | 850 | 13 | 30.95 | 1,086 | 16 | 33.69 |
| 28 | 837 | 12 | 30.42 | 1,070 | 16 | 33.28 |
| 29 | 825 | 11 | 29.86 | 1,054 | 16 | 32.68 |

Table XVII.—*continued.*

| | Males. | | | Females. | | |
|---|---|---|---|---|---|---|
| Age. | Living. | Decr. | Expectat. | Living. | Decr. | Expectat. |
| 30 | 814 | 10 | 29.25 | 1,038 | 13 | 32.27 |
| 31 | 804 | 9 | 28.61 | 1,025 | 13 | 31.68 |
| 32 | 795 | 10 | 27.93 | 1,012 | 13 | 31.58 |
| 33 | 785 | 10 | 27.29 | 999 | 13 | 30.47 |
| 34 | 775 | 10 | 26.63 | 986 | 13 | 29.87 |
| 35 | 765 | 11 | 25.96 | 973 | 14 | 29.26 |
| 36 | 754 | 11 | 25.34 | 959 | 14 | 28.58 |
| 37 | 743 | 12 | 24.71 | 945 | 14 | 28.10 |
| 38 | 731 | 12 | 24.11 | 931 | 14 | 27.52 |
| 39 | 719 | 13 | 23.50 | 917 | 15 | 26.93 |
| 40 | 706 | 13 | 22.92 | 902 | 15 | 26.37 |
| 41 | 693 | 14 | 22.33 | 887 | 15 | 25.80 |
| 42 | 679 | 14 | 21.65 | 872 | 15 | 25.48 |
| 43 | 665 | 15 | 21.24 | 857 | 14 | 24.67 |
| 44 | 650 | 15 | 20.72 | 843 | 15 | 24.07 |
| 45 | 635 | 15 | 20.20 | 828 | 15 | 23.50 |
| 46 | 620 | 15 | 19.67 | 813 | 15 | 22.93 |
| 47 | 605 | 15 | 19.15 | 798 | 15 | 22.35 |
| 48 | 590 | 16 | 18.62 | 783 | 16 | 21.75 |
| 49 | 574 | 16 | 18.13 | 767 | 15 | 21.21 |
| 50 | 558 | 16 | 17.64 | 752 | 15 | 20.61 |
| 51 | 542 | 16 | 17.14 | 737 | 14 | 20.03 |
| 52 | 526 | 16 | 16.65 | 723 | 14 | 19.41 |
| 53 | 510 | 16 | 16.15 | 709 | 14 | 18.78 |
| 54 | 494 | 15 | 15.66 | 695 | 14 | 18.31 |
| 55 | 479 | 14 | 15.14 | 681 | 13 | 17.52 |
| 56 | 465 | 14 | 14.58 | 668 | 13 | 16.85 |
| 57 | 451 | 14 | 14.01 | 655 | 13 | 16.17 |
| 58 | 437 | 14 | 13.22 | 642 | 15 | 15.49 |
| 59 | 423 | 16 | 12.88 | 627 | 15 | 14.85 |
| 60 | 407 | 19 | 12.37 | 612 | 20 | 14.20 |
| 61 | 388 | 22 | 11.94 | 592 | 25 | 13.66 |
| 62 | 366 | 22 | 11.63 | 567 | 25 | 13.24 |
| 63 | 344 | 22 | 11.34 | 542 | 25 | 12.83 |
| 64 | 322 | 20 | 11.09 | 517 | 21 | 12.43 |
| 65 | 302 | 16 | 10.78 | 496 | 17 | 11.93 |
| 66 | 286 | 13 | 10.36 | 479 | 15 | 11.34 |
| 67 | 273 | 11 | 9.83 | 464 | 15 | 10.69 |
| 68 | 262 | 11 | 9.22 | 449 | 16 | 10.03 |

Table XVII.—*continued.*

| | Males. | | | Females. | | |
|---|---|---|---|---|---|---|
| Age. | Living. | Decr. | Expectat. | Living. | Decr. | Expectat. |
| 69 | 251 | 13 | 8.61 | 433 | 20 | 9.38 |
| 70 | 238 | 16 | 8.05 | 413 | 25 | 8.81 |
| 71 | 222 | 22 | 7.60 | 388 | 30 | 8.35 |
| 72 | 200 | 22 | 7.38 | 358 | 30 | 8.01 |
| 73 | 178 | 21 | 7.23 | 328 | 30 | 7.69 |
| 74 | 157 | 18 | 7.13 | 298 | 27 | 7.42 |
| 75 | 139 | 15 | 6.99 | 271 | 23 | 7.11 |
| 76 | 124 | 12 | 6.77 | 248 | 22 | 6.72 |
| 77 | 112 | 11 | 6.44 | 226 | 21 | 6.33 |
| 78 | 101 | 11 | 6.19 | 205 | 21 | 5.93 |
| 79 | 90 | 10 | 5.77 | 184 | 21 | 5.55 |
| 80 | 80 | 10 | 5.43 | 163 | 21 | 5.20 |
| 81 | 70 | 10 | 5.14 | 142 | 21 | 4.89 |
| 82 | 60 | 9 | 4.91 | 121 | 21 | 4.66 |
| 83 | 51 | 8 | 4.69 | 100 | 21 | 4.53 |
| 84 | 43 | 7 | 4.47 | 79 | 18 | 4.60 |
| 85 | 36 | 6 | 4.25 | 61 | 12 | 4.81 |
| 86 | 30 | 5 | 4.00 | 49 | 8 | 4.86 |
| 87 | 25 | 4 | 3.70 | 41 | 6 | 4.72 |
| 88 | 21 | 4 | 3.30 | 35 | 4 | 4.44 |
| 89 | 17 | 3 | 2.97 | 31 | 4 | 3.95 |
| 90 | 14 | 3 | 2.50 | 27 | 4 | 3.46 |
| 91 | 11 | 3 | 2.04 | 23 | 4 | 2.98 |
| 92 | 8 | 3 | 1.62 | 19 | 4 | 2.50 |
| 93 | 5 | 2 | 1.30 | 15 | 4 | 2.03 |
| 94 | 3 | 2 | 0.83 | 11 | 4 | 1.59 |
| 95 | 1 | 1 | 0.50 | 7 | 3 | 1.21 |
| 96 | - - | - - | - - | 4 | 3 | 0.75 |
| 97 | - - | - - | - - | 1 | 1 | 0.50 |

# TABLE XVIII.

he present Value of £1 to be received at the End of any Number of Years, not exceeding 100; discounting at the Rates of 3, 4, 5, and 6 per Cent. Compound Interest.

| Years. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 1 | .970874 | .961538 | .952381 | .943396 |
| 2 | .942596 | .924556 | .907029 | .889996 |
| 3 | .915142 | .888996 | .863838 | .839619 |
| 4 | .888487 | .854804 | .822702 | .792094 |
| 5 | .862609 | .821927 | .783526 | .747258 |
| 6 | .837484 | .790315 | .746215 | .704961 |
| 7 | .813092 | .759918 | .710681 | .665057 |
| 8 | .789409 | .730690 | .676839 | .627412 |
| 9 | .766417 | .702587 | .644609 | .591898 |
| 10 | .744094 | .675564 | .613913 | .558395 |
| 11 | .722421 | .649581 | .584679 | .526788 |
| 12 | .701380 | .624597 | .556837 | .496969 |
| 13 | .680951 | .600574 | .530321 | .468839 |
| 14 | .661118 | .577475 | .505068 | .442301 |
| 15 | .641862 | .555265 | .481017 | .417265 |
| 16 | .623167 | .533908 | .458112 | .393646 |
| 17 | .605016 | .513373 | .436297 | .371364 |
| 18 | .587395 | .493628 | .415521 | .350344 |
| 19 | .570286 | .474642 | .395734 | .330513 |
| 20 | .553676 | .456387 | .376889 | .311805 |
| 21 | .537549 | .438834 | .358942 | .294155 |
| 22 | .521893 | .421955 | .341850 | .277505 |
| 23 | .506692 | .405726 | .325571 | .261797 |
| 24 | .491934 | .390121 | .310068 | .246979 |
| 25 | .477606 | .375117 | .295303 | .232999 |
| 26 | .463695 | .360689 | .281241 | .219810 |
| 27 | .450189 | .346817 | .267848 | .207368 |
| 28 | .437077 | .333477 | .255094 | .195630 |
| 29 | .424346 | .320651 | .242946 | .184557 |
| 30 | .411987 | .308319 | .231377 | .174110 |
| 31 | .399987 | .296460 | .220359 | .164255 |
| 32 | .388337 | .285058 | .209866 | .154957 |

## Table XVIII.—*continued.*

| Years. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 33 | .377026 | .274094 | .199873 | .146186 |
| 34 | .366045 | .263552 | .190355 | .137912 |
| 35 | .355383 | .253415 | .181290 | .130105 |
| 36 | .345032 | .243669 | .172657 | .122741 |
| 37 | .334983 | .234297 | .164436 | .115793 |
| 38 | .325226 | .225285 | .156605 | .109239 |
| 39 | .315754 | .216621 | .149148 | .103056 |
| 40 | .306557 | .208289 | .142046 | .097222 |
| 41 | .297628 | .200278 | .135282 | .091719 |
| 42 | .288959 | .192575 | .128840 | .086527 |
| 43 | .280543 | .185168 | .122704 | .081630 |
| 44 | .272372 | .178046 | .116861 | .077009 |
| 45 | .264439 | .171198 | .111297 | .072650 |
| 46 | .256737 | .164614 | .105997 | .068538 |
| 47 | .249259 | .158283 | .100949 | .064658 |
| 48 | .241999 | .152195 | .096142 | .060998 |
| 49 | .234950 | .146341 | .091564 | .057546 |
| 50 | .228107 | .140713 | .087204 | .054288 |
| 51 | .221463 | .135301 | .083051 | .051215 |
| 52 | .215013 | .130097 | .079096 | .048316 |
| 53 | .208750 | .125093 | .075330 | .045582 |
| 54 | .202670 | .120282 | .071743 | .043001 |
| 55 | .196767 | .115656 | .068326 | .040567 |
| 56 | .191036 | .111207 | .065073 | .038271 |
| 57 | .185472 | .106930 | .061974 | .036105 |
| 58 | .180070 | .102817 | .059023 | .034061 |
| 59 | .174825 | .098863 | .056212 | .032133 |
| 60 | .169733 | .095060 | .053536 | .030314 |
| 61 | .164789 | .091404 | .050986 | .028598 |
| 62 | .159990 | .087889 | .048558 | .026980 |
| 63 | .155330 | .084508 | .046246 | .025453 |
| 64 | .150806 | .081258 | .044044 | .024012 |
| 65 | .146413 | .078133 | .041946 | .022653 |
| 66 | .142149 | .075128 | .039949 | .021370 |
| 67 | .138009 | .072238 | .038047 | .020161 |
| 68 | .133989 | .069460 | .036235 | .019020 |
| 69 | .130086 | .066788 | .034509 | .017943 |
| 70 | .126297 | .064219 | .032866 | .016927 |

Table XVIII.—*continued.*

| Years. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 71 | .122619 | .061749 | .031301 | .015969 |
| 72 | .119047 | .059374 | .029811 | .015065 |
| 73 | .115580 | .057091 | .028391 | .014213 |
| 74 | .112214 | .054895 | .027039 | .013408 |
| 75 | .108945 | .052784 | .025752 | .012649 |
| 76 | .105772 | .050754 | .024525 | .011933 |
| 77 | .102691 | .048801 | .023357 | .011258 |
| 78 | .099700 | .046924 | .022245 | .010620 |
| 79 | .096796 | .045120 | .021186 | .010019 |
| 80 | .093977 | .043384 | .020177 | .009452 |
| 81 | .091240 | .041716 | .019216 | .008917 |
| 82 | .088582 | .040111 | .018301 | .008412 |
| 83 | .086002 | .038569 | .017430 | .007936 |
| 84 | .083497 | .037085 | .016600 | .007487 |
| 85 | .081065 | .035659 | .015809 | .007063 |
| 86 | .078704 | .034287 | .015056 | .006663 |
| 87 | .076412 | .032969 | .014339 | .006286 |
| 88 | .074186 | .031701 | .013657 | .005930 |
| 89 | .072026 | .030481 | .013006 | .005595 |
| 90 | .069928 | .029309 | .012387 | .005278 |
| 91 | .067891 | .028182 | .011797 | .004979 |
| 92 | .065914 | .027098 | .011235 | .004697 |
| 93 | .063994 | .026056 | .010700 | .004432 |
| 94 | .062130 | .025053 | .010191 | .004181 |
| 95 | .060320 | .024090 | .009705 | .003944 |
| 96 | .058563 | .023163 | .009243 | .003721 |
| 97 | .056858 | .022272 | .008803 | .003510 |
| 98 | .055202 | .021416 | .008384 | .003312 |
| 99 | .053594 | .020592 | .007985 | .003124 |
| 100 | .052033 | .019800 | .007604 | .002947 |

# TABLE XIX.

The present Value of an Annuity of One Pound for any Number of Years not exceeding 100, at the several Rates of 3, 4, 5, and 6*l.* per Cent.

| Years. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 1 | .9708 | .9615 | .9523 | .9433 |
| 2 | 1.9134 | 1.8861 | 1.8594 | 1.8333 |
| 3 | 2.8286 | 2.7751 | 2.7232 | 2.6730 |
| 4 | 3.7170 | 3.6299 | 3.5459 | 3.4651 |
| 5 | 4.5797 | 4.4518 | 4.3294 | 4.2123 |
| 6 | 5.4171 | 5.2421 | 5.0756 | 4.9173 |
| 7 | 6.2302 | 6.0020 | 5.7863 | 5.5823 |
| 8 | 7.0196 | 6.7327 | 6.4632 | 6.2097 |
| 9 | 7.7861 | 7.4353 | 7.1078 | 6.8016 |
| 10 | 8.5302 | 8.1109 | 7.7217 | 7.3600 |
| 11 | 9.2526 | 8.7605 | 8.3064 | 7.8868 |
| 12 | 9.9540 | 9.3850 | 8.8632 | 8.3838 |
| 13 | 10.6349 | 9.9856 | 9.3935 | 8.8526 |
| 14 | 11.2960 | 10.5631 | 9.8986 | 9.2949 |
| 15 | 11.9379 | 11.1184 | 10.3796 | 9.7122 |
| 16 | 12.5611 | 11.6523 | 10.8377 | 10.1058 |
| 17 | 13.1661 | 12.1656 | 11.2740 | 10.4772 |
| 18 | 13.7535 | 12.6593 | 11.6895 | 10.8276 |
| 19 | 14.3238 | 13.1339 | 12.0853 | 11.1581 |
| 20 | 14.8774 | 13.5903 | 12.4622 | 11.4699 |
| 21 | 15.4150 | 14.0291 | 12.8211 | 11.7640 |
| 22 | 15.9369 | 14.4511 | 13.1630 | 12.0415 |
| 23 | 16.4436 | 14.8568 | 13.4885 | 12.3033 |
| 24 | 16.9355 | 15.2469 | 13.7986 | 12.5503 |
| 25 | 17.4131 | 15.6220 | 14.0939 | 12.7833 |
| 26 | 17.8768 | 15.9827 | 14.3751 | 13.0031 |
| 27 | 18.3270 | 16.3295 | 14.6430 | 13.2105 |
| 28 | 18.7641 | 16.6630 | 14.8981 | 13.4061 |
| 29 | 19.1884 | 16.9837 | 15.1410 | 13.5907 |
| 30 | 19.6004 | 17.2920 | 15.3724 | 13.7648 |

Table XIX.—*continued.*

| Years. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 31 | 20.0004 | 17.5884 | 15.5928 | 13.9290 |
| 32 | 20.3887 | 17.8735 | 15.8026 | 14.0840 |
| 33 | 20.7657 | 18.1476 | 16.0025 | 14.2302 |
| 34 | 21.1318 | 18.4111 | 16.1929 | 14.3681 |
| 35 | 21.4872 | 18.6646 | 16.3741 | 14.4982 |
| 36 | 21.8322 | 18.9082 | 16.5468 | 14.6209 |
| 37 | 22.1672 | 19.1425 | 16.7112 | 14.7367 |
| 38 | 22.4924 | 19.3678 | 16.8678 | 14.8460 |
| 39 | 22.8082 | 19.5844 | 17.0170 | 14.9490 |
| 40 | 23.1147 | 19.7927 | 17.1590 | 15.0462 |
| 41 | 23.4124 | 19.9930 | 17.2943 | 15.1380 |
| 42 | 23.7013 | 20.1856 | 17.4232 | 15.2245 |
| 43 | 23.9819 | 20.3707 | 17.5459 | 15.3061 |
| 44 | 24.2542 | 20.5488 | 17.6627 | 15.3831 |
| 45 | 24.5187 | 20.7200 | 17.7740 | 15.4558 |
| 46 | 24.7754 | 20.8846 | 17.8800 | 15.5243 |
| 47 | 25.0247 | 21.0429 | 17.9810 | 15.5890 |
| 48 | 25.2667 | 21.1951 | 18.0771 | 15.6500 |
| 49 | 25.5016 | 21.3414 | 18.1687 | 15.7075 |
| 50 | 25.7297 | 21.4821 | 18.2559 | 15.7618 |
| 51 | 25.9512 | 21.6174 | 18.3389 | 15.8130 |
| 52 | 26.1662 | 21.7475 | 18.4180 | 15.8613 |
| 53 | 26.3749 | 21.8726 | 18.4934 | 15.9069 |
| 54 | 26.5776 | 21.9929 | 18.5651 | 15.9499 |
| 55 | 26.7744 | 22.1086 | 18.6334 | 15.9905 |
| 56 | 26.9654 | 22.2198 | 18.6985 | 16.0288 |
| 57 | 27.1509 | 22.3267 | 18.7605 | 16.0649 |
| 58 | 27.3310 | 22.4295 | 18.8195 | 16.0989 |
| 59 | 27.5058 | 22.5284 | 18.8757 | 16.1311 |
| 60 | 27.6755 | 22.6234 | 18.9292 | 16.1614 |
| 61 | 27.8403 | 22.7148 | 18.9802 | 16.1900 |
| 62 | 28.0003 | 22.8027 | 19.0288 | 16.2170 |
| 63 | 28.1556 | 22.8872 | 19.0750 | 16.2424 |
| 64 | 28.3064 | 22.9685 | 19.1191 | 16.2664 |
| 65 | 28.4528 | 23.0466 | 19.1610 | 16.2891 |

Table XIX.—*continued.*

| Years. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 66 | 28.5950 | 23.1218 | 19.2010 | 16.3104 |
| 67 | 28.7330 | 23.1940 | 19.2390 | 16.3306 |
| 68 | 28.8670 | 23.2635 | 19.2753 | 16.3496 |
| 69 | 28.9971 | 23.3302 | 19.3098 | 16.3676 |
| 70 | 29.1234 | 23.3945 | 19.3426 | 16.3845 |
| 71 | 29.2460 | 23.4562 | 19.3739 | 16.4005 |
| 72 | 29.3650 | 23.5156 | 19.4037 | 16.4155 |
| 73 | 29.4806 | 23.5727 | 19.4321 | 16.4297 |
| 74 | 29.5928 | 23.6276 | 19.4592 | 16.4431 |
| 75 | 29.7018 | 23.6804 | 19.4849 | 16.4558 |
| 76 | 29.8076 | 23.7311 | 19.5094 | 16.4677 |
| 77 | 29.9102 | 23.7799 | 19.5328 | 16.4790 |
| 78 | 30.0099 | 23.8268 | 19.5550 | 16.4896 |
| 79 | 30.1067 | 23.8720 | 19.5762 | 16.4996 |
| 80 | 30.2007 | 23.9153 | 19.5964 | 16.5091 |
| 81 | 30.2920 | 23.9571 | 19.6156 | 16,5180 |
| 82 | 30.3805 | 23.9972 | 19.6339 | 16.5264 |
| 83 | 30.4665 | 24.0357 | 19.6514 | 16.5343 |
| 84 | 30.5500 | 24.0728 | 19.6680 | 16.5418 |
| 85 | 30.6311 | 24.1085 | 19.6838 | 16.5489 |
| 86 | 30.7098 | 24.1428 | 19.6988 | 16.5556 |
| 87 | 30.7862 | 24.1757 | 19.7132 | 16.5618 |
| 88 | 30.8604 | 24.2074 | 19.7268 | 16.5678 |
| 89 | 30.9324 | 24.2379 | 19.7398 | 16.5734 |
| 90 | 31.0024 | 24.2672 | 19.7522 | 16.5787 |
| 91 | 31.0703 | 24.2954 | 19.7640 | 16.5836 |
| 92 | 31.1362 | 24.3225 | 19.7752 | 16.5883 |
| 93 | 31.2002 | 24.3486 | 19.7859 | 16.5928 |
| 94 | 31.2623 | 24.3736 | 19.7961 | 16.5969 |
| 95 | 31.3226 | 24.3977 | 19.8058 | 16.6009 |
| 96 | 31.3812 | 24.4209 | 19.8151 | 16.6046 |
| 97 | 31.4380 | 24.4431 | 19.8239 | 16.6081 |
| 98 | 31.4932 | 24.4646 | 19.8323 | 16.6114 |
| 99 | 31.5468 | 24.4852 | 19.8403 | 16.6145 |
| 100 | 31.5989 | 24.5050 | 19.8479 | 16.6175 |
| Perpetuity. | 33.3333 | 25.0000 | 20.0000 | 16.6666 |

# TABLE XX.

Shewing the Sum to which £1 Principal will increase at Compound Interest in any Number of Years not exceeding a Hundred.

| Yrs. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 1 | 1.030,000 | 1.040,000 | 1.050,000 | 1.060,000 |
| 2 | 1.060,900 | 1.081,600 | 1.102,500 | 1.123,600 |
| 3 | 1.092,727 | 1.124,864 | 1.157,625 | 1.191,016 |
| 4 | 1.125,508 | 1.169,858 | 1.215,506 | 1.262,476 |
| 5 | 1.159,274 | 1.216,652 | 1.276,281 | 1.338,225 |
| 6 | 1.194,052 | 1.265,319 | 1.340,095 | 1.418,519 |
| 7 | 1.229,873 | 1.315,931 | 1.407,100 | 1.503,630 |
| 8 | 1.266,770 | 1.368,569 | 1.477,455 | 1.593,848 |
| 9 | 1.304,773 | 1.423,311 | 1.551,328 | 1.689,478 |
| 10 | 1.343,916 | 1.480,244 | 1.628,894 | 1.790,847 |
| 11 | 1.384,233 | 1.539,454 | 1.710,339 | 1.898,298 |
| 12 | 1.425,760 | 1.601,032 | 1.795,856 | 2.012,196 |
| 13 | 1.468,533 | 1.665,073 | 1.885,649 | 2.132,928 |
| 14 | 1.512,589 | 1.731,676 | 1.979,931 | 2.260,903 |
| 15 | 1.557,967 | 1.800,943 | 2.078,928 | 2.396,558 |
| 16 | 1.604,706 | 1.872,981 | 2.182,874 | 2.540,351 |
| 17 | 1.652,847 | 1.947,900 | 2.292,018 | 2.692,772 |
| 18 | 1.702,433 | 2.025,816 | 2.406,619 | 2.854,339 |
| 19 | 1.753,506 | 2.106,849 | 2.526,950 | 3.025,599 |
| 20 | 1.806,111 | 2.191,123 | 2.653,297 | 3.207,135 |
| 21 | 1.860,294 | 2.278,768 | 2.785,962 | 3.399,563 |
| 22 | 1.916,103 | 2.369,918 | 2.925,260 | 3.603,537 |
| 23 | 1.973,586 | 2.464,715 | 3.071,523 | 3.819,749 |
| 24 | 2.032,794 | 2.563,304 | 3.225,099 | 4.048,934 |
| 25 | 2.093,777 | 2.665,836 | 3.386,354 | 4.291,870 |
| 26 | 2.156,591 | 2.772,469 | 3.555,672 | 4.549,382 |
| 27 | 2.221,289 | 2.883,368 | 3.733,456 | 4.822,345 |
| 28 | 2.287,927 | 2.998,703 | 3.920,129 | 5.111,686 |
| 29 | 2.356,565 | 3.118,651 | 4.116,135 | 5.418,387 |
| 30 | 2.427,262 | 3.243,397 | 4.321,942 | 5.743,491 |
| 31 | 2.500,080 | 3.373,133 | 4.538,039 | 6.088,100 |
| 32 | 2.575,082 | 3.508,058 | 4.764,941 | 6.453,386 |
| 33 | 2.652,335 | 3.648,381 | 5.003,188 | 6.840,589 |
| 34 | 2.731,905 | 3.794,316 | 5.253,347 | 7.251,025 |
| 35 | 2.813,862 | 3.946,088 | 5.516,015 | 7.686,086 |
| 36 | 2.898,278 | 4.103,932 | 5.791,816 | 8.147,252 |

## Table XX.—*continued.*

| Yrs. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 37 | 2.985,226 | 4.268,089 | 6.081,406 | 8.636,087 |
| 38 | 3.074,783 | 4.438,813 | 6.385,477 | 9.154,252 |
| 39 | 3.167,026 | 4.616,365 | 6.704,751 | 9.703,507 |
| 40 | 3.262,037 | 4.801,020 | 7.039,988 | 10.285,717 |
| 41 | 3.359,898 | 4.993,061 | 7.391,988 | 10.902,861 |
| 42 | 3.460,695 | 5.192,783 | 7.761,587 | 11.557,032 |
| 43 | 3.564,516 | 5.400,495 | 8.149,666 | 12.250,454 |
| 44 | 3.671,452 | 5.616,515 | 8.557,150 | 12.985,481 |
| 45 | 3.781,595 | 5.841,175 | 8.985,007 | 13.764,610 |
| 46 | 3.895,043 | 6.074,822 | 9.434,258 | 14.590,487 |
| 47 | 4.011,895 | 6.317,815 | 9.905,971 | 15.465,916 |
| 48 | 4.132,251 | 6.570,528 | 10.401,269 | 16.393,871 |
| 49 | 4.256,219 | 6,833,349 | 10.921,333 | 17.377,504 |
| 50 | 4.383,906 | 7.106,683 | 11.467,399 | 18.420,154 |
| 51 | 4.515,423 | 7.390,950 | 12.040,769 | 19.525,363 |
| 52 | 4.650,885 | 7.686,588 | 12.642,808 | 20.696,885 |
| 53 | 4.790,412 | 7.994,052 | 13.274,948 | 21.938,698 |
| 54 | 4.934,124 | 8.313,814 | 13.938,696 | 23.255,020 |
| 55 | 5.082,148 | 8.646,366 | 14.635,630 | 24.650,321 |
| 56 | 5.234,613 | 8.992,221 | 15.367,412 | 26.129,340 |
| 57 | 5.391,651 | 9.351,910 | 16.135,783 | 27.697,101 |
| 58 | 5.553,400 | 9.725,986 | 16.942,572 | 29.358,927 |
| 59 | 5.720,003 | 10.115,026 | 17.789,700 | 31.120,463 |
| 60 | 5.891,603 | 10.519,627 | 18.679,185 | 32.987,690 |
| 61 | 6.068,351 | 10.940,412 | 19.613,145 | 34.966,952 |
| 62 | 6.250,401 | 11.378,029 | 20.593,802 | 37.064,969 |
| 63 | 6.437,913 | 11.833,150 | 21.623,492 | 39.288,867 |
| 64 | 6.631,051 | 12.306,476 | 22.704,667 | 41.646,199 |
| 65 | 6.829,982 | 12.798,735 | 23.839,900 | 44.144,971 |
| 66 | 7.034,882 | 13.310,684 | 25.031,895 | 46.793,669 |
| 67 | 7.245,928 | 13.843,112 | 26.283,490 | 49.601,290 |
| 68 | 7.463,306 | 14.396,836 | 27.597,664 | 52.577,367 |
| 69 | 7.687,205 | 14.972,709 | 28.977,548 | 55.732,009 |
| 70 | 7.917,821 | 15.571,618 | 30.426,425 | 59.075,930 |
| 71 | 8.155,356 | 16.194,483 | 31.947,746 | 62.620,485 |
| 72 | 8.400,017 | 16.842,262 | 33.545,134 | 66.377,715 |
| 73 | 8.652,017 | 17.515,952 | 35,222,390 | 70.360,378 |
| 74 | 8.911,578 | 18.216,591 | 36.983,510 | 74.582,000 |
| 75 | 9.178,925 | 18.945,254 | 38.832,685 | 79.056,920 |
| 76 | 9.454,293 | 19.703,064 | 40.774,320 | 83.800,336 |
| 77 | 9.737,922 | 20.491,187 | 42.813,036 | 88.828,356 |

Table XX.—*continued.*

| Yrs. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 78 | 10.030,059 | 21.310,834 | 44.953,688 | 94.158,057 |
| 79 | 10.330,961 | 22.163,268 | 47.201,372 | 99.807,541 |
| 80 | 10.640,890 | 23.049,799 | 49.561,441 | 105.795,993 |
| 81 | 10.960,117 | 23.971,791 | 52.039,513 | 112.143,753 |
| 82 | 11.288,920 | 24.930,662 | 54.641,488 | 118.872,378 |
| 83 | 11.627,588 | 25.927,889 | 57.373,563 | 126.004,720 |
| 84 | 11.976,416 | 26.965,004 | 60.242,241 | 133.565,004 |
| 85 | 12.335,708 | 28.043,604 | 63.254,353 | 141.578,904 |
| 86 | 12.705,779 | 29.165,349 | 66.417,071 | 150.073,638 |
| 87 | 13.086,953 | 30.331,963 | 69.737,924 | 159.078,057 |
| 88 | 13.479,561 | 31.545,241 | 73.224,820 | 168.622,740 |
| 89 | 13.883,948 | 32.807,051 | 76.886,061 | 178.740,104 |
| 90 | 14.300,467 | 34.119,333 | 80.730,365 | 189.464,511 |
| 91 | 14.729,481 | 35.484,106 | 84.766,883 | 200.832,381 |
| 92 | 15.171,365 | 36.903,470 | 89.005,227 | 212.882,324 |
| 93 | 15.626,506 | 38.379,609 | 93.455,488 | 225.655,264 |
| 94 | 16.095,301 | 39.914,794 | 98.128,263 | 239.194,580 |
| 95 | 16.578,160 | 41.511,385 | 103.034,676 | 253.546,254 |
| 96 | 17.075,505 | 43.171,841 | 108.186,410 | 268.759,030 |
| 97 | 17.587,770 | 44.898,715 | 113.595,730 | 284.884,572 |
| 98 | 18.115,403 | 46.694,663 | 119.275,517 | 301.977,646 |
| 99 | 18.658,866 | 48.562,450 | 125.239,293 | 320.096,305 |
| 100 | 19.218,631 | 50.504,948 | 131.501,257 | 339.302,083 |

# TABLE XXI.

Shewing the Sum to which £1 per Annum will increase at Compound Interest in any Number of Years not exceeding a Hundred.

| Yrs. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 1 | 1.000,000 | 1.000,000 | 1.000,000 | 1.000,000 |
| 2 | 2.030,000 | 2.040,000 | 2.050,000 | 2.060,000 |
| 3 | 3.090,900 | 3.121,600 | 3.152,500 | 3.183,600 |
| 4 | 4.183,627 | 4.246,464 | 4.310,125 | 4.374,616 |
| 5 | 5.309,135 | 5.416,322 | 5.525,631 | 5.637,092 |
| 6 | 6.468,409 | 6.632,975 | 6.801,912 | 6.975,318 |
| 7 | 7.662,462 | 7.898,294 | 8.142,008 | 8.393,837 |
| 8 | 8.892,336 | 9.214,226 | 9.549,108 | 9.897,467 |
| 9 | 10.159,106 | 10.582,795 | 11.026,564 | 11.491,315 |
| 10 | 11.463,879 | 12.006,107 | 12.577,892 | 13.180,794 |
| 11 | 12.807,795 | 13.486,351 | 14.206,787 | 14.971,642 |
| 12 | 14.192,029 | 15.025,805 | 15.917,126 | 16.869,941 |
| 13 | 15.617,790 | 16.626,837 | 17.712,982 | 18.882,137 |
| 14 | 17.086,324 | 18.291,911 | 19.598,631 | 21.015,065 |
| 15 | 18.598,913 | 20.023,587 | 21.578,563 | 23.275,969 |
| 16 | 20.156,881 | 21.824,531 | 23.657,491 | 25.672,528 |
| 17 | 21.761,587 | 23.697,512 | 25.840,366 | 28.212,879 |
| 18 | 23.414,435 | 25.645,412 | 28.132,384 | 30.905,652 |
| 19 | 25.116,868 | 27.671,229 | 30.539,003 | 33.759,991 |
| 20 | 26.870,374 | 29.778,078 | 33.065,954 | 36.785,591 |
| 21 | 28.676,485 | 31.969,201 | 35.719,251 | 39.992,726 |
| 22 | 30.536,780 | 34.247,969 | 38.505,214 | 43.392,290 |
| 23 | 32.452,883 | 36.617,888 | 41.430,475 | 46.995,827 |
| 24 | 34.426,470 | 39.082,604 | 44.501,998 | 50.815,577 |
| 25 | 36.459,264 | 41.645,908 | 47.727,098 | 54.864,512 |
| 26 | 38.553,042 | 44.311,744 | 51.113,453 | 59.156,382 |
| 27 | 40.709,633 | 47.084,214 | 54.669,126 | 63.705,765 |
| 28 | 42.930,922 | 49.967,582 | 58.402,582 | 68.528,111 |
| 29 | 45.218,850 | 52.966,286 | 62.322,711 | 73.639,798 |
| 30 | 47.575,415 | 56.084,937 | 66.438,847 | 79.058,186 |
| 31 | 50.002.678 | 59.328,335 | 70.760,789 | 84.801,677 |
| 32 | 52.502,758 | 62.701,468 | 75.298,829 | 90.889,778 |
| 33 | 55.077,841 | 66.209,527 | 80.063,770 | 97.343,164 |
| 34 | 57.730,176 | 69.857,908 | 85.066,959 | 104.183,754 |
| 35 | 60.462,081 | 73.652,224 | 90.320,307 | 111.434,779 |
| 36 | 63.275,944 | 77.598,313 | 95.836,322 | 119.120,866 |

Table XXI.—*continued.*

| Yrs. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 37 | 66.174,222 | 81.702,246 | 101.628,138 | 127.268,118 |
| 38 | 69.159,449 | 85.970,336 | 107.709,545 | 135.904,205 |
| 39 | 72.234,232 | 90.409,149 | 114.095,023 | 145.058,458 |
| 40 | 75.401,259 | 95.025,515 | 120.799,774 | 154.761,965 |
| 41 | 78.663,297 | 99.826,536 | 127.839,762 | 165.047,683 |
| 42 | 82.023,196 | 104.819,597 | 135.231,751 | 175.950,544 |
| 43 | 85.483,892 | 110.012,381 | 142.993,338 | 187.507,577 |
| 44 | 89.048,409 | 115.412,876 | 151.143,005 | 199.758,031 |
| 45 | 92.719,861 | 121.029,392 | 159.700,155 | 212.743,513 |
| 46 | 96.501,457 | 126.870,567 | 168.685,163 | 226.508,124 |
| 47 | 100.396,500 | 132.945,390 | 178.119,421 | 241.098,612 |
| 48 | 104.408,395 | 139.263,206 | 188.025,392 | 256.564,528 |
| 49 | 108.540,647 | 145.833,734 | 198.426,662 | 272.958,400 |
| 50 | 112.796,867 | 152.667,083 | 209.347,995 | 290.335,904 |
| 51 | 117.180,773 | 159.773,767 | 220.815,395 | 308.756,058 |
| 52 | 121.696,196 | 167.164,717 | 232.856,165 | 328.281,422 |
| 53 | 126.347,082 | 174.851,306 | 245.498,973 | 348.978,307 |
| 54 | 131.137,494 | 182.845,358 | 258.773,922 | 370.917,006 |
| 55 | 136.071,619 | 191.159,173 | 272.712,618 | 394.172,026 |
| 56 | 141.153,768 | 199.805,539 | 287.348,249 | 418.822,348 |
| 57 | 146.388,381 | 208.797,761 | 302.715,661 | 444.951,689 |
| 58 | 151.780,032 | 218.149,672 | 318.851,444 | 472.648,790 |
| 59 | 157.333,433 | 227.875,658 | 335.794,017 | 502.007,717 |
| 60 | 163.053,436 | 237.990,685 | 353.583,717 | 533.128,180 |
| 61 | 168.945,039 | 248.510,312 | 372.262,903 | 566.115,871 |
| 62 | 175.013,391 | 259.450,725 | 391.876,048 | 601.082,824 |
| 63 | 181.263,792 | 270.828,754 | 412.469,851 | 638.147,793 |
| 64 | 187.701,706 | 282.661,904 | 434.093,343 | 677.436,661 |
| 65 | 194.332,757 | 294.968,380 | 456.798,011 | 719.082,860 |
| 66 | 201.162,740 | 307.767,115 | 480.637,911 | 763.227,832 |
| 67 | 208.197,622 | 321.077,800 | 505.669,807 | 810.021,502 |
| 68 | 215.443,551 | 334.920,912 | 531.953,297 | 859.622,792 |
| 69 | 222.906,858 | 349.317,748 | 559.550,962 | 912,200,160 |
| 70 | 230.594,063 | 364.290,458 | 588.528,510 | 967.932,169 |
| 71 | 238.511,885 | 379.862,077 | 618.954,936 | 1027.008,099 |
| 72 | 246.667,242 | 396.056,560 | 650.902,683 | 1089.628,585 |
| 73 | 255.067,259 | 412.898,822 | 684.447,817 | 1156.006,300 |
| 74 | 263.719,277 | 430.414,775 | 719.670,208 | 1226.366,679 |
| 75 | 272.630,855 | 448.631,366 | 756.653,718 | 1300.948,679 |
| 76 | 281.809,781 | 467.576,621 | 795.486,404 | 1380.005,600 |
| 77 | 291.264,074 | 487.279,686 | 836.260,724 | 1463.805,936 |
| 78 | 301.001,996 | 507.770,873 | 879.073,760 | 1552.634,292 |

## Table XXI.—*continued.*

| Yrs. | 3 per Cent. | 4 per Cent. | 5 per Cent. | 6 per Cent. |
|---|---|---|---|---|
| 79 | 311.032,056 | 529.081,708 | 924.027,448 | 1646.792,350 |
| 80 | 321.363,018 | 551.244,976 | 971.228,821 | 1746.599,891 |
| 81 | 332.003,909 | 574.294,775 | 1020.790,262 | 1852.395,884 |
| 82 | 342.964,026 | 598.266,566 | 1072.829,775 | 1964.539,637 |
| 83 | 354.252,947 | 623.197,229 | 1127.471,264 | 2083.412,016 |
| 84 | 365.880,535 | 649.125,118 | 1184.844,827 | 2209.416,737 |
| 85 | 377.856,951 | 676.090,123 | 1245.087,068 | 2342.981,741 |
| 86 | 390.192,660 | 704.133,728 | 1308.341,422 | 2484.560,645 |
| 87 | 402.898,440 | 733.299,077 | 1374.758,493 | 2634.634,284 |
| 88 | 415.985,393 | 763.631,040 | 1444.496,418 | 2793.712,341 |
| 89 | 429.464,955 | 795.176,282 | 1517.721,238 | 2962.335,082 |
| 90 | 443.348,903 | 827.983,333 | 1594.607,300 | 3141.075,187 |
| 91 | 457.649,370 | 862.102,667 | 1675.337,665 | 3330.539,698 |
| 92 | 472.378,851 | 897.586,773 | 1760.104,549 | 3531.372,080 |
| 93 | 487.550,217 | 934.490,244 | 1849.109,776 | 3744.254,405 |
| 94 | 503.176,723 | 972.869,854 | 1942.565,265 | 3969.909,669 |
| 95 | 519.272,025 | 1012.784,648 | 2040.693,528 | 4209.104,249 |
| 96 | 535.850,186 | 1054.296,034 | 2143.728,205 | 4462.650,504 |
| 97 | 552.925,692 | 1097.467,875 | 2251.914,615 | 4731.409,534 |
| 98 | 570.513,462 | 1142.366,590 | 2365.510,346 | 5016.294,106 |
| 99 | 588.628,866 | 1189.061,254 | 2484.785,863 | 5318.271,753 |
| 100 | 607.287,732 | 1237.623,704 | 2610.025,156 | 5638.368,058 |

# APPENDIX.

THE last four tables may be found in most of the books on compound interest; and though it forms no part of my design in this work to enter fully into that subject, it may not be improper to give some explanation of the principles on which they are computed; more especially as constant recourse is had to one or other of them in the solution of the different problems in the doctrine of life annuities and reversions.

Let $r$ be £1 increased by its interest for a year, then will £1 be the present value of $r$ to be received at the end of that time; and as $r$ is to 1, so is 1 to $\frac{1}{r}$*, the present value of £1 to be received at the end of a year. If the payment is postponed to the end of two years, $r$ must be increased by its interest for a year, and will amount to $r + r \times \overline{r - 1} = r^2$. And as $r^2$ is to 1, so is 1 to $\frac{1}{r^2}$, the present value of £1 to be received at the end of two years. In like manner, if the payment is deferred for three years,

* If $r$ denote the *interest* only of £1, the value of £1 payable at the end of 1, 2, 3, &c. years, will be $\frac{1}{1+r}$, $\frac{1}{\overline{1+r}|^2}$, $\frac{1}{\overline{1+r}|^3}$, &c.; and the amount of £1 improved, at compound interest, at the end of 1, 2, 3, &c. years, will be $\overline{1+r}$, $\overline{1+r}|^2$, $\overline{1+r}|^3$, &c.

$r^2$ must be increased by its interest for a year, and will amount to $r^2 + r^2 \times \overline{r-1} = r^3$; and $r^3$ will be to 1 as 1 to $\frac{1}{r^3}$, the present value of £1 to be received at the end of three years. By proceeding in this manner, the value of £1 to be received at the end of 4, 5, 6, &c. years, will be $\frac{1}{r^4}, \frac{1}{r^5}, \frac{1}{r^6}$, &c. The numbers, therefore, in the different columns of the 18th Table are found by dividing £1 by $r, r^2, r^3$, &c.: thus, if the rate of interest be 3 per cent., the first number is $\frac{1}{1.03} = .9708$; the second number $\frac{1}{\overline{1.03}|^2} = .9426$; the third number $\frac{1}{\overline{1.03}|^3} = .9151$, and so on.

As the value of an annuity certain for any number of years consists of the values of £1 payable at the end of each year in that term, the numbers in the 19th Table are the sums of the values in the 18th Table; hence the value of an annuity certain for three years at 3 per cent. is the sum of the three first numbers in the 18th Table, that is $.9708 + .9426 + .9151 = 2.8285$. For four years it is $2.8285 + .8885 = 3.7170$; and so on for a longer term, and at a different rate of interest. The numbers in the 20th Table, giving the amount of £1 with its accumulated interest in any number of years, are the several powers of $r$, which have been found above to express that amount; that is, they are $r, r^2, r^3, r^4$, &c., according as the term is 1, 2, 3, 4, &c. years. Thus, for the term of four years, and at 4 per cent. interest, the fourth number in the second column is $\overline{1.04}|^4 = 1.16986$; for the term of six years it is $\overline{1.04}|^6 = 1.26532$, &c. And the numbers in the 21st Table, giving the amount of an annuity with its accumulated interest in any number of years, are evidently the

*sum* of the different powers of $r$ given in Tab. 19th; thus, the amount of £1 per annum for three years is $1 + r + r^2$; for four years $1 + r + r^2 + r^3$, and so on. Supposing then the term to be three years, and the rate of interest 3 per cent., the amount will be 3.0909, as in Tab. 21st, or the sum of £1 added to the first two numbers in the 20th Table. If the term be four years, it will be 4.18363, as in Tab. 21st, or £1 added to the first three numbers in Tab. 20th.

By the assistance of the foregoing tables, an answer to most of the cases in compound interest may be easily obtained; and in order to exemplify this, I shall just insert the following:

I. Supposing an estate subject to a fine of £100, to be paid at the end of every 20 years from the present time, and it were required to know the value of those fines for 100 years, at 5 per cent.—By Tab. 18th, the values of £1 payable at the end of 20, 40, 60, 80, and 100 years, are respectively equal to .3769, .1420, .0535, .0202, and .0076, amounting to .5002; which being multiplied into 100, produces £50.02 for the value of those fines. If their value for ever is required, it may be obtained from the 20th Table, by dividing unity by the amount of £1 in twenty years, lessened by unity, and multiplying the quotient into the given fine. Thus, the amount of £1 in 20 years by Table 20th is 2.6533, and unity divided by 1.6533 quotes .6048; the value, therefore, of the fines for ever is £60.48.

II. Supposing the value were required of an annuity of £100 at 4 per cent., for a term of 20 years, to commence at the expiration of 10 years. By Tab. 19th, the value of an annuity for 30 years, at 4 per cent., is 17.292, and the value of an annuity for 10 years is 8.111; their difference

is 9.181, and consequently the value of this reversionary annuity is £918.100. If the value of this annuity for ever after a term of ten years had been required, 8.111 must have been deducted from 25 (the perpetuity), and the answer in that case would have been £1688.9.

III. Supposing it were required to ascertain the sum which should be annually applied to the extinction of a debt of £1000 in fifteen years, at 5 per cent. In this case, recourse must be had to Tab. 21st, from which it appears that £1 per annum in fifteen years, at 5 per cent., will amount to 21.758. Dividing, therefore, 1000 by 21.758, we have £45.96 for the answer.

A great variety of other cases might be stated; but these are sufficient for my present purpose. If the values are required at a rate of interest different from either of those rates at which the tables are computed, it becomes necessary to adopt other methods; and the following theorems, containing the solution of some of the most important cases, are inserted with the view of giving a general, rather than a complete view of the subject *.

Let $p$ be the present value of an annuity $a$, $n$ the number of payments at $r$ interest; 1st. If $a$, $n$, and $r$ are given, $p$ will be $= \frac{a.\overline{1+r}|^{n}-1}{r.\overline{1+r}|^{n}}$. 2d. If $n$, $p$, $r$ are given, $a$ will

* The values in the 19th and 21st Tables are found by a shorter process than by the addition of the several terms in the 18th and 20th Tables. If $r$ be £1 increased by its interest for a year, the value of an annuity certain of £1 for $n$ years will be $= \frac{1}{r-1} - \frac{1}{r^{n}.\overline{r-1}}$; and the amount of an annuity of £1 in $n$ years will be $= \frac{r^{n}-1}{r-1}$.

be $= \frac{pr.\overline{1+r}|^n}{\overline{1+r}|^n-1}$. 3d. If $a$, $p$, $r$ are given, $n$ will be $=$ $\frac{\text{Log. B}}{\text{Log. }\overline{1+r}}$, B being $= \frac{a}{a-pr}$. 4th. If $a$, $p$, $n$ are given, $r$ will be $= \frac{6-\sqrt{36-\overline{n-1}.12.\overline{G-1}}}{n-1}$, G being $= \overline{\frac{an}{p}}\big|^{\frac{2}{n+1}}$.

Again, let $m$ be the amount of an annuity $a$ for $n$ years at $r$ interest; 1st. Supposing $a$, $n$, $r$ to be given, $m$ will be $= \frac{a.\overline{1+r}|^n-1}{r}$; 2d. Supposing $m$, $p$, $r$ to be given, $a$ will be $= \frac{mr}{\overline{1+r}|^n-1}$; 3d. Supposing $a$, $m$, $r$ to be given, $n$ will be $= \frac{\text{Log. C}}{\text{Log. }\overline{1+r}}$, (C being $= \frac{mr+a}{a}$); and 4th. Supposing $a$, $m$, $n$ to be given, $r$ will be $= \frac{\sqrt{36+\overline{n+1}.12.\overline{D-1}}-6}{n+1}$, D being $= \overline{\frac{m}{an}}\big|^{\frac{2}{n-1}}$.

EXAMPLE I. Let $a$ the annuity be 10, $r$ the interest .045, and $n$ the time, twenty years, then will $p$ be $=$ $\frac{10\times\overline{1.045}|^{20}-1}{.045\times\overline{1.045}|^{20}} = \frac{10\times1.41171}{.045\times2.41171} = 130.079$. If $p$, $n$, and $r$ be 130.079, 20, and .045 respectively, $a$ will be $=$ $\frac{130.079\times.045\times\overline{1.045}|^{20}}{\overline{1.045}|^{20}-1} = \frac{14.117}{1.4117} = 10$. If $a$, $p$, and $r$ be 10, 130.079, and .045 respectively, $n$ will be $=$ Log. $\frac{10}{10-.045\times130.079} \div \text{Log. } 1.045 = \frac{.3823260}{.0191163} = 20$. And if $a$, $p$, and $n$ respectively be 10, 130.079, and 20, $\overline{\frac{an}{p}}\big|^{\frac{2}{n+1}}$ will be $= \overline{\frac{200}{130.079}}\big|^{\frac{2}{21}} = \overline{1.5375}\big|^{\frac{2}{21}} = 1.04181$, and $r$ will be $= \frac{6-\sqrt{36-12\times19\times.04181}}{19} = \frac{6-\sqrt{36-9.5327}}{19} =$ $\frac{6-\sqrt{26.4673}}{19} = \frac{6-5.1446}{19} = .045$.

EXAMPLE II. Let $a$ the annuity be $= 10$, $r$ the interest $= .035$, and $n$ the time be 30 years, $m$ the amount will then be $\frac{10 \times \overline{1.035}|^{30} - 1}{.035} = \frac{10 \times 1.80678}{.035} = 516.227$. Let $n$, $m$, and $r$ respectively be 30, 516.227, and .035, and $a$ will be $= \frac{.035 \times 516.227}{\overline{1.035}|^{30} - 1} = \frac{18.0678}{1.80678} = 10$. If $a$, $m$, and $r$ be 10, 516.227, and .035, $\frac{mr+a}{a}$ will be 2.80678, and $n$ $=$ Log. 2.80678 $\div$ Log. 1.035 $= \frac{.4482090}{.0149403} = 30$. Lastly, if $a$, $m$, $n$, be respectively $= 10$, 516,227, and 30; $\overline{\frac{m}{an}}\Big|^{\frac{2}{n-1}}$ will be $= \overline{\frac{516.227}{300}}\Big|^{\frac{2}{29}} = \overline{1.720756}\Big|^{\frac{2}{29}} = 1.03814$, and $r = \frac{\sqrt{36 + 12 \times 31 \times .03814} - 6}{31} = \frac{\sqrt{50.188} - 6}{31} = \frac{7.0843 - 6}{31} = .035$. *

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* It should be observed, that although $r$ has been taken for the interest of £1 for a year in these examples, the rules for determining the values of $a$, $p$, $m$, or $n$, would have answered equally well if $r$ had been taken for any fraction of a year, and $n$ had been taken for the number of *payments* instead of the number of *years*: thus, if the payments instead of being made yearly had been supposed in these examples to be made quarterly, $r$ in the former would have been $= \frac{.045}{4}$, and in the latter $= \frac{.035}{4}$; and $n$, instead of being $= 20$ and 30, would have been $= 80$ and 120: that is, $p$ in the first example would have been $= \frac{10 \times \overline{1.01125}|^{80} - 1}{.045 \times \overline{1.01125}|^{80}} = 131.418$, and $m$ in the second example $= \frac{10 \times \overline{1.00875}|^{120} - 1}{.035} = 527.048$; and $n$ in the former would have been $=$ Log. of $\frac{10}{10 - .045 \times 131.418} \div$ Log. of $1.01125 = 80$, and in the latter $=$ Log. of $\frac{.035 \times 527.048 + 10}{10} \div$ Log. of 1.00875 $= \frac{.454032}{.0037836} = 120$.

In most books on the subject of annuities certain, and compound interest, it has been usual to combine the five quantities $a$, $m$, $n$, $p$, and $r$, and to give theorems for determining the value of either of them when the other four are known. But it seldom or ever happens that $m$ the amount, and $p$ the value of the annuity, are both known in the same case, and therefore it will not be necessary to give those theorems here.

As the following problems have occurred in practice, and I am not aware that solutions of them have been given in any book on the subject, it may not be improper to insert them in this place, although not immediately connected with the object and design of this work.

PROBLEM I. To find the value of an annuity for $n$ years, supposing the first payment to be $a$, the 2d $\overline{a-d}$, the 3d $\overline{a-2d}$, - - - the $n$th $\overline{a-\overline{n-1}d}$.

SOLUTION. The series expressing the value of this annuity, supposing $r$ to be £1 increased by its interest for a year, will be $\frac{a}{r}+\frac{\overline{a-d}}{r^2}+\frac{\overline{a-2d}}{r^3}$ - - - $+\frac{\overline{a-\overline{n-1}.d}}{r^n}$. Let A be the value of £1 *per annum* for $n$ years, and this series will be found $= a\times A+\frac{\overline{n-1}.d}{r^n.\overline{r-1}}-\frac{\overline{r^{n-1}-1}.d}{r^{n-1}.\overline{r-1}|^2}$.

EXAMPLE. Let the annuity be £40 in the first year, decreasing £1 4s. in the 2d and subsequent years for a term of 14 years, and let $r$ be $= 1.05$. Then will the above expression be $40\times 9.8986+\frac{13\times 1.2}{1.9799\times .05}-\frac{.88565\times 1.2}{1.8856c\times .0025}=395.944+157.58-225.45=$ £328.074.

PROBLEM II. To determine the sum to which an annuity will accumulate at the end of $n$ years, supposing

such annuity to be received *yearly*, and its interest to be improved *half-yearly*.

SOLUTION. Let $a$ be the given annuity, and $r$ the interest of £1 for a year; then will the sum to which the first payment of the annuity at the end of the first year, will accumulate, together with its interest, improved *half-yearly* during $n-1$. years, be $= a.\ \overline{1+\frac{r}{2}}\Big|^{2n-2}$; the sum to which the second payment will accnmulate in $n-2$. years, be $a.\ \overline{1+\frac{r}{2}}\Big|^{2n-4}$; the sum to which the third payment will accumulate in $n-3$ years be $= a.\ \overline{1+\frac{r}{2}}\Big|^{2n-6}$; and so on. The whole sum therefore will be expressed by the series $a\left(\overline{1+\frac{r}{2}}\Big|^{2} + \overline{1+\frac{r}{2}}\Big|^{4} + \overline{1+\frac{r}{2}}\Big|^{6} \cdots\cdots + \overline{1+\frac{r}{2}}\Big|^{2n-2}\right) = a \times$

$$\frac{\overline{1+\frac{r}{2}}\Big|^{2n} - 1}{\overline{1+\frac{r}{2}}\Big|^{2} - 1}.$$

EXAMPLE. Let $a$ be $= 500$, $r = .05$, and $n = 30$. The above expression will then be $= 500 \times \frac{\overline{1.025}|^{60} - 1}{\overline{1.025}|^{2} - 1}$ $= 500 \times 67.1567 = 33{,}578.350$. Were the above sum improved annually, it would amount to 33,219.4. And were the annuity received and improved half-yearly, it would amount to 33,998.

PROBLEM III. To determine the value of £1 *per annum* for $n$ years after $n$ years, of £2 *per annum* for $n$ years after $2n$ years, of £3 *per annum* for $n$ years after $3n$ years, and so on increasing after the expiration of each $n$ years for ever.

SOLUTION. Supposing in this case $r$ to be £1 increased by its interest for a year, and the value of the annuity will

be $= \frac{1}{r^{n+1}} + \frac{1}{r^{n+2}} + \frac{1}{r^{n+3}} + \&c. --- \left(\frac{1}{r^{2n}}\right) --- + \frac{2}{r^{2n+1}} + \frac{2}{r^{2n+2}} + \frac{2}{r^{2n+3}} + \left(\frac{2}{r^{3n}}\right) --- + \frac{3}{r^{3n+1}} + \frac{3}{r^{3n+2}} + \frac{3}{r^{3n+3}} + --- \left(\frac{3}{r^{4n}}\right) ---$ &c. &c. &c. The sums of these several series are $\frac{1}{r^{n}.\,\overline{r-1}} - \frac{1}{r^{2n} \times \overline{r-1}} --- \frac{2}{r^{2n}.\,\overline{r-1}} - \frac{2}{r^{3n}.\,\overline{r-1}} --- \frac{3}{r^{3n}.\,\overline{r-1}} - \frac{3}{r^{4n}.\,\overline{r-1}} ---$ &c. &c. respectively. The whole value therefore will be expressed by the two series $\frac{1}{\overline{r-1}}\left(\frac{1}{r^{n}} + \frac{2}{r^{2n}} + \frac{3}{r^{3n}} + \&c.\right) - \frac{1}{\overline{r-1}}\left(\frac{1}{r^{2n}} + \frac{2}{r^{3n}} + \frac{3}{r^{4n}} + \frac{4}{r^{5n}} + \&c.\right) = \frac{1}{\overline{r-1}.\,\overline{r^{n}-1}}$.

Corollary. If $n$ be $= 1$, the value of the above annuity will become $= \frac{1}{\overline{r-1}|^{2}}$ which is known to be the true value from other principles.

Example. Supposing A to hold a lease for 21 years, on condition that at the end of that term the rent should be increased 5 guineas for the next 21 years, 10 guineas for the following 21 years, and so increasing 5 guineas *per annum* every 21 years for ever; what is the value of such increasing rent at £5 *per cent.* *Answer.* In this case $n$ is $= 21$, and $r = 1.05$; the value therefore will be $= 5.25 \times \frac{1}{.05 \times \overline{\overline{1.05}|^{21} - 1}} = 5.25 \times 11.2 = 58.8$.

Problem IV. To determine the sum which should be paid for any given annuity for $n$ years, so as to secure the purchaser the return of his capital at the expiration of the term, supposing him to have the means of reproducing that capital at $\varrho$ *per cent.*, and that the value of the annuity is computed at $r$ *per cent.*

Solution. Let $a$ be the given annuity, and $x$ the capital to be reproduced at the end of $n$ years, or the sum

which should be paid for the annuity on the above conditions. Since $\frac{\overline{1+\varrho}|^{n}-1}{\varrho}$ is the amount of £1 *per annum* at $\varrho$ interest in $n$ years, $\overline{a-rx} \times \frac{\overline{1+\varrho}|^{n}-1}{\varrho}$ will be equal to $x$; from which equation $x$ is easily found $= \frac{a \times \overline{1+\varrho}|^{n}-1}{\varrho + \overline{\overline{1+\varrho}|^{n}-1} \cdot r}$.

EXAMPLE. *A* purchases an annuity of £65 for 10 years, and is to be allowed £9 *per cent.* in the purchase, but being unable to improve the difference between £65 and the interest at £9 *per cent.* on the capital at no higher rate than £3 *per cent.*; it is proposed to make him such allowance in the purchase money as shall enable him to replace his capital at the end of the term by improving it at this reduced interest. In this case $\varrho$ is $= .03$, $r = .09$, $n = 10$, and $a = 65$; and $x$ will therefore be $=$ $\frac{65 \times \overline{1.03}|^{10}-1}{.03 + \overline{\overline{1.03}|^{10}-1} \times .09} = \frac{65 \times .344}{.03 + .344 \times .09} = 366.710$. In other words; $366.710 \times .09 = 33.004$, and $65 - 33.004$ ($= 31.096$) multiplied into 11.464 the amount of £1 *per annum* in 10 years, by Tab. 21, gives 366.710.

COROLLARY. When $\varrho = r$, the above expression becomes $\frac{\overline{1+r}|^{n}-1}{r \cdot \overline{1+r}|^{n}}$ which is known to express the amount of £1 *per annum* in $n$ years.

## OBSERVATION.

The values of annuities on two joint lives in the 4th Table are given only for every five years difference of age, and in the 14th Table for every six years difference. But the values for the intermediate ages may be found with sufficient accuracy by the method of interpolation. Thus, supposing it were required to find from the 4th Table the

value at 5 *per cent.* of two joint lives aged 44 and 52; we shall have 7.875 for the value of two joint lives aged 52 and 42, and 7.582 for the value of two joint lives aged 52 and 47. If the difference, or .292 be divided by 5, the quotient or .0586, will be the 5th mean between these two values. Deducting .117, or two of these means, from 7.875, the remainder, or 7.758 will be the value required. In like manner, if it were required to determine the value at 4 *per cent.* by the 14th Table of an annuity on two joint lives age 63 and 54, the value of two joint lives aged 63 and 51 is 6.505, the value of two joint aged 63 and 57 is 6,045, and their difference, or .460, being divided by 6, gives .0767 for the 6th mean between those two values, Three of these means, or .230, being subtracted from 6.505, will leave 6.275 for the value of the two joint lives aged 63 and 54 years.

# POSTSCRIPT.

THE following *nosological* table, though it may not have an immediate connection with the doctrine of life annuities and reversions, or perhaps lead to any certain conclusions respecting the proportion in which different disorders prevail among the human species, cannot, however, be considered as altogether unimportant, or forming an improper addition to this work.

It should be observed, that the diseases of which the members of the Equitable Society are stated to have died, are all certified by gentlemen of the medical profession, and therefore that the present table is so far correct. It contains also an account of all the deaths that have happened in the Society during the course of the last 20 years, among a population exceeding 150,000 persons; so that it is not improbable but that, as to the diseases of which they have died, it gives results not very different from those which take place in general among the inhabitants of this country.

The number of deaths appears to have been much less than they should have been by the *Northampton*, or indeed by any other table of observations, with which I am acquainted. How far this circumstance may have had any effect in changing the proportions of the diseases to each other, I must leave for the investigation of the medical profession, as this may possibly be a question of greater difficulty than it is to ascertain the effect which it has had on the *funds* of this Society.

| Disease. | 10 to 20. | 20 to 30. | 30 to 40. | 40 to 50. | 50 to 60. | 60 to 70. | 70 to 80. | 60, &c. | Total. |
|---|---|---|---|---|---|---|---|---|---|
| Angina Pectoris - | — | — | 5 | 11 | 12 | 9 | 4 | 3 | 44 |
| Apoplexy - - | 1 | 3 | 19 | 38 | 69 | 69 | 38 | 5 | 242 |
| Asthma - - | — | — | — | 2 | 19 | 19 | 11 | 2 | 53 |
| Atrophy - - | — | — | 3 | 4 | 6 | 11 | 1 | — | 25 |
| Cancer - - | — | — | 1 | 4 | 10 | 8 | 1 | 1 | 25 |
| Child Birth - - | — | — | 2 | 2 | — | — | — | — | 4 |
| Consumption - | 2 | 9 | 34 | 31 | 44 | 28 | 5 | — | 153 |
| Convulsion Fits - | — | — | 3 | 4 | 1 | 3 | — | — | 11 |
| Decay (Natural) and Old Age | — | — | — | — | 5 | 72 | 127 | 58 | 262 |
| Diabetes - - | — | — | — | 2 | 2 | — | 1 | 1 | 6 |
| Dropsy - - | 1 | — | 7 | 28 | 38 | 41 | 20 | 2 | 137 |
| Dropsy in the Chest | — | 1 | 3 | 18 | 34 | 28 | 16 | — | 100 |
| Dysentery - - | — | — | 1 | 1 | 2 | 4 | 4 | — | 12 |
| Disease of the Stomach and Digestive Organs | — | — | 5 | 4 | 8 | 8 | 1 | — | 26 |
| Diseased Liver - | — | 2 | 5 | 24 | 23 | 21 | 4 | — | 79 |
| Disease of the Bladder and Urinary Passages - | — | — | 2 | 4 | 15 | 23 | 15 | — | 59 |
| Epilepsy - - | — | — | 2 | 3 | 2 | 1 | 2 | — | 10 |
| Erysipelas - - | — | 1 | — | 2 | 3 | 2 | 2 | — | 10 |
| Fevers, General - | — | 6 | 18 | 33 | 33 | 39 | 15 | 2 | 146 |
| —— Bilious - | — | 1 | 4 | 8 | 9 | 4 | 1 | 1 | 28 |

## TABLE—*continued*.

| Disease. | 10 to 20. | 20 to 30. | 30 to 40. | 40 to 50. | 50 to 60. | 60 to 70. | 70 to 80. | 80, &c. | Total. |
|---|---|---|---|---|---|---|---|---|---|
| Fevers, Nervous - | — | 3 | 3 | 13 | 6 | 8 | 3 | — | 36 |
| —— Inflammatory | — | — | — | 4 | 6 | 3 | 2 | — | 15 |
| —— Putrid - | — | 2 | 7 | 4 | 6 | 7 | — | — | 26 |
| Gout - - - | — | — | 1 | 4 | 4 | 11 | 6 | — | 26 |
| Inflammation of the Bowels - | 1 | 2 | 11 | 13 | 15 | 25 | 9 | 1 | 77 |
| Inflammation of the Lungs - | — | — | 9 | 4 | 24 | 22 | 12 | 2 | 73 |
| Inflammation of the Brain - - | — | 3 | 7 | 5 | 5 | 3 | — | — | 23 |
| Inflammation of the Chest, and Peripneumony | 1 | 1 | 1 | 1 | 6 | 7 | 4 | 1 | 22 |
| Palsy - - | — | 1 | 3 | 8 | 26 | 42 | 34 | 2 | 116 |
| Quincy - - | — | — | — | 1 | 1 | 1 | — | — | 3 |
| Rupture of a Blood Vessel - - | — | — | 7 | 14 | 13 | 12 | 3 | — | 49 |
| Slain in War - | 1 | 1 | 1 | 1 | — | — | — | — | 4 |
| Stone - - | — | — | — | — | 1 | 2 | 4 | 1 | 8 |
| Suicide - - | — | 1 | 2 | 3 | 7 | 2 | — | — | 15 |
| Water on the Brain | — | — | — | 1 | 3 | 1 | — | — | 5 |
| | 7 | 37 | 166 | 299 | 458 | 536 | (345 - - 82) 427 | | 1,930 |
| Number assured during the last 20 years - | 1,494 | 8,996 | 33,850 | 45,429 | 36,489 | 19,042 | 6454 from 70, &c. | | 151,754 |